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AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
PAPERS

WIND STRESS ANALYSIS BY THE *K*-PERCENTAGE METHOD

By F. P. Witmer,[1] M. Am. Soc. C. E.

Synopsis

The purpose in preparing this paper was twofold:

(1) To describe in detail a procedure for applying the K-percentage method of wind stress analysis which was presented[2] in 1939, in order to facilitate its practical utilization; and

(2) To present a method of design, using the aforementioned principles, but with the assumption that vertical wind reactions and direct wind stresses in columns must be in accordance with the cantilever relation, thus eliminating all secondary moments due to change in length of columns under their direct wind stresses.

Present Methods of Design

In the analysis and design of a building bent of the usual type, having girders framed between columns, with small connections designed to resist pure bending, the engineer is confronted at the outset with two alternative methods of procedure, for the determination of wind stresses:

(1) An "approximate" method, in which the distribution of vertical wind shear across the bent is assumed to occur in some arbitrarily assigned relation, without consideration of the elastic behavior of the various members of the bent; and

(2) One of the so-called "exact" methods, in which the elastic action is considered in a more or less accurate theoretical manner.

The approximate methods are much more readily and quickly performed, and, for buildings of usual proportions, are quite satisfactory, both practically and economically, although their resultant stresses may be considerably in error. However, they ignore the secondary bending moments resulting from

Note.—Written comments are invited for immediate publication; to insure publication the last discussion should be submitted by **October 15, 1941.**

[1] Director, Civ. Eng., Univ. of Pennsylvania, Philadelphia, Pa.

[2] "Sixth Progress Report of Sub-Committee No. 31, Committee on Steel of the Structural Division, on Wind Bracing in Steel Buildings," *Proceedings*, Am. Soc. C. E., June, 1939, p. 975.

change in length of columns under direct wind stresses and, for extreme proportions of height to width of bent, these moments may attain serious magnitude.

The exact methods give correct values for primary wind stresses, but, as usually applied, they also ignore these secondary moments. In one case, however, that of the "Spurr method,"[3] this procedure is entirely justified, because, since the bent is assumed to act as a cantilever fixed at the base, and is carefully designed elastically upon this premise, all joints in each floor will remain in a plane under wind load, and these secondary moments, therefore, will be non-existent.

By any method other than the Spurr method, it is thus seen that these effects of column axial wind distortion are habitually neglected, unless a separate additional analysis of them is made by some more or less laborious process.

The principal approximate methods in use have been two—the "Portal Method" and the "Cantilever Method"—both described[4] by Robins Fleming in 1913. The portal method assumes that vertical wind reactions and direct wind stresses for all interior columns are equal to zero. The cantilever method assumes the direct-wind unit stresses in columns to be proportional to the distance of the column from the neutral axis of the bent. Neither method takes into account the fact that these particular relations of reactions and stresses cannot exist unless the relative sizes of columns and girders are such as to develop them by elastic action. The reactions assumed in the portal method are theoretically correct for primary wind moments if all sizes have been computed from vertical floor loads alone, assuming discontinuous girders, and also assuming: That the floor load is constant throughout any floor; that girders in any floor are of the same depth throughout; and that column areas are always proportional to their moments of inertia.[5] If these assumptions are not made, the portal method is only approximately correct, the degree of accuracy depending upon the degree of departure from these assumptions. The cantilever method can only give correct results if the relation of sizes happens to be such as to develop the assumed reactions and direct column stresses when the bent distorts elastically. This condition will obviously be only of accidental occurrence.

Of the so-called "exact" methods, the "slope deflection method," introduced in 1915 by W. M. Wilson and G. A. Maney,[6] Members, Am. Soc. C. E., and the method of end-moment distribution, presented by Hardy Cross,[7] M. Am. Soc. C. E., are the most outstanding. Each will produce a correct analysis of primary wind moments for a bent whose members are of any assumed size. Both are laborious, that of Professor Cross being far more practical in the case of bents many stories high. Neither method, as before stated, takes account of secondary bendings from column, axial, wind distortion, without an additional analysis made for this purpose.

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[3] "Wind Bracing," by Henry V. Spurr, M. Am. Soc. C. E., McGraw-Hill Book Co., Inc.

[4] *Engineering News-Record*, March 13, 1913.

[5] *Proceedings*, Am. Soc. C. E., November, 1936, p. 1496.

[6] *Bulletin No. 80*, Univ. of Illinois Experiment Station, Urbana, Ill., 1915.

[7] "Analysis of Continuous Frames by Distributing Fixed-End Moments," by Hardy Cross, *Transactions*, Am. Soc. C. E., Vol. 96 (1932), p. 1.

Proposed Method of Design

If it is considered necessary that these secondary moments be not neglected, the only method heretofore available is the Spurr method which is also very

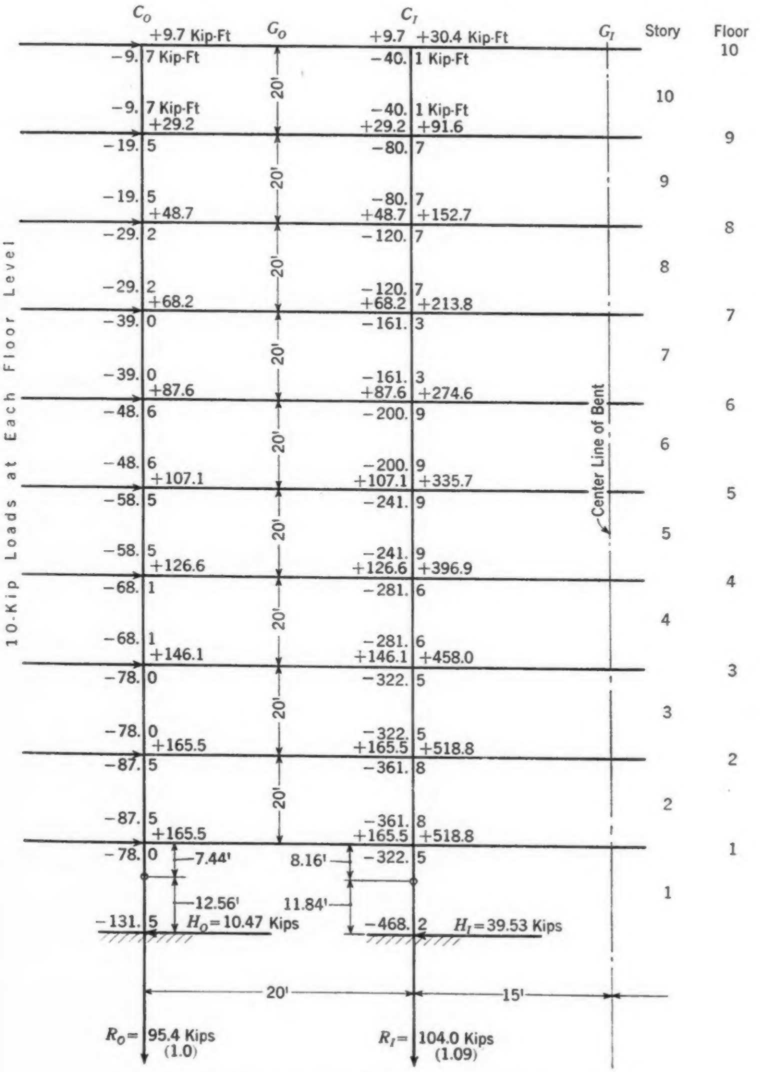

Fig. 1.—Wind Moments, in Kip-Feet (Cantilever Reactions)

time-consuming. In the present paper a new method is presented which is both rapid and convenient, at the same time insuring practical agreement with the cantilever relation of column, vertical, wind reactions which is necessary to

justify ignoring the secondary wind effects. The proposed method is based upon the "method of K-percentages" first reported in 1939,[2] and particularly utilizes the relation[8] that ΣI varies as $V_g L_g^2$, in which ΣI = approximate sum of moments of inertia of all girders in a bay; V_g = relative vertical wind shear in the bay as determined from relative wind reactions required by the cantilever theory; and L_g = length of girder in the bay.

A bent of assumed dimensions, carrying a specified floor load and wind load, was designed by the proposed method. For this purpose, a bent of three bays was assumed, the outer ones each being 20 ft wide and the inner one 30 ft wide (see Fig. 1). Ten equal stories of 20-ft height were assumed and bents were assumed to be 15 ft apart. Floor loads (inclusive of all dead load) were assumed equal to 200 lb per sq ft for all floors and a horizontal wind force of $33\frac{1}{3}$ lb per sq ft was used. Assuming a parapet 10 ft high produced a uniform wind force of 10 kips at each floor (1 kip equals 1 "kilo-pound," or 1,000 lb), which was adopted for convenience in calculations.

A unit fiber stress of 18 kips per sq in. was used for girders and 12 kips per sq in. in compression for all columns, regardless of their slenderness ratios. At the ends of columns, however, where no buckling need be considered, 18 kips per sq in. were allowed for compression, inclusive of wind bending stress. One third of the vertical load stress was deducted when wind stress was included, for both girders and columns; and for girders, the end moment for vertical loads was taken at two thirds that at the center of a simple span of the same length.

The first step is to compute all girder moments and column stresses from the vertical floor load of 200 lb per sq ft, assuming girders to be non-continuous. The tentative column areas of first-story columns for this stress are as follows —

$$\text{Column } C_O\text{: } 300 \div 12 = 25.0 \text{ sq in. required}$$

$$\text{Use 12-in., 85-lb H} = 24.98 \text{ in.}^2$$

$$\text{Column } C_I\text{: } 750 \div 12 = 62.5 \text{ sq in. required}$$

$$\text{Use 14-in., 215-lb H} = 63.21 \text{ sq in.}$$

The maximum moments, in kip-inches, at the centers of all girders are as follows:

Location	Girders G_O	Girders G_I
At center	1,800	4,050
At ends	1,200	2,700

The column stresses due to vertical loads are given in Table 1.

TABLE 1.—COLUMN STRESSES, IN KIPS PER SQUARE INCH, DUE TO VERTICAL LOADS

Column (see Fig. 1)	Floor No.:									
	1	2	3	4	5	6	7	8	9	
C_O	300	270	240	210	180	150	120	90	60	30
C_I	750	675	600	525	450	375	300	225	150	75

[8] *Proceedings*, Am. Soc. C. E., June, 1939, p. 977.

Assume that the unit stresses in columns are proportional to their distance from the neutral axis of the bent, which, in this case, is at the center. Then, if A_O and A_I are the tentative areas of the outer and inner first-story columns respectively, and the vertical wind reaction R_O at the outer column is taken at unity, the inner-column wind reaction will be (see Fig. 1): $R_I = \frac{15}{35} \times \frac{A_I}{A_O} = \frac{15}{35} \times \frac{63.21}{24.98} = 1.09$. If $R_O = 1.0$, this also equals the vertical wind shear V_O in the outer bay. Then the vertical wind shear in the inner bay will be $V_I = R_O + R_I = 1.0 + 1.09 = 2.09$.

Hence, if I_O equals the moment of inertia of the outer girder in any floor, that of inner girder in the same floor will be $I_I = I_O \times 2.09 \times \frac{30^2}{20^2} = 4.70$.

Thus the moment of inertia of any inner girder should be equal to 4.70 times that of outer girder in the same floor in order to produce the cantilever relation of vertical wind reactions. This ratio will be subject to verification and possible change after the actual design of girders and columns for the inclusion of wind stress.

The next step is to compute wind moments in the girders, assuming vertical, column, wind, reactions and direct stresses in the various stories to be in the proportion of 1.0 and 1.09 for outer and inner columns respectively, and assuming all points of contraflexure at the mid-point of the member except first-story columns. For the latter it is better to assume the point of contraflexure at sixth tenths of the story height above the bottom. Each of the outer girder moments is taken as equal to the product of one half the girder length center to center of columns by the shear received from the column at its outer end. This shear equals the difference between outer column direct wind stresses above and below the floor in question.

The direct wind stress in the outer column of any story will be found by the following moment equation:

$$M = 70\, C_O + (1.09 \times 30)\, C_O = 102.7\, C_O \qquad (1)$$

from which

$$C_O = \frac{M}{102.7} \qquad (2)$$

In Eqs. 1 and 2 M equals the wind moment about the point of contraflexure of the column in the story in question and C_O is the stress in the outer column of this story.

It is not actually necessary to compute these wind moments.[9] The horizontal wind shears in all stories are always readily available and the difference between M_9 and M_{10}, for instance, in stories 9 and 10, will be

$$M_{10} - M_9 = \frac{H_9\, h_9}{2} + \frac{H_{10}\, h_{10}}{2} \qquad (3)$$

in which H_9 and H_{10} are the story, horizontal, wind shears, and h_9 and h_{10} are

[9] *Proceedings*, Am. Soc. C. E., June, 1939, p. 980.

the story heights. This relation is easily shown to be true. Hence, the wind moment in the outer girder of floor 9 (between stories 9 and 10) is equal to $\frac{1}{102.7}\left(\frac{H_9\,h_9}{2} + \frac{H_{10}\,h_{10}}{2}\right) \times 10$. For the top girder, this moment becomes $\frac{1}{102.7}\left(\frac{H_{10}\,h_{10}}{2}\right) \times 10 = \frac{1}{102.7}(10 \times 10) \times 10 = +9.7$ kip-ft. For the ninth floor girder, it is $\frac{1}{102.7}(10 \times 10 + 20 \times 10) \times 10 + 29.2$ kip-ft and similarly for the other floors above the first floor. For the first story, the point of contraflexure being at six tenths of the story height above the base, or four tenths of this height below the floor, $M_1 - M_2 = \frac{H_2\,h_2}{2} + H_1 \times 8$, from which the moment in the girder is $\frac{1}{102.7}\left(\frac{H_2\,h_2}{2} + H_1 \times 8\right) \times 10$ $= \frac{1}{102.7}\left(90 \times \frac{20}{2} + 100 \times 8\right) \times 10 = +165.5$ kip-ft.

For inner girders, the moments will be equal to those for the corresponding outer girders, multiplied by $\frac{V_I}{V_O} \times \frac{15}{10}$, in which V_I and V_O are, respectively, the vertical wind shears in the inner and outer bays. As found before, if $V_O = 1.0$, $V_I = 2.09$. Hence, if M_O and M_I are moments in outer and inner girders, respectively, $M_I = M_O \times \frac{2.09}{1.0} \times \frac{15}{10}$. All inner girder moments are thus quickly found.

By the condition that $\Sigma M = 0$ at each joint, and working from the top of bent downward, column moments are all readily determined from the girder moments, except those at the bottom of the first story. Thus (in kip-ft):

Story		C_O
10		-9.7
9		$-(29.2 - 9.7) = -19.5$
: : :	etc.	: : :

For the bottom of the first-story columns, the total wind moment in the first story (that is, 100 × 20 ft = 2,000 kip-ft) is assumed divided among the four columns in proportion to their K-values, or, as all have the same length, in proportion to their moments of inertia. The tentative areas of C_O and C_I are, respectively, 24.98 sq in. and 63.21 sq in. Assuming these columns to be tentatively 12-in., 85-lb H ($I = 723.3$) and 14-in., 215-lb H ($I = 2{,}730.0$) these columns will have total moments of $\frac{2{,}000}{2 \times 3{,}453.3} \times 723.3 = 209.5$ kip-ft and $\frac{2{,}000}{2 \times 3{,}453.3} \times 2{,}730.0 = 790.7$ kip-ft, respectively. Deducting from these moments the moments previously found at the top of columns in first story, the base moments given in Fig. 1 are determined, this figure also giving the wind moments throughout, computed as described herein. Wind reactions in Fig. 1 are found as follows:

By taking moments about the contraflexure point in the first story, assumed to be 8 ft below the first floor,

$$R_O = \frac{(10 \times 10)\,(20 \text{ ft} \times 4.5 + 8 \text{ ft})}{102.7} = 95.42 \text{ kips}$$

and

$$R_I = R_O \times 1.09 = 104.0 \text{ kips}.$$

The total moment in the outer column of the first story was found to be 209.5 kip-ft. Hence, $H_O = \frac{209.5}{20} = 10.47$ kips.

$$\text{Similarly, } H_I = \frac{790.7}{20} = 39.53 \text{ kips}.$$

The corresponding location of points of contraflexure will be at distances above the base of $\frac{131{,}500}{10{,}470} = 12.56$ ft and $\frac{468{,}200}{39{,}530} = 11.84$ ft respectively, instead of sixth tenths of 20 ft = 12 ft as was assumed originally in computing wind moments.

The next step is to proportion columns and outer girders for vertical loads, or for vertical loads combined with wind stresses, whichever will require the greater section. Inner girders are then to be proportioned by making the moment of inertia of any girder, as nearly as practicable, equal to 4.70 times that of the outer girder in the same floor, unless its stress from vertical and wind loads requires a larger section. In the latter case the corresponding outer girder must be increased until its moment of inertia is equal to $\frac{1}{4.70}$ times that

TABLE 2.—Sizes of H-Beams After Proportioning for Wind and Vertical Loads

Floor or story	Columns		Girders	
	Outer (C_O)	Inner (C_I)	Outer (G_O)	Inner (G_I)
10	12 in., 32 lb	14 in., 58 lb	21 in., 59 lb	33 in., 125 lb
9	12 in., 32 lb	14 in., 58 lb	21 in., 59 lb	33 in., 125 lb
8	12 in., 45 lb	14 in., 111 lb	21 in., 59 lb	33 in., 125 lb
7	12 in., 45 lb	14 in., 111 lb	21 in., 59 lb	33 in., 125 lb
6	12 in., 58 lb	14 in., 158 lb	21 in., 59 lb	33 in., 125 lb
5	12 in., 58 lb	14 in., 158 lb	21 in., 59 lb	33 in., 125 lb
4	12 in., 79 lb	14 in., 211 lb	21 in., 68 lb	33 in., 132 lb
3	12 in., 79 lb	14 in., 211 lb	24 in., 74 lb	36 in., 160 lb
2	12 in., 85 lb	14 in., 237 lb	24 in., 74 lb	36 in., 160 lb
1	12 in., 85 lb[a]	14 in., 237 lb[b]	24 in., 74 lb	36 in., 160 lb

[a] With two plates, 9 in. by $\frac{3}{8}$ in. by 3 ft at bottom. [b] With two plates, 11 in. by $\frac{11}{16}$ in. by 4.0 ft at bottom.

of the inner girder. In other words, the relation $I_I = 4.70\, I_O$ must be maintained in order to produce the assumed cantilever relation of wind reactions, while, at the same time, no girder or column may be of less section than is required by its own maximum stresses.

In proportioning columns, the ratio of the moments of inertia in inner and outer columns should be maintained as nearly constant as practicable for the same reason as was stated in the case of girders. This condition, however, is not nearly so essential as is the proper relation of girder moments of inertia. The effect upon the wind-reaction ratio of a change in the relative size of girders is much more pronounced than is the effect of a similar change in columns.

Table 2 shows sizes of all members after proportioning them for vertical loads and wind. This operation, in some detail, is as follows:

Design of Outer Girders G_O, First Floor.—

Maximum moment at center = 1,800 kip-in. at 18 kips per sq in. = 100 (minimum section modulus required)

Maximum moment at end	= 1,200 kip-in.
Less one third	= 400 kip-in.
Net moment from vertical load	= 800 kip-in.
Wind moment 165.5 kip-ft × 12 in.	= 1,986 kip-in.
Total end moment	= 2,786 kip-in.; at 18 kips per sq in. = 154.8 section modulus required

Use 24-in. 74-lb H (170.4 section modulus; I = 2,033.8)

A similar procedure is followed for each floor to determine the sizes shown in Table 2.

Design of Inner Girders G_I, First Floor.—

Maximum moment at center = 4,050 kip-in., at 18 kips per sq in. = 225 (minimum section modulus required)

Maximum moment at end	= 2,700 kip-in.
Less one third	= 900 kip-in.
Net moment from vertical load	= 1,800 kip-in.
Wind moment 518.8 kip-ft × 12 in.	= 6,225 kip-in.
Total end moment	= 8,025 kip-in.; at 18 kips per sq in. = 445.8 section modulus required

A 33-in. 141-lb H (446.8 section modulus; I = 7,442.2) is sufficient for stress. However, the designer must consider the relative moments of inertia of girders G_O and G_I:

$$I \text{ of } G_O = 2{,}033.8; \text{ hence, the required—}$$
$$I \text{ of } G_I = 2{,}033.8 \times 4.70 = 9{,}560 \text{ in.}^4$$

Therefore, for G_I, use a 36-in. 160-lb H (I = 9,738.8).

In a similar manner sizes for all inner girders are found, as given in Table 2.

Design of Outer Columns C_O, First Story.—

Vertical load	= 300 kips, at 12 kips per sq in.	= 25.0 sq in.
Less one third	= 100 kips	
Net vertical load	= 200 kips	

Wind Load (Top).—

$$78 \text{ kip-ft} \times 12 \text{ in.} \times \frac{c}{r^2} = 200 \ (c = 6.25 \text{ in. and } r = 5.38 \text{ in.})$$

Total at end = 400 kips, at 18 kips per sq in. = 22.2 sq in.
Use a 12-in. 85-lb H (24.98 sq in.; I = 723.3).

Wind Load (Bottom).—

$$131.5 \text{ kip-ft} \times 12 \text{ in.} \times \frac{c}{r^2} = 343 \text{ kips}$$

Net vertical load = 200 kips

Total at end = 543 kips, at 18 kips per sq in. = 30.2 sq in.
Use a 12-in. 85-lb H (24.98 sq in.), with two plates
9 in. by $\frac{3}{8}$ in. by 3.0 ft = 6.75 (at base)

Total at base = 31.73 sq in.

Design of Outer Columns C_O, Second Story.—

Vertical load	= 270 kips, at 12 kips per sq in.	= 22.5 sq in.
Less one third	= 90 kips	
Net vertical load	= 180 kips	

$$\text{Wind } 87.5 \text{ kip-ft} \times 12 \text{ in.} \times \frac{c}{r^2} = 227 \text{ kips } (c = 6.25 \text{ in. and } r = 5.38 \text{ in.})$$

Total at end = 407 kips, at 18 kips per sq in. = 22.6 sq in.
Use 12-in. 85-lb H (24.98 sq in.) for first and second stories.

Similarly sizes for other stories are obtained as given in Table 2.

Design of Inner Columns C_I, First Story.—

Vertical load	= 750 kips, at 12 kips per sq in.	= 62.5 sq in.
Less one third	= 250 kips	
Net vertical load	= 500 kips	

Wind Load (Top).—

$$322.5 \text{ kip-ft} \times 12 \text{ in.} \times \frac{c}{r^2} = 709 \text{ kips } (c = 8.06 \text{ in. and } r = 6.65 \text{ in.})$$

Total at end = 1,209 kips, at 18 kips = 67.2 sq in.
Use a 14-in. 237-lb H (69.69 sq in.; I = 3,080.9 in.^{4}).

Wind Load (Bottom).—

$468.2 \text{ kip-ft} \times 12 \text{ in.} \times \frac{c}{r^2} = 1{,}024$ kips

Net vertical load $= 500$ kips

Total at end $= 1{,}524$ kips, at 18 kips per sq in. $= 84.8$ sq in.

Use a 14-in. 237-lb **H** $= 69.69$ sq in.

Plus two plates, 11 by $\frac{11}{16}$ by 4.0 ft $= 15.14$ sq in. (at base)

Total at base $= 84.83$ sq in.

Second Story.—

Vertical load $=$ 675 kips, at 12 kips per sq in. $= 56.25$ sq in.

Less one third $=$ 225 kips

Net vertical load $=$ 450 kips

Wind $361.8 \text{ kip-ft} \times 12 \text{ in.} \times \frac{c}{r^2} =$ 793 kips

Total at end $= 1{,}243$ kips, at 18 kips per sq in. $= 69.1$ sq in.

Use 14-in. 237-lb **H** (69.69 sq in.) for the first and second stories.

Similarly, sizes are obtained for other stories, as given in Table 2.

All sizes having been determined, a check for the reaction ratio $\frac{R_I}{R_O}$ will now be made by the *K*-percentage method:

Stories	Moments of inertia, *I*, for: C_O	C_I
1st and 2d	723.3	3,080.9
3d and 4th	663.0	2,671.4
5th and 6th	476.1	1,900.6
7th and 8th	350.8	1,266.5
9th and 10th	246.8	597.9
	2,460.0	9,517.3

Average $I = 492.0 = 1{,}903.5$

Average $K = 24.6 = 95.17$

Ratio $\frac{C_I}{C_O} = 3.87$

Although this ratio differs from the ratio 2.50 upon which the original computations of column areas were made, the difference will probably have little effect upon resulting wind stresses, these being influenced mainly by the relation between girder moments of inertia in the different bays. This relation is found as follows:

Floor	Moments of inertia, I, for: G_O	G_I
1	2,033.8	9,738.8
2	2,033.8	9,738.8
3	2,033.8	9,738.8
4	1,478.3	6,856.8
5	1,246.8	6,354.7
6	1,246.8	6,354.7
7	1,246.8	6,354.7
8	1,246.8	6,354.7
9	1,246.8	6,354.7
10	1,246.8	6,354.7
	15,060.5	74,201.4

$$\text{Average } I = 1{,}506.05 = 7{,}420.14$$

$$\text{Average } K = 75.30 \qquad 247.30$$

$$\text{Ratio } \frac{G_I}{G_O} = \frac{7{,}420.14}{1{,}506.05} = 4.92$$

This value checks reasonably well with 4.70 which was previously determined and used in proportioning the inner girders.

Analysis of a "Compromise 3-Story Bent"

The computation of the compromise 3-story bent shown in Fig. 2 is as follows:

Determination of K-Percentages.—From the K-values at the joints:

75.30	99.90	75.30	
24.60	24.60	247.30	417.77
99.90)75.30	124.50)75.30	95.17	95.17
0.754	0.605	417.77)75.30	512.94)75.30
		0.181	0.147
		)247.30	)247.30
		0.592	0.482

Determination of Reaction Ratio R.—

0.181	0.754
0.147	0.605
0.147	0.605
	1.836

$$0.475 \times \frac{95.17}{24.60} = 3.800 \div 20 = 0.1900 = R_O = V_O$$

0.592
0.484
0.484

$$1.560 \times 2 \times \frac{95.17}{24.60} = 12.120 \div 30 = 0.4040 = V_I$$

$$V_I - V_O = 0.2140 = R_I$$

$$R = \frac{R_I}{R_O} = 1.12. \quad \text{Hence, if } R_O = 1.0,\ R_I = 1.12.$$

This checks closely with 1.09 in original calculations. The design is therefore consistent with the cantilever reaction relation.

Should a discrepancy be found great enough to necessitate further computation, the wind reactions for the new reaction ratio R may be found readily as

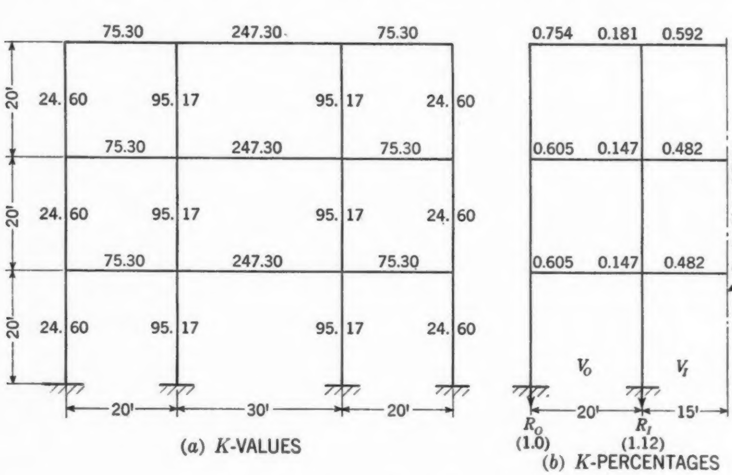

FIG. 2.—COMPROMISE THREE-STORY BENT

was done before, and the new wind moments in girders found by simple proportion from those in Fig. 1, the outer girder moments being obtained by multiplying the former moments by the ratio of the respective outer vertical wind shears V_O $(= R_O)$ and the inner girder moments by multiplying the former moments by the ratio of respective inner vertical wind shears V_I $(= R_O + R_I)$. Column moments result, as before, from the condition that $\Sigma M = 0$ at each joint, with special procedure in the first story.

The sizes of girders and columns may then be checked against these new moments, and changes made where required. Generally if the first design was made carefully to accord with the proper ratio of I's for inner and outer girders, found as described, such corrections will be very slight, even if required at all.

The proposed method of designing for cantilever behavior, and the K-percentage method for obtaining wind reaction ratio R for any design, may be applied to bents of any number and spacing of bays. The latter principle is particularly useful in the investigation of wind stresses in existing structures or in designs that have been made without due consideration of their elastic action.

As a final check on the accuracy of the foregoing wind-moment computation, a moment-distribution analysis[7] was made for the design. In Table 3, comparisons are made with the average of the Cross moments at the two ends of any member; in other words, the total moments in the member are compared. These show a very good agreement. All differences are of negligible proportions except in one bottom column (16%) and in certain members in the

ninth and tenth floors and stories. The latter show differences as high as 26%; but as the actual moments are very small the sizes used are ample without change. Such a result may generally be relied upon. Ordinarily, large percentage discrepancies in wind moments near the top of the bent are of but little practical importance in the design of members.

TABLE 3.—CHECK COMPUTATIONS OF MOMENTS FOR COMPARISON

(C = Cross Method; W = Witmer Method; L = left end; R = right end; D = percentage difference; T = top; B = bottom)

Floor or story	Outer Girder, G_O				Inner Girder, G_I			Outer Column, C_O				Inner Column, C_I			
	C		W	D	C	W	D	C		W	D	C		W	D
	L	R						T	B			T	B		
10	15.0	11.1	9.7	−26	28.5	30.4	+ 7	14.2	11.4	9.7	−24	38.4	35.9	40.1	+ 8
9	37.0	30.7	29.2	−14	81.8	91.6	+12	26.8	25.7	19.5	−26	74.8	71.8	80.7	+10
8	56.4	49.1	48.7	− 8	147.6	152.7	+ 3	29.8	28.3	29.2	+ 1	124.6	117.2	120.7	0
7	72.4	68.2	68.2	− 3	222.0	213.8	− 4	42.7	41.1	39.0	− 7	173.3	143.1	161.3	+ 2
6	88.0	83.2	87.6	+ 2	270.5	274.6	+ 2	47.3	45.4	48.6	+ 5	210.0	197.2	200.9	− 1
5	104.5	101.3	107.1	+ 4	340.6	335.9	− 1	59.1	59.1	58.5	− 1	245.8	235.9	241.9	0
4	126.3	125.4	126.6	+ 1	385.3	396.9	− 3	66.9	69.3	68.6	+ 1	275.9	287.8	281.6	0
3	147.0	147.9	146.1	− 1	460.7	458.0	− 1	78.5	77.3	78.0	0	321.5	322.6	322.5	0
2	164.7	165.3	165.5	0	521.5	518.8	− 1	87.8	85.8	87.5	+ 1	364.4	361.1	361.8	0
1	171.9	167.0	165.5	− 2	557.7	518.8	− 7	86.4	96.7	106.5	+16	364.2	452.6	393.5	− 4
$\frac{\Sigma M}{20}$ =	96.6		95.4	$\frac{\Sigma M}{30}$ =	201.4	199.4	....	....	....	....	....	....	....	....	....

The design was made using moments at the center of intersections for all girders and columns. If it is desired to use moments at the face of columns or at the top or bottom of girders, corresponding corrections may always be made, in proportioning the members. The most practicable way to do the original computation is to consider moments at the centers of intersections of members.

If it is found that the horizontal deflection of the building exceeds a proper assigned proportion of its height, a correction may be applied readily by making a proportional reduction in the allowed column unit stress, keeping the stiffness relation unchanged between outer and inner columns. A change in girder stiffness will have little effect upon deflection. The deflection can be computed from column wind moments at either inner or outer column, in the manner described elsewhere.[10]

In general, a design based on the assumption of a cantilever relation between vertical wind reactions may not be the most economical as to weight although it is to be recommended because it eliminates secondary wind bending moments due to axial distortion of columns under direct wind stresses.

For 3-bay bents, the loss in economy will be greater when the width of the inner bay is relatively greater. This is apparent since in such cases the inner girders G_I are longer; and the foregoing computations show that, in order to

[10] *Proceedings*, Am. Soc. C. E., February, 1932, p. 229.

produce the cantilever relation of wind reactions, the sections of these girders must be increased arbitrarily above the requirements for vertical loads alone.

General Applicability of K-Percentage Method

It should be remembered that the K-percentage method is not limited to the type of case considered herein—namely, one in which vertical wind reactions are made to accord with the cantilever relation—but is applicable to any sizes that may exist in the bent, and to a bent of any number and any relative width of bays. It is particularly useful in the investigation of wind stresses in an existing bent, for which the assumptions as to wind distribution which were made in its design are unknown.

If it is not considered necessary to maintain the cantilever relation of vertical wind reactions, the bent may first be designed throughout for vertical loads alone. Using the sizes thus obtained, wind reaction ratios are found by the K-percentage method and from these ratios wind moments are computed. The members are then re-designed to include these wind moments and finally a K-percentage check is made on wind-reaction ratios. These operations are similar to those described in the foregoing example. If the reaction ratios last found do not vary too greatly from those used in computing the wind moments, no further change in sizes is required. In this connection it should be noted that a considerable variation may occur in the values of wind reaction ratios without materially changing the wind moments.[11] For this reason a second revision of design will generally be unnecessary.

[11] *Proceedings*, Am. Soc. C. E., June, 1936, p. 974.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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PAPERS

SALTS IN IRRIGATION WATER

By Raymond A. Hill,[1] M. Am. Soc. C. E.

Synopsis

Those concerned with domestic and industrial water supplies were the first to appreciate the importance of the mineral salts invariably present in natural stream and ground waters. Irrigation engineers later began to realize that water in its pure form is never found in nature, and that natural water may be properly termed a mineral, of which H_2O is the principal constituent.

Unfortunately, there has been a tendency for irrigation engineers to disregard the character of the other constituents of natural water, limiting themselves largely to consideration of the total amount of dissolved solids. The interpretation of chemical analyses of irrigation water generally has been left to the agriculturalist. In an effort to bridge this gap between the engineer and the agriculturalist, the writer developed a geochemical chart. This he has found most useful for the classification of irrigation waters and for the solution of problems having to do with mixtures of water and with changes in quality resulting from the use of water for irrigation.

Geochemical Classifications

In 1911 Chase Palmer presented[2] a method for geochemical classification of natural waters. In his description he stated:

> "Nearly all terrestrial waters have two general properties, salinity and alkalinity, on whose relative proportions their fundamental characters depend. Salinity is caused by salts that are not hydrolyzed; alkalinity is attributed to free alkaline bases produced by the hydrolytic action of water on solutions of bicarbonates and on solutions of salts of other weak acids.
>
> "All the positive radicals, including hydrogen, may participate in producing salinity; but of the negative radicals only those of the actively strong acids can perform a similar function. The principal strong acids in natural waters are represented by the sulphates, chlorides, and nitrates."

Note.—Written comments are invited for immediate publication; to insure publication the last discussion should be submitted by **October 15, 1941.**

[1] Cons. Engr. (Leeds, Hill, Barnard & Jewett), Los Angeles, Calif.

[2] "The Geochemical Interpretation of Water Analyses," by Chase Palmer, *Bulletin No. 479*, U. S. Geological Survey, 1911.

In accordance with the prevalence of the reacting values of the groups of positive radicals in the system, five special properties are possible, according to Mr. Palmer—namely, (1) primary salinity (alkali salinity); (2) primary alkalinity (permanent alkalinity); (3) secondary salinity (permanent hardness); (4) secondary alkalinity (temporary alkalinity); and (5) tertiary salinity (acidity). He made the following statement with reference to these five special properties:

> "The character of natural waters with reference to the lithology of the region from which they are derived, to their solvent action on minerals with which they may come in contact, to sedimentary deposits that they are likely to form, to their effect on industrial processes, and to their chemical action in general can best be portrayed by a statement of as many of the five special properties * * * as may be found, expressed in percentages of their totality."

In any system of geochemical classification it is necessary that the reacting values be used—that is, the equivalent weights of hydrogen, usually expressed as milligram equivalents of hydrogen per liter. Substantially, 1 mg of hydrogen per liter is 1 ppm of hydrogen; consequently, "milligrams equivalent per liter" can be termed "equivalents per million," abbreviated to "epm." The equivalent weights of the more common ions are as follows:

Calcium (Ca), 20.04	Carbonate (CO_3), 30.00
Magnesium (Mg), 12.16	Bicarbonate ($H\,CO_3$), 61.01
Sodium (Na), 23.00	Sulfate (SO_4), 48.03
Potassium (K), 39.10	Chloride (Cl), 35.46
Nitrate (NO_3), 62.02	

Mr. Palmer computed the percentage reacting values of each ion from the ratio of the equivalent weight of that ion to the equivalent weight of all of the ions. However, because half of the ions are positive and half are negative, it is now more usual to express the percentage reacting value of any positive ion as the ratio of the equivalent weight of that ion to the total equivalent weight of all cations. Likewise, it is customary to compute the equivalent weight of any negative ion, such as the sulfate radical, as the ratio of the equivalent weight of SO_4 to the total equivalent weight of all anions. Regardless of the manner of computing the individual percentage reacting values, the geochemical groups are reported as percentages which themselves total 100%.

Tertiary salinity, as classified by Mr. Palmer, is rarely found in irrigation waters. The magnitude of the other four geochemical classifications suggested by him can be computed by grouping the ions normally present in natural waters in the following manner:

Primary Salinity.—Maximum grouping of Na, K, SO_4, Cl, and NO_3 ions—that is, the greatest possible amount of sodium sulfate, sodium chloride, sodium nitrate, potassium sulfate, potassium chloride, and potassium nitrate that could be in the solution.

Primary Alkalinity.—Minimum grouping of Na, K, $H\,CO_3$, and CO_3 ions—that is, the smallest possible amount of sodium bicarbonate and carbonate, and of potassium bicarbonate and carbonate, which could be in the solution.

Secondary Salinity.—Minimum grouping of Ca, Mg, SO_4, Cl, and NO_3 ions—that is, the smallest possible amount of calcium sulfate, calcium chloride, calcium nitrate, magnesium sulfate, magnesium chloride, and magnesium nitrate which could be in the solution.

Secondary Alkalinity.—Maximum grouping of Ca, Mg, H CO_3, and CO_3 ions—that is, the greatest possible amount of calcium bicarbonate and carbonate and of magnesium bicarbonate and carbonate that could be in the solution.

This system of geochemical classification was particularly adapted to the study of waters used for domestic and industrial purposes. For the interpretation of analyses of water used in agriculture it was felt desirable to group the sulfates with the carbonates rather than with the chlorides and nitrates. This resulted in the following four geochemical groups of salts that can be used to classify all but definitely acid waters:

Z_1.—Maximum grouping of Na, K, Cl, and NO_3 ions—that is, the greatest possible amount of sodium chloride, sodium nitrate, potassium chloride, and potassium nitrate that could be in the solution.

Z_2.—Minimum grouping of Na, K, SO_4, H CO_3, and CO_3 ions—that is, the smallest possible amount of sodium sulfate, sodium bicarbonate and carbonate, potassium sulfate, and potassium bicarbonate and carbonate that could be in the solution.

Z_3.—Minimum grouping of Ca, Mg, Cl, and NO_3 ions—that is, the smallest possible amount of calcium chloride, calcium nitrate, magnesium chloride, and magnesium nitrate that could be in the solution.

Z_4.—Maximum grouping of Ca, Mg, SO_4, H CO_3, and CO_3 ions—that is, the greatest possible amount of calcium sulfate, calcium bicarbonate and carbonate, magnesium sulfate, and magnesium bicarbonate and carbonate that could be in the solution.

The designations Z_1, Z_2, Z_3, and Z_4 were used rather than names to avoid confusion with the classification suggested by Mr. Palmer. Group Z_1 is composed largely of sodium chloride; hence it is termed the Common Salt Group. The second group includes those salts commonly called "white alkali" and "black alkali," and this might be called the Alkali Group. Group Z_3 is sometimes referred to as the Bittern Group. Group Z_4 is termed the Hardness Group because it is made up of those salts which most frequently produce hardness in water.

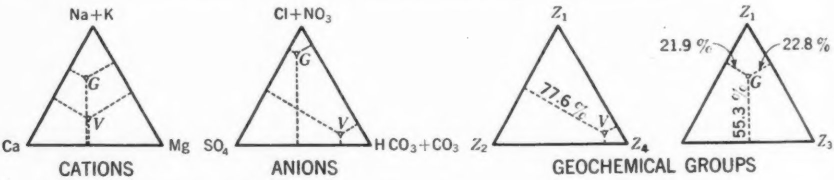
Development of Geochemical Chart

From the nature of the geochemical groups, as defined, there never can be more than three in any sample of water. Of these, Group Z_1 is almost always present, as is Group Z_4, but it is not possible to have salts of both Group Z_2 and Group Z_3 in the solution. Usually, the equivalent weight of sodium and potassium exceeds the amount of chlorides and nitrates. In such event, all of the latter are assigned to Group Z_1; should the amount of chlorides exceed the amount of the alkali bases, all of the sodium and potassium would be included in Group Z_1; in either case, nothing remains to form the fourth group.

The sum of the percentage values of the three groups present being equal to unity, the geochemical character of any water can be represented by a point within an equilateral triangle located so that the perpendiculars to that point are proportional to these percentages. A similar method of plotting can be used to show the relative proportion of the principal ions found in natural water, because there are usually only three cations present in substantial quantities, and likewise the sulfate, bicarbonate, and chloride ions usually constitute almost all of the anions. However, because of the limitation on the number of variables that can be plotted in this manner, it is necessary to combine the percentages of potassium and sodium, and to lump carbonates with bicarbonates, and nitrates with chlorides.

Typical plottings of two waters, the analyses of these waters, and the computation of the geochemical groups are shown in Table 1. The points marked

TABLE 1.—ILLUSTRATIVE ANALYSIS, TRILINEAR PLOTTING OF CHEMICAL ANALYSES



Line	Unit	Cations			Σ	Anions			Groups			
		Ca	Mg	Na		H CO$_3$	SO$_4$	Cl	Z_1	Z_2	Z_3	Z_4
POINTS *V*, VERDE RIVER BELOW BARTLETT DAM, ARIZONA												
1	epm	2.33	1.87	1.20	5.40	3.90	0.99	0.51	0.51	0.69	0	4.20
2	%	43.1	34.5	22.4	100.0	72.2	18.3	9.5	9.5	12.9	0	77.6
POINTS *G*, GILA RIVER ABOVE SALT RIVER, ARIZONA												
3	epm	15.4	8.9	30.1	54.4	4.0	8.4	42.0	30.1	0	11.9	12.4
4	%	28.3	16.4	55.3	100.0	7.4	15.4	77.2	55.3	0	21.9	22.8

"*V*" represent a sample of the Verde River below Bartlett Dam in Arizona. This water is generally characteristic of natural streams. The points marked "*G*" are for return flow in the Gila River in Arizona immediately above its confluence with the Salt River. The method of plotting is illustrated by the diagram showing the proportions of the various geochemical groups found in the Gila River water. If all of the salts were sodium chloride, the point would have fallen at the Z_1 vertex. Actually, 55.3% of the salts were of the Z_1 Group, and the plotted position is therefore 55.3% of the distance from the base opposite the Z_1 vertex toward that vertex of the equilateral triangle. Likewise, the plotted position is 22.8% of the distance from the base opposite the Z_4 vertex toward that vertex, and 21.9% toward the Z_3 vertex from the

opposite base. The positions of the points V and G on the cation triangle and on the anion triangle were determined in the same manner by plotting the percentage reacting values of the respective ions.

Although the chemical character of almost any natural water could be illustrated thus by the use of four equilateral triangles, the same data can be shown more effectively on a chart that combines these four triangles in the manner shown in Fig. 1.

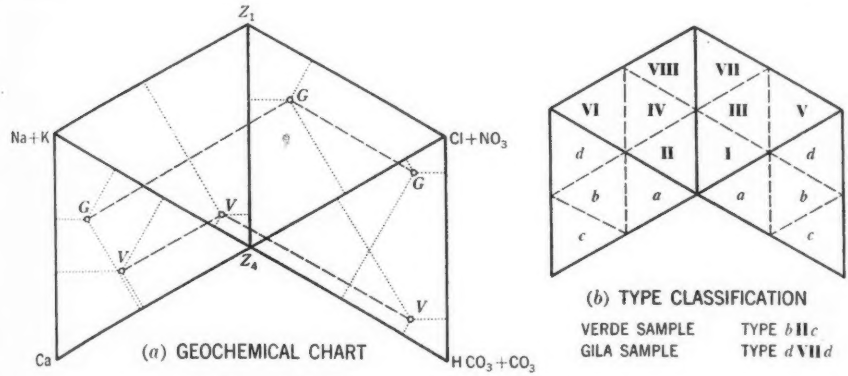


FIG. 1.—TRILINEAR PLOTTING OF ANALYSES ON GEOCHEMICAL CHART

Each of the triangles used for plotting the geochemical groups (Fig. 1(a)) has a Z_1 vertex and a Z_4 vertex; in other words, the $Z_1 - Z_4$ base is common to both. Consequently, by superimposing these vertexes a diamond is formed, the bisector of which is the common base. The lower sides of the diamond can now be treated as oblique coordinates, with the origin at the Z_4 vertex, reading in percentage of chloride to the right and percentage of sodium to the left.

By definition, a point plotted at the Z_4 vertex would represent a water containing neither sodium nor chloride, whereas a point plotted along the side of the diamond connecting the Z_2 vertex and the Z_1 vertex would represent a water in which all salts were those of sodium and potassium. Similarly, a point on the line connecting the Z_3 with the Z_1 vertex of the diamond would indicate that all the salts were chlorides or nitrates. A point plotted on the cation triangle at the sodium vertex represents a water in which all of the positive ions are those of sodium and potassium, and a point along the opposite base would be for a water in which there was no sodium or potassium. Therefore, by bringing the cation triangle into contact with the diamond so that the sodium vertex of the cation triangle is coincident with the Z_2 vertex of the diamond and the magnesium vertex of the cation triangle is coincident with the Z_4 vertex of the diamond, a common scale of sodium percentages can be used. In like manner, the anion triangle can be placed beside the diamond with the chloride and nitrate vertex coincident with the Z_3 vertex of the diamond and with the sulfate vertex coincident with the Z_4 vertex of the diamond. When this is done, a common scale of chlorides plus nitrates can be used for the anion triangle and for the geochemical groups.

In Fig. 1 are shown the same points as those in Table 1 for samples of water from the Verde River and the Gila River in Arizona. It will be noted that the points in the cation triangle and those in the anion triangle can be projected parallel to the coordinate axes to intersections that coincide with the points plotted in the diamond. In fact, in the actual use of the geochemical chart it is customary to plot the points representing the water in the cation triangle and in the anion triangle from the percentage reacting values of the ions, and then to plot the percentage of sodium plus potassium against the percentage of chloride plus nitrate, using the sides of the diamond as oblique coordinates, only checking the position of the point by computation of the Z_2 or Z_3 perpendicular, as the case may be.

Types of Water

Waters of different chemical characteristics can be classified readily from their plotted positions on the geochemical chart. When lines are drawn parallel to each base at 50% of the distance toward the opposite vertex, each of the four fundamental triangles is divided into four secondary triangles. These have been numbered arbitrarily in the manner shown in Fig. 1(*b*), following a pattern that can be remembered easily. The sample of water from the Verde River is characteristic of many natural streams and would usually be referred to as Type II water. Were more precise description desirable, it would be termed a Type *b*II*c* water. The Roman numeral II indicates a water in which most of the salts fall in Group Z_4 and with some salts in Group Z_2 but none in Group Z_3. The prefix "*b*" indicates a water in which no one of the positive ions exceeds 50% of the total. The suffix "*c*" defines a water in which more than 50% of the anions are bicarbonates or carbonates. The sample from the Gila River is a Type VII water; for consistency, the prefix "*d*" and the suffix "*d*" could be added, although by the nature of the diagram any water of Type VII contains more than 50% sodium and more than 50% chloride. This water contains salts of Group Z_3, but not of Group Z_2, and might well be called "irrigation sewage."

Eight primary types are thus available for cataloging irrigation waters, which can be broken down into thirty-two subtypes by use of letters indicating the preponderance of certain positive and negative ions. No attempt will be made herein to list these thirty-two different types of water in the order of their desirability for use in irrigation agriculture. Suffice it to state that, in the case of waters of equal concentration, those in Type II are preferable, and that those of Type VI should be avoided. It is hoped that some discusser will proceed further and describe the effect of different kinds of water on soils and on plant growth.

Mixtures of Different Waters

It can be demonstrated that the plotted position of a mixture of two waters must lie on a straight line connecting the plotted positions of each of the waters comprising the mixture. It can be demonstrated further that the distance of the plotted position of the mixture from either of the two waters making up the mixture depends directly upon the ratio between the total quantity of

salts contributed by one water and that contributed by the other. When the respective products of the concentrations of salts in equivalents of hydrogen and the quantities of water are treated as forces normal to the plane of the paper, the position and magnitude of the mixture correspond to the resultant of the forces. Such products of concentration and quantity can be computed conveniently as equivalent tons of hydrogen, or tons-equivalent, abbreviated to "T_e." When the quantity of water is given in acre-feet, the quantity of any ion in tons-equivalent is equal to the quantity times the concentration in milligram equivalents of hydrogen per liter divided by 735, or algebraically:

$$T_e = \frac{C\,Q}{735} \qquad (1)$$

in which C is concentration of salts in equivalents per million; and Q is quantity of water in acre-feet.

When it is known that one water is in fact a true mixture of two or even three different waters, the amount of each contribution to the mixture can be computed analytically by use of simultaneous equations. However, when it is only suspected that one water is a mixture of several different waters, or when the amount and character of one of the contributing supplies is unknown, an analytical solution becomes impracticable.

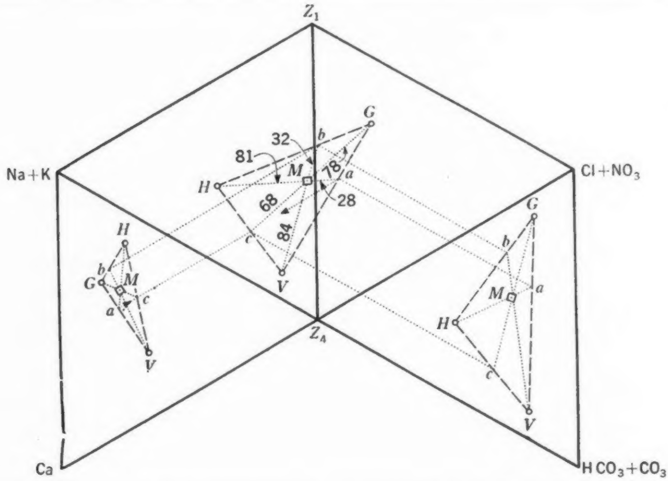
The first step in the solution of such a problem is to determine whether or not the presumed mixture is in fact a mixture. This can be readily ascertained graphically by use of the geochemical chart. A typical solution for a true mixture of three waters is shown in Table 2; in this it is assumed that the amount of water from each source is unknown.

For purposes of illustration, the sample of Verde River water (V) and the sample of Gila River water (G) described in connection with Table 1 have been used. The third water (H) contributing to the presumed mixture is somewhat similar to ground waters found in the same region; in fact, the mixture (M) is quite representative of water being used for irrigation in the Gila River Valley west of Phoenix, Ariz. The known data are shown in Table 2 and are limited to the amount of water in the mixture and to analyses of samples of the mixture, of Verde River water, of Gila River water, and of the third water. It is to be noted that the total amount of salt in each water, expressed in equivalents per million and in tons-equivalent, is the sum of the positive ions, or the sum of the negative ions, but is not the sum of both.

In each of the three figures in Table 2 the point M falls within the triangle formed by connecting points V, G, and H. This indicates, but does not prove, that the water represented by points M is a mixture of the other three waters. If the presumed mixture M is in fact a mixture of all of the three, it can be considered as being made up of any one of the three and a mixture of the other two. Consequently, if a line is drawn from point G, for example, through the point M to its intersection with the line HV at point c, the latter point represents a mixture of water V and water H in the same proportion as these two occur in the mixture of all three waters. The projection of point c in the cation triangle, therefore, must interest the projection of point c in the anion triangle

at point *c* within the geochemical diamond. The same is necessarily true for points *a* and *b*, determined by drawing lines from the plotted positions of water *H* and water *V* through the plotted positions of the mixture.

TABLE 2.—ANALYSIS OF A MIXTURE OF THREE WATERS;[a] KNOWN DATA AND COMPUTED VALUES



Line	Unit	Cations			Σ	Anions		
		Ca	Mg	Na		H CO_3	SO_4	Cl
				POINTS *V*, VERDE RIVER (1,800 ACRE-FT)				
1	epm	2.33	1.87	1.20	5.40	3.90	0.99	0.51
2	T_e	(5.70)	(4.57)	(2.94)	(13.21)	(9.54)	(2.42)	(1.25)
3	%	43.1	34.5	22.4	100.0	72.2	18.3	9.5
				POINTS *G*, GILA RIVER (300 ACRE-FT)				
4	epm	15.4	8.9	30.1	54.4	4.0	8.4	42.0
5	T_e	(6.28)	(3.62)	(12.28)	(22.18)	(1.63)	(3.42)	(17.13)
6	%	28.3	16.4	55.3	100.0	7.4	15.4	77.2
				POINTS *H*, HYPOTHETICAL WATER (900 ACRE-FT)				
7	epm	1.00	2.60	6.40	10.00	2.80	4.60	2.60
8	T_e	(1.22)	(3.18)	(7.83)	(12.23)	(3.42)	(5.63)	(3.18)
9	%	10.0	26.0	64.0	100.0	28.0	46.0	26.0
				POINTS *M*, MIXTURE (3,000 ACRE-FT)				
10	epm	3.24	2.79	5.66	11.69	3.58	2.82	5.29
11	T_e	13.20	11.37	23.05	47.62	14.59	11.47	21.56
12	%	27.7	23.8	48.5	100.0	30.6	24.1	45.3

[a] Computed values shown in parentheses.

When it has been determined that the presumed mixture is in fact a true mixture, the salt, in tons-equivalent, contributed by each of the three waters comprising the mixture can be computed, with an accuracy commensurate with that of the analyses themselves, by measuring, at some convenient scale, the distance from point M to each of the aforementioned points, and by treating these distances as lever arms of forces normal to the plane of the paper. For example, if moments are taken about the line HG:

$$\begin{aligned}(84 + 32)\, V &= 32\, M \\ 116\, V &= 32 \times 47.62 \\ V &= 13.21\end{aligned}$$

In the foregoing, 47.62 is the number of tons-equivalent in the mixture, so that 13.21 is the number of tons-equivalent in the Verde River water contributing to that mixture. The difference (that is, 34.41 T_e) represents the tons-equivalent of salts contributed by the Gila River water and by the hypothetical water, in such a proportion that the mixture of these two alone would plot at point b. In similar manner, the tons-equivalent of salt contributed by each of these waters is determined. Knowing the total amount of salt in each water making up the mixture, and the concentration of salt in samples of each, the number of acre-feet of water from each source can then be computed directly by multiplying the tons-equivalent by 735 and by dividing the product by the concentration in equivalents per million. For example, in the case of the Verde River water, the concentration was 5.40 epm and the total amount of salts was 13.21 T_e; hence, the Verde River contributed 1,800 acre-ft of water to the mixture. From the other data shown in Table 2, it was found that the Gila River furnished 300 acre-ft, and that the remaining 900 acre-ft were from the hypothetical well.

Changes in Quality

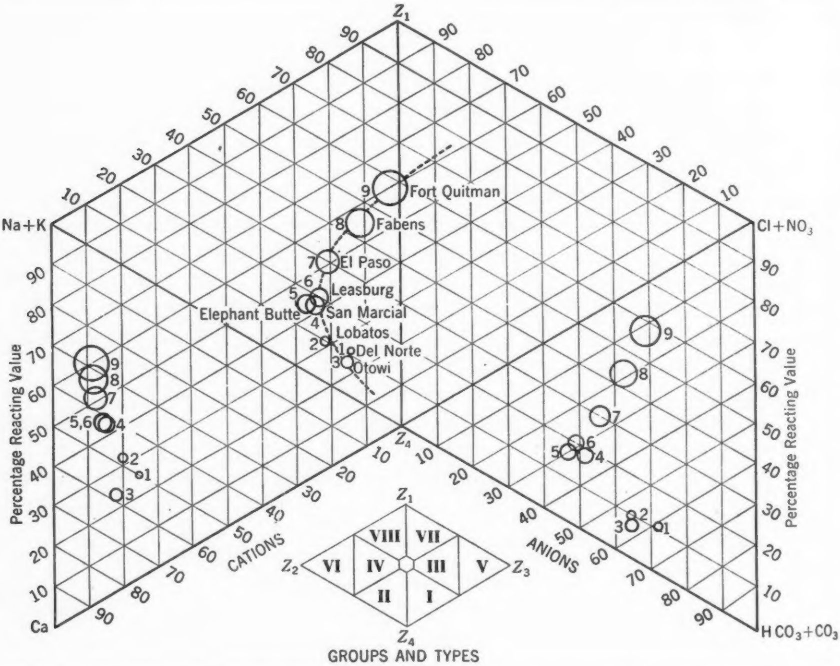
The foregoing has been descriptive of the manner in which the geochemical chart was developed and of the fundamental principles involved in its use. One of the many applications to which the chart can be put is that shown in Table 3. On the graph have been plotted characteristic analyses of samples taken from the Rio Grande at various stations along its course from the mountains in Colorado to Fort Quitman, Tex. The data are from the 1936 report of the Rio Grande Joint Investigation. The chart itself is a reduction of a standard commercial form originally printed with 2% graduations at a scale of about 1 in. equals 20%. The weighted average concentration of the ions in equivalents per million is also given in Table 3.

As an aid in visualization of the changes that occur in the quality of the waters of the Rio Grande along its course, the analyses have been indicated by circles whose areas are proportional to the concentration of salts, ranging from the small circle representing samples from the Rio Grande at Del Norte, Colo., to the large circle corresponding to the highly concentrated waters found in the Rio Grande at Fort Quitman.

It is to be noted from the plotting of these data that the water coming out of the mountains is of Type *c*II*c*—that is, a water in which calcium bicarbonate

predominates. Below Del Norte lies the San Luis Valley in Colorado within which there are several hundred thousand acres of irrigated land. The change in quality resulting from irrigation use is reflected by the shift from point 1 to point 2, the latter being for the Lobatos station near the Colorado-New Mexico

TABLE 3.—GEOCHEMICAL CLASSIFICATION OF THE WATER OF THE RIO GRANDE ABOVE FORT QUITMAN, TEX.



No.	Location	Ca	Mg	Na+K	Total anions	CO_3 + HCO_3	SO_4	Cl + NO_3
1	Del Norte, Colo.	0.56	0.27	0.30	1.14	0.70	0.32	0.12
2	Lobatos, Colo.	1.39	0.58	0.91	2.83	1.54	1.00	0.29
3	Otowi, N. Mex.	2.19	0.69	0.90	3.71	2.12	1.28	0.31
4	San Marcial, N. Mex.	3.76	1.27	3.62	8.66	2.96	4.11	1.59
5	Elephant Butte, N. Mex.	3.75	1.28	3.88	8.96	2.66	4.76	1.54
6	Leasburg, N. Mex.	4.05	1.35	4.20	9.63	2.82	4.88	1.93
7	El Paso, Tex.	4.95	1.70	6.98	13.65	3.55	6.07	4.03
8	Fabens, Tex.	6.90	2.75	12.40	22.12	4.10	8.26	9.76
9	Fort Quitman, Tex.	9.54	3.75	19.78	33.20	4.01	10.17	19.02

boundary. Below this station there are other tributary streams, particularly Rio Chama, draining the high mountains in northern New Mexico. Some of these waters obviously contain more of the Z_4 Group of salts than does the water of the Rio Grande at Del Norte, because the analysis for Otowi, N. Mex. (point 3), plots between the Z_4 vertex and point 2 and closer to that vertex than the Del Norte water.

Below Otowi, which is northwest of Santa Fe, N. Mex., there is the Middle Rio Grande Valley which extends along the river to San Marcial, N. Mex., near the head of Elephant Butte Reservoir. The marked reduction in quality that occurs between Otowi and San Marcial is perhaps due most to the influence of tributary streams, such as Rio Puerco, draining the plateaus of western New Mexico. The water released from Elephant Butte (point 5) differs somewhat from the inflow at San Marcial because the supply drawn out of the reservoir in 1936 included water that came into the reservoir in the preceding year. The reductions in quality of water from Elephant Butte to Fort Quitman were due almost entirely to the use and re-use of the stream waters for irrigation of the Rincon, N. Mex.; the Mesilla, N. Mex.; the El Paso, Tex.; and the Juarez, Mexico, valleys.

The significance of these changes in total concentration and in the relative proportions of the various ions and geochemical groups of salts will be better appreciated if the successive analyses are thought of in terms of mixtures. Salts that pass out of any valley are one part of a mixture; salts left behind as a result of irrigation constitute the other part; the water entering the valley is the principal source of both. For example, practically all of the water measured at El Paso was released from Elephant Butte Reservoir. The salts carried out of the Mesilla Valley are represented by point 7; the salts left behind, if known, could be shown by a point on or near the prolongation of the line from point 7 back through point 5.

From this it will be seen that the salts left in the soils of the San Luis Valley were largely carbonates and that the same was generally true through the Middle Rio Grande Valley. Below Elephant Butte Dam, however, a different situation prevailed; here the residual salts contained 30% to 40% of the Alkali Group.

This progressive depreciation of the quality of water is quite natural where the stream flow is used and re-used for irrigation. When water is applied to land, a large part of it is transpired by plants and evaporated from moist soil. Very little of the salt in the water is taken up by the plants; consequently, there is a much greater concentration of salts in the soil solution than in the irrigation supply. Since the carbonates of calcium and magnesium are sparingly soluble, they are precipitated first. The irrigation percolate, which finds its way back into the river either through drains or by natural seepage, thus contains a higher proportion of sulfates, chlorides, and sodium than the waters that were diverted for irrigation. Progressively downstream, H_2O is lost to the atmosphere by evaporation and the less soluble salts are left behind by precipitation, until only irrigation sewage remains.

Somewhat similar variations in quality exist along the Colorado River from its headwaters in the Rocky Mountains to the Gulf of California, but the degree of change is smaller, due to the fact that the Colorado River is less developed relatively than the Rio Grande above Fort Quitman. In spite of this, however, the supply released from Lake Mead contains 30% more salt per acre-foot than water released at Elephant Butte; and, although the sodium percentage is not quite as high, the proportion of chlorides and sulfates is greater in the

water discharged from Lake Mead than it is in the Rio Grande below Elephant Butte.

Valley Salt Balance

In addition to presenting a picture of what happens along the course of a river whose waters are used and re-used in the irrigation of land, the geochemical chart can be used to determine whether any salt is being accumulated in the lands irrigated.

Whenever the total amount of salt removed from an area exceeds the salt load of the incoming supply, it would appear that a favorable salt balance exists; however, this is not necessarily true. In many river valleys the ground waters are much more saline than the surface stream waters, as, for example, along the Rio Grande below El Paso. Water seeping from irrigation canals and percolating from fields tends to displace previous accumulations of ground water and to cause lateral movement toward any drains. Consequently, the drainage outflow is usually more saline than the contribution to the ground water.

Wherever this situation exists, the total quantity of salt in the effluent from a valley is likely to be greater than that in the influent to the same area, and this condition will hold until all the saline ground waters have been displaced. Therefore, it is necessary to do more than compute the difference between the salt load of the irrigation water and that of the drainage water.

Table 4 shows the solution of such a problem for a 15,000-acre unit of the El Paso Valley. In the year 1934, which has been taken for illustration, 69,000 acre-ft of water, carrying 78,000 tons of dissolved solids of a type similar to that represented by point 7 in the graph insert of Table 3, were delivered into that area. The drainage effluent amounted to 27,000 acre-ft with a salt load of 140,000 tons. There was thus an apparent favorable salt balance amounting to 62,000 tons.

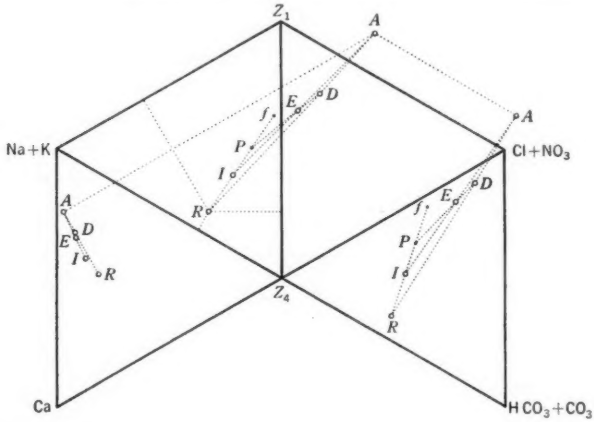
From a number of separate analyses made during that year, the total amount of each ion in the influent to that area, and in the effluent from it, were computed. These data are shown in Table 4. All other values given in the table are computed. The apparent gain (that is, the amount by which the salts in the effluent exceed the salts in the influent) is merely the difference between these quantities. The net gain in positive ions amounted to 244 T_e of calcium, 29 T_e of magnesium, and 806 T_e of sodium, making a total of 1,079 T_e removed from the area with the drainage water in excess of the amount brought in with the irrigation water.

The simple difference between the amount of chloride brought in and the amount removed shows an apparent gain of 1,261 T_e, which is 182 T_e greater than the net amount of cations removed. This excess is offset by the apparent accumulation of 122 T_e of HCO_3 and 60 T_e of SO_4

In any quantity of water there must be the same equivalent weight of positive ions as there is of negative ions. It follows, therefore, that, with the 182 T_e of chlorides in excess of the total of the positive ions, there were 182 T_e of cations in the effluent that were not brought into the area during the same year. Furthermore, with these 182 T_e of cations displaced from some source

within the area, there must have been another 182 T_e of anions. In these anomalies lies the solution of the problem of how much of the influent salt was left in this part of El Paso Valley and how much of the salt already accumulated was removed from it.

TABLE 4.—VALLEY SALT BALANCE; KNOWN DATA AND COMPUTED VALUES[a]



Line	Unit	Cations			Σ	Anions		
		Ca	Mg	Na		H CO₃	SO₄	Cl
				INFLUENT SALT, *I*				
1	T_e	465	166	664	1,295	332	576	387
2	%	35.9	12.8	51.3	100.0	25.6	44.5	29.9
				EFFLUENT SALT, *E*				
3	T_e	709	195	1,470	2,374	210	516	1,648
4	%	29.9	8.2	61.9	100.0	8.8	21.8	69.4
				APPARENT GAIN, *A*				
5	T_e	244	29	806	1,079	−122	−60	1,261
6	%	22.6	2.7	74.7	100.0	−11.3	−5.6	116.9
				DISPLACED SALT, *D*				
7	T_e	458	130	1,036	1,624	92	216	1,316
8	%	28.2	8.0	63.8	100.0	5.7	13.3	81.0
				RESIDUAL SALT, *R*				
9	T_e	214	101	230	545	214	276	55
10	%	39.3	18.5	42.2	100.0	39.3	50.7	10.0

[a] All values for displaced and residual salts are computed.

The necessity of considering more than just the difference between the amount of salt brought in and that removed becomes more apparent when the data are plotted on the geochemical chart. Point I, Table 4, represents the influent, point E the effluent, and point A the apparent gain. Point A, however, falls outside of the anion triangle, and likewise outside of the geochemical diamond. This could not be the case if the excess amount of salt in the effluent resulted only from the removal of previous accumulations, because no water can contain less than zero of any ion. The apparent gain must, in itself, be the difference between the total salts displaced from the area and the amount of influent salts left behind in the soil or in the ground water.

The approximate magnitude and character of the displaced salts and the residual salts can be computed from the following relationships:

$$I = R + P \qquad (2a)$$

$$E = P + D \qquad (2b)$$

$$E - I = A \text{ or } E = I + A \qquad (2c)$$

$$P + D - R - P = A \qquad (2d)$$

and

$$D - R = A \text{ or } D = R + A \qquad (2e)$$

in which: I is the amount of influent salts; E is the amount of effluent salts; P is the amount of the influent salts that passes through the area within the period of time under consideration; R is the amount of salts left behind in the soil or in the ground water; and D is the amount of salts displaced from the soil or from the ground water.

From Eqs. 2, E thus can be considered a mixture of I and A; likewise, D can be treated as a mixture of R and A. Consequently, the points I, E, and A must lie on a straight line, and the points R, D, and A must also lie on a straight line. Furthermore, the positions of the points along these lines must be such that there will be equilibrium of forces when the equivalent total weight of each is treated as a force normal to the paper.

If moments are taken about the base opposite the vertex corresponding to any ion or geochemical group, the percentage of that ion or group can be used as the lever arm of the force. In Eqs. 3 such percentages are designated by the letter C with appropriate subscripts:

$$C_i \times I + C_a \times A = C_e \times E \qquad (3a)$$

and

$$C_r \times R + C_a \times A = C_d \times D \qquad (3b)$$

There are four unknowns in Eqs. 3—namely, C_r, R, C_d, and D. Eq. 3(b) and the equation of mass (that is, Eq. 2(e)) are the only simultaneous equations available for determining these unknowns. Consequently, an infinite number of solutions of the problem are possible. The position of points R and D on the geochemical chart are circumscribed by other factors, however, so that the magnitude of these values can vary only within limits.

The smallest possible amount of residual salt was 182 T_e; if this were the correct value, point D would be at the chloride vertex of the anion triangle,

indicating the displacement of no other ion from the soil or from the ground water; it also would require that the salts left behind include no chlorides. Neither presumption is warranted. There was probably no percolate from some irrigated fields, so that a part of the residual was substantially of the same character as the influent salt. All of the canal seepage could not have reached the effluent drains within the period of time under consideration; this residual was also of the same character as the influent. In other parts of the area where there was free drainage, probably no chlorides were left behind. It is reasonable, therefore, to assume that the residual salts contained substantially 10% of chlorides. Under this assumption, point *R* must lie on a line parallel to the base opposite the chloride vertex of the anion triangle and 10% of the distance from that base toward the chloride vertex.

The residual being one element of a mixture which is represented by the influent, the other element of the mixture (that is, point *P*) must lie on the prolongation of the line *RI*. Point *P* is in itself a mixture of canal water and field percolate. The canal water is the same as the influent; therefore, the position representing the field percolate must lie on the prolongation of the line *IP* at some such point as *f*, shown in Table 4. Any field percolate will be low in carbonates but presumably will contain some carbonates.

Point *D*, Table 4, is indicative of the character of the salts displaced from the soil and the ground water. These are naturally high in chlorides, and it is reasonable to assume ordinarily that only about 30% of the salts are sulfates and carbonates. In the solution of the particular problem used for illustration, certain analyses of well waters were available which indicated that the displaced salts contained more than 80% chlorides.

The location of points *R* and *D* in Table 4 were selected with consideration to these factors; the magnitude of the quantities were computed from Eqs. 2 and 3. The resulting values are shown in Table 4. It will be noted that the apparent gain of 1,079 T_e of salts during 1934 was probably brought about by the displacement of 1,624 T_e of salts from the soil and the ground water and the accumulation of 545 T_e of the salts in the irrigation supply. Other reasonable solutions of this problem are possible, but the magnitude of the residual cannot be changed to any large extent without making extreme assumptions. There is ample reason to assume that the true residual was between 450 T_e and 650 T_e and that the large net gain in chlorides resulted from the displacement of saline ground water.

The character of salts left behind as a result of irrigation in this part of the El Paso Valley can be computed from the chart. The perpendiculars to point *R* on the geochemical diamond are proportional to the magnitude of the various groups. These magnitudes are as follows:

Group	Tons-equivalent
Z_1	54.5
Z_2	175.5
Z_4	315.0

Substantially all of Group Z_1 was sodium chloride, so that the total amount of this group left behind can be computed by multiplying 54.5 T_e by the sum of

the combining weights of sodium and chloride (namely, 23.0 + 35.5), giving roughly 3,200 tons. In Group Z_2 there was evidently no sodium bicarbonate because there was ample calcium present to combine with the carbonates. Accordingly, it was found that 12,500 tons of sodium sulfate were left in the area as a result of irrigation. Computation of the amount of the different salts included in Group Z_4 involves the assumption that certain combinations will be formed. These hypothetical combinations are quite uncertain, but to complete the picture it can be estimated that 6,100 tons of magnesium sulfate and 10,700 tons of calcium carbonate were included in the residual.

Approximately 32,500 tons of various salts out of the 78,000 tons brought in by the irrigation water in 1934 were thus left behind as a result of the irrigation of less than 15,000 acres. From this it follows that two thirds of the salt load of the drainage water was derived from the saline waters underlying the area, and that the salt balance within the root zone was probably adverse.

In addition to this accumulation of mineral salts in the soils of the area, it appears that there was some accumulation of sodium as a result of base exchange. The displaced salts contained 17.2% of the Z_3 Group, which is made up primarily of the chlorides of calcium and magnesium. The amount of magnesium displaced was only 8% of the cations so that about one half of the Z_3 Group of displaced salts was calcium chloride. It is unlikely that calcium chloride would be present unless there had been base exchange, because of the character of the influent water. The probability that calcium was displaced from the soil colloids by sodium is also indicated by the proximity of point *D* on the cation triangle to point *E*.

The assumption that a favorable salt balance is established whenever the total of the effluent salts exceeds the total of the influent salts may thus be erroneous. It is important to ascertain in fact whether a favorable salt balance is being maintained within the root zone of the irrigated fields, because any accumulation of salt in the upper soil horizons, no matter how slow, must eventually render irrigation agriculture unprofitable.

Necessity for Further Study

This paper has been presented for two purposes: First, to offer to the engineering profession a convenient tool for the interpretation of chemical analyses of stream and ground waters; and second, to stimulate irrigation engineers to consider quality of water to be as important as quantity.

The data presented were selected primarily in illustration of the geochemical chart developed by the writer and of the methods used in its application to irrigation problems. Much more study of these problems by irrigation engineers and agriculturalists is needed, to the end that successful irrigation agriculture may be permanent rather than temporary.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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PAPERS

COST OF PUBLIC SERVICES IN RESIDENTIAL AREAS

By F. Dodd McHugh,[1] Esq.

The Problem

Material available, although not conclusive, shows that the United States is faced with a slowing up in the rate of growth. The greatest intensity of this change is felt in cities. During the decade 1910–1920 the population of New York, N. Y., increased 17.8% and that of the nation 14.9%. The increase was 23.3% for the city and 6.1% for the nation during the period 1920–1930. The U. S. Census Bureau reports an increase (in 1940) of only 6.5% for New York City and an increase of 7% for the continental United States since 1930.

Modern trends reveal a shifting of people from the older sections of the city to newer residential developments; but the older sections are only partly vacated and are rarely reoccupied by an influx of people or by new commercial activities. The preliminary report of the Census Bureau shows that Manhattan's population remained practically unchanged between 1930 and 1940, whereas the population of the four other boroughs of New York City increased from 4% to 20%. It now appears that, since the 1934 Real Property Inventory of New York City, some of the older sections of Manhattan have continued to lose population. The new developments in the northern part of the borough have apparently attracted a sufficient number of residents to offset the loss in older sections. The exact extent of this population shift cannot be determined until detailed information by census tracts for 1940 has been analyzed.

In 1934 the Real Property Inventory revealed that about 12% of all dwelling units in New York City were vacant, but that about 5% of Manhattan's dwellings were unoccupied. This inventory also shows that only four fifths of the dwelling units were occupied in the older apartment sections

Note.—Written comments are invited for immediate publication; to insure publication the last discussion should be submitted by **October 15, 1941**.

[1] Director of Planning, Dept. of City Planning, New York, N. Y.

of Manhattan covered by this study. The Census Bureau's tentative report shows that 7.6% of all dwelling units were vacant in New York City as of April, 1940, and that about 11% were unoccupied in Manhattan. The census shows a gain in the occupancy ratio for the entire city as compared with 1934, but a decline for Manhattan. The meager data now at hand do not suggest that the older apartment districts of this borough have made any considerable gains in population subsequent to 1934. In fact, the preliminary census figures along with building permit, foreclosure, tax delinquent, and rental reports indicate that since 1934 the occupancy of dwellings in the older districts of Manhattan has probably not improved in relation to the city-wide occupancy ratio. Lacking any definite information to the contrary, it may be expected, therefore, that people are still leaving these older apartment districts to live in newer developments.

Why do people move from the older sections of cities? One explanation, given by Clarence A. Perry,[2] is applicable to practically all of the residential sections of New York City and other cities:

> "A prominent New York real estate man recently expressed the opinion that the 'normal life expectancy' of an ordinary dwelling was two or three times that of its neighborhood. In other words, the quality or desirability of a particular district tends to decline long before its component houses begin to wear out. Even so, this observation acquires a general significance only if it is discovered that the neighborhood deterioration affects the value of its individual houses. When the owner of a dwelling finds he can keep it occupied only by progressively lowering its rent, then its actual value is declining and the matter is of importance to the public, since that property is no longer able to bear its original share of the tax burden.
>
> "If this view is correct then the unsatisfactory conditions that lead families to give up their domiciles and move to new quarters become of private and public concern. If the dissatisfaction has arisen from something in the environment, it should be of special interest to the community because the matter is then beyond the power of the individual to correct and must receive attention from the public. That neighborhood conditions do cause people to move is a common view, but there is no unanimity as to just what these conditions are."

When old sections lose population, public services in these sections must be maintained and are usually operated below actual capacity. Meanwhile, additional public services must be supplied to the newer residential developments that are drawing population from the older sections of the city.

A survey made for the (New York City) Department of City Planning shows that one third of the city's gross area was vacant and unimproved in 1938. The 1940 federal census reports that in most of the larger cities of the United States, as well as in New York City, the rate of growth is slowing down. It is apparent from these facts and other considerations that the city has more than enough land available to house its present residential population and commercial activities, as well as any increase to be reasonably expected in the future. This outlook does not evidence a positive need to continue the extension of the

[2] "Housing for the Machine Age," by Clarence A. Perry, Russell Sage Foundation, New York, N. Y., 1939, p. 15.

city's physical growth to outlying undeveloped sections. The present economic and social conditions found in New York City obviously exist in many other cities. The prevalence of such unstable conditions would seem to require careful re-examination of the urban structure with particular reference to the possibilities of revitalizing the older sections of American cities.

Constitutional and statutory provisions usually limit the debt-incurring and real-estate taxing powers of municipalities to a fixed percentage of the assessed valuation of real estate. In general, assessed values are not rising, and the increase of urban government income from all sources, including real-estate taxes, has failed to keep pace with the growing demand for expansion of public services. Consequently the cities' financial capacity to meet changing urban conditions is threatened seriously.

Some of the conditions underlying this study are peculiar to New York City. To the extent that such factors as high density of buildings and population reflect New York characteristics, the findings are not directly applicable to other municipalities. It seems, however, that the methods employed may be useful in measuring the magnitude of a problem that is common to most American cities, and that any differences between one city and another as may be revealed by similar studies will be mainly a matter of degree.

Purpose of Study

An attempt is made in this paper to examine the problem in terms of city expenditures for permanent improvements and public services in old and new residential areas. Two types of residential areas are investigated: (1) A specimen community that is typical of the centrally located, existing residential development; and (2) a newer type of "neighborhood unit" development that is applicable to the rebuilding of old sections or in developing outlying vacant areas.

Costs are estimated for the public services found in the specimen community that represents existing conditions and for replacing the obsolete part of existing public facilities in this old area. The study also explores the public costs involved in rebuilding the old area on a neighborhood unit pattern, and in an identical new development to house the same population on raw land.

The sums that the private developer must spend for land and construction and carrying charges are not considered. Extensive studies by Messrs. Thomas Adams, M. Am. Soc. C. E., and the late Robert Whitten,[3] and Clarence Perry,[4] the President's Conference on Home Building and House Ownership,[5] the National Resources Committee, and others,[6] have approached the problem from planning and housing viewpoints with particular reference to the charges borne by the individual home owner and tenant.

[3] "Neighborhoods of Small Homes," by Robert Whitten and Thomas Adams, Harvard Univ. Press, 1931; and "Design of Residential Areas," by Thomas Adams, Harvard Univ. Press, 1934.

[4] "Housing for the Machine Age," by Clarence Arthur Perry, Russell Sage Foundation, New York, N. Y., 1939; and "Neighborhood and Community Planning," Regional Survey, Vol. VII, Regional Plan of New York and Its Environs, 1929.

[5] "Planning for Residential Districts," Vol. I, and "Slums, Large-Scale Housing and Decentralization," Vol. III, the President's Conference on Home Building and Home Ownership, Washington, D. C., 1932.

[6] "Land, Materials and Labor Costs," *Housing Monograph Series, No. 3*, National Resources Planning Board, Washington, D. C., 1939; see also "Rehousing Urban America," by Henry Wright, Columbia Univ. Press, 1935.

Procedure

A study of this nature is necessarily developed from, and limited by, hypotheses that may not be accepted in their entirety. It is believed, however, that the principles underlying this study furnish a sound approach to a complex problem.

The procedures used in this study involve the selection of the types and geographic location of specimen residential communities; the determination of land uses and building bulk in a specimen community representing old sections, and the allocation of land uses and fixing of building bulk in the "neighborhood units" as the basis for estimating the population that can be accommodated in each type of specimen community; the determination of the character and extent of public services existing in old sections and the needs for public services required to serve the residents of neighborhood units; and estimating the costs to the city for permanent improvements and annual operating expenses in each community.

Character of Development

The several community studies undertaken by the Mayor's Committee on City Planning prior to 1938 supply the most recent information on the character of development existing in the fully developed, high-density apartment districts located in the old central parts of the city. It should be noted that these communities are predominantly residential districts. In addition to dwellings, they have parks, schools, churches, local business, and other community facilities that serve residential needs. There are, however, cemeteries and industries in some of these communities that are not considered desirable within residential districts.

Existing Multi-Family Areas.—The Mayor's Committee Studies designated as East Harlem, East Side, and Yorkville are typical of existing conditions in the older apartment districts of Manhattan. Of their aggregate land area, 35.55% is used for streets on the gridiron system; only 2.7% is not built upon; 3.6% is parks; about 38.5% is residential; and the remainder is made up of business, industry, public buildings, and private institutions (see Table 1).[7] About 95% of the population lives in apartments and more than 87% of the residential land is used for multi-family housing.

The population was reported as 678,446 by the 1934 Real Property Inventory, but the total available family quarters were only 79.6% occupied. With full occupancy of all family quarters, 851,900 persons could be accommodated in the housing existing in these communities in 1934. The land areas expressed as acres per 1,000 persons in Table 1 are based on this estimate of "population provided for in existing housing."

If all apartments were occupied, the 851,900 persons housed on the 1,254.4 residential acres would produce a density of 678 persons per net acre! Allowing 70% coverage of residential land and 242 sq ft of building area per person, apartments would average 5.3 stories in height to house this population. As a rough check on this value for average height, it is only necessary to inspect

[7] Data from East Harlem, East Side, and Yorkville Community Studies of the Mayor's Committee on City Planning (1935).

these communities. One finds that most of the buildings are old "walk-up" flats or brownstone houses converted into apartments. Relatively few "new law" multi-family buildings of more than six stories will be seen and most of them are found on main thoroughfares. In Yorkville, for example, such apart-

TABLE 1.—"BUILT-UP EXPERIENCE" IN CENTRAL APARTMENT COMMUNITIES OF NEW YORK CITY

Land Use	Area		Percentage of:	
	In acres	In acres per 1,000 persons	Gross area	Developed area
Developed Area:				
(a) Net Usable—				
Residential	1,254.4	1.47	38.49	39.56
Business	276.8	0.32	8.50	8.73
Industry	149.8	0.17	4.60	4.72
Institutions	209.4	0.28	6.42	6.61
Parks and Playgrounds	119.9	0.14	3.67	3.78
Cemeteries	1.5	......	0.04	0.05
Total net area	2,011.8	2.36	61.72	63.45
(b) Streets in use	1,158.8	1.36	35.55	36.55
Total developed	3,170.6	3.72	97.27	100.00
Vacant land	89.1	0.10	2.73	......
Gross area	3,259.7	3.82	100.00	......

ments are situated along the north-and-south avenues and the major east-and-west streets.

The 1934 Real Property Inventory shows that about 85% of the buildings in East Harlem were erected prior to 1900, and further, that about 14% of these structures are between twenty and thirty-five years old. More than one fifth of the residential buildings need major repairs and two thirds require minor repairs; only 9% are in first-class condition. The 1935 survey of the Mayor's Committee on City Planning discloses that (1) nearly 23% of the properties in East Harlem were tax delinquent, and (2) that of a total assessed valuation of $263,000,000, land accounted for $125,000,000 or 47% of the total.

New Residential Units.—The diverse and sporadic development of individual properties under built-up experience usually produces the situation reflected in the foregoing examination of existing multi-family areas.

It is believed that urban blight may be combated successfully through large-scale planning and development, both in older, central, deteriorated districts and in new outlying districts. If such large-scale developments are not to become obsolete faster than the individual buildings therein, they must be of a size sufficient to permit the establishment of a self-contained neighborhood that can create a desirable residential environment and can maintain this character by preventing the occurrence of conditions that have caused people to move away from existing residential communities, leaving vacant dwellings and partly used public improvements that must be duplicated elsewhere. These new neighborhoods must also offer the amenities for urban living that will enable them to compete successfully with the newer developments that attract people from older sections of the city to suburban districts.

The Neighborhood Unit.—It is not the purpose of this study to formulate means nor to evaluate procedures suggested by others to promote the development of the newer type of residential neighborhood. Mr. Perry has presented [2] possible approaches to the problems inherent in development of neighborhoods. The neighborhood unit concept, as presented by Mr. Perry, sets the pattern for self-contained communities, and was adopted with minor modifications as the basis for ascertaining the costs of public improvements and services in newer types of residential developments. "The neighborhood unit—a scheme for arrangement for the family-life community" consists of six principles:[8]

(1) *Size.*—A neighborhood unit should provide housing for that population for which one elementary school is ordinarily required, its actual area depending upon its population density.

(2) *Boundaries.*—The unit should be bounded on all sides by arterial streets, sufficiently wide to facilitate its by-passing, instead of penetration, by through traffic.

(3) *Open Spaces.*—A system of small parks and recreation spaces, planned to meet the needs of the particular neighborhood, should be provided.

(4) *Institution Sites.*—Sites for the school and other institutions having service spheres coinciding with the limits of the unit should be suitably grouped about a central point.

(5) *Local Shops.*—One or more shopping districts, adequate for the population to be served, should be laid out in the circumference of the unit, preferably at traffic junctions and adjacent to similar districts of adjoining neighborhoods.

(6) *Internal Street System.*—The unit should be provided with a special street system, each highway being proportioned to its probable traffic load, and the street net as a whole being designed to facilitate circulation within the unit and to discourage its use by through traffic.

In addition to the foregoing principles the neighborhood unit formula as used in this study implies that: (1) Building density should be limited to insure light and air to residents and to prevent unnecessary overcrowding of the land; (2) the unit should be fully developed in order to utilize land, buildings, and public services efficiently; and (3) public and private facilities should be adequate to meet the needs of the people living in such self-contained residential neighborhoods. These principles of the unit scheme can ordinarily be fully applied to new developments only. Consequently, the neighborhood unit principle may be limited to outlying unbuilt areas and to the rebuilding of centrally located deteriorated districts.

Types of Development Studied

This neighborhood unit formula seems to present a sound technique for the development of communities that will avoid the incipient blight and early deterioration experienced in existing residential sections. If this new type of development can establish and maintain a desirable environment for residential

[8] "Neighborhood and Community Planning," Regional Survey, Vol. VII, Regional Plan of New York and Its Environs, 1929.

neighborhoods, stabilize development, and conserve values by helping to prevent unnecessary shifting of population, then the relative costs of public improvements in such neighborhoods are of considerable interest. This paper, therefore, undertakes to examine two types of development—(1) an area characteristic of existing development, and (2) a new neighborhood unit that could be used in rebuilding the old area or for a new development on vacant land.

Existing Area Selected for Study.—The character and extent of development in specimen Area *A*, Fig. 1, is determined by the "built-up experience" exhibited in the three multi-family communities surveyed by the Mayor's Committee on City Planning. (The locations shown in Fig. 1 are not intended to

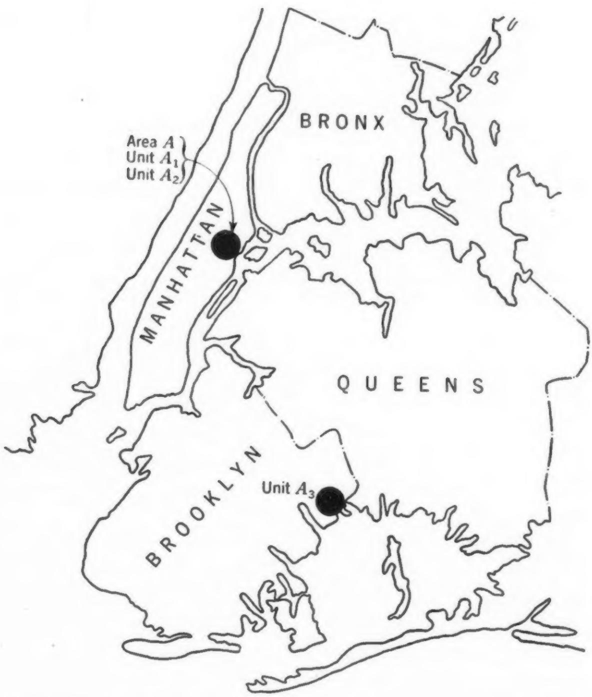

FIG. 1.—APPROXIMATE LOCATION OF THE 160-ACRE COMMUNITIES, NEW YORK, N. Y.

designate specific areas but merely to show the approximate locations.) Area *A* is a fully developed apartment district situated in the central city; it is typical of the "built-up experience" in existing multi-family areas that are developed on a gridiron street plan. Such areas are usually found in the older sections of most large cities.

New Units Selected for Study.—Each of the neighborhood unit types of development selected for study is predicated upon the principles underlying the unit formula and upon the following minimum "standards":

Unit A_1.—This development represents an attempt to apply the neighborhood unit scheme in the rebuilding of Area A on a super block plan, which involves the conversion of the "normal" street system shown in Fig. 2 to the master block plan in Fig. 3. It provides sufficient new apartments to rehouse the present population of Area A, and the necessary private facilities and public services required to serve this population.

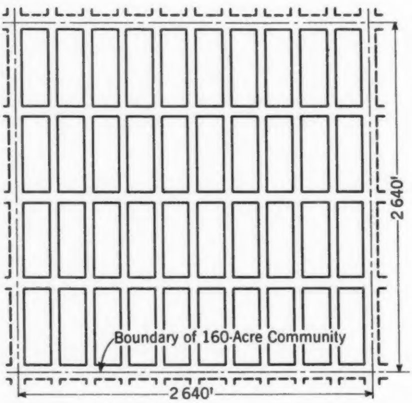

FIG. 2.—"NORMAL" GRIDIRON STREET SYSTEMS (AREA A)

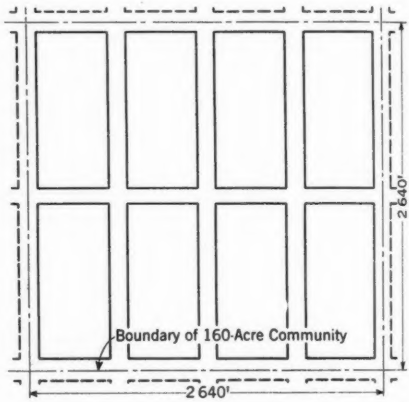

FIG. 3.—MASTER BLOCK STREET SYSTEMS (UNITS A_1 AND A_2)

If the present population of Area A is rehoused on the same site, under minimum standards for attendant facilities, there would be 15.11 acres less land available for dwellings in Unit A_1 than is now used for apartments in Area A (see Table 2). Consequently, the building bulk of the new apartments in Unit A_1 would exceed the bulk of buildings existing in old Area A. At an average height of six stories, apartments in Unit A_1 must cover 68% of the residential land.

Unit A_2.—This development also applies the neighborhood unit formula in the rebuilding of Area A on a master block plan. In Unit A_2, however, apartments averaging six stories cover only 50% of the residential land. The new buildings in Unit A_2 would have less bulk than the apartments and would, therefore, house fewer people than live in Area A.

It is believed that the building bulk resulting from 68% cover, six story apartments, which is required in Unit A_1 to rehouse the present population of Area A on the same site, would make that new unit an obsolete development. The apartments in Unit A_2, covering only 50% of the residential land and averaging six stories in height, provide more open space and permit a better layout which would probably be subject to a much lower rate of obsolescence than the bulkier buildings of Unit A_1.

Unit A_3.—Unit A_3 is a new apartment development on raw land; it is identical to Unit A_2 and houses the same population; but entirely new public facilities are needed in the undeveloped outlying section selected as a site for Unit A_3. This development is predicated on six-story buildings covering 50%

of the residential land. Such a building bulk is not desirable in outlying districts; nor should it be implied that this study so recommends. The purpose of this theoretical development, the same as the other units, is to afford a direct comparison of the results obtained.

Land Area of Speciman Communities.—The specimen area and units studied are considered predominantly residential in character with only such local business, recreational, and cultural facilities within their boundaries as are required for the convenient service of the persons housed in each community. In order to measure public services in the existing area and to ascertain needs in the new units that could be reduced to comparable terms, it was necessary to find a common denominator. Accordingly, each of the communities studied is a type or specimen development predicated on a square containing 160 acres, measured to the center line of boundary streets, in order to secure comparability in one respect—namely, land area. It should be noted that the area of 160 acres, developed with apartments of the character assumed in this study, would house more people than could be efficiently served by one elementary school. Under the foregoing principles of the neighborhood unit, such a development might be served by several elementary schools.

Location of Communities Studied.—There is no particular advantage, however, in examining an area of 160 acres within designated boundary streets. The value of the findings could be easily prejudiced by conditions peculiar to a specific location. It is true that a number of unit costs vary, as between the boroughs of New York City, due to differences in construction practices and in the extent to which certain city services are normally provided, but there is no significant variation within a borough. Approximate locations, within a well-defined section of a borough, furnish ample bases for determining the extent of public development existing in an area typical of that section and the costs of public facilities prevailing in the vicinity. The resulting cost estimates are more representative of average conditions for large sections of the city and, therefore, have broader implications.

Accordingly, Area A (see Fig. 1) is located within the East Harlem section of Manhattan; Units A_1 and A_2, the two schemes for rebuilding multi-family Area A, of course, are on the same site as Area A in East Harlem; and Unit A_3 is located on vacant land, adjacent to Jamaica Bay in Brooklyn, N. Y.

Land Use, Building Development, and Population

The area of land used for various purposes and the bulk of buildings determine the number of persons that can be accommodated in these communities. The population of the specimen communities must be ascertained before the extent and cost of public facilities can be investigated.

Land Use.—Two major factors are involved in allocating the 160 acres of these typical communities to street, residence, institution, park, and business uses—namely, area and population. The land required for street purposes is determined by the street plan, and in the old area both vacant land and streets are based on the extent of existing development. The land area needed for residence, parks, institutions, and business, however, is dependent upon the number of persons to be housed in a given community; hence, the extent of these

land uses is predicated upon the service requirements of the community's population.

The "Area" Factor.—As shown in Figs. 2 and 3, the communities studied are developed on the two types of street systems: (1) The "normal" gridiron street plan; and (2) a master block arrangement.

Gridiron Street Plan.—The usual gridiron street plan existing in New York City is not directly adaptable to a theoretical community of 160 acres, in the form of a square measuring one-half mile on a side. Minor adjustments in the occurrence of the wider streets and avenues in relation to the local streets of standard width are, however, satisfactory for the purposes of this study. The gridiron street system was standardized in a "normal" pattern to permit direct comparisons between communities (see Fig. 2). The amount of land in Area A that is allocated to the gridiron street plan on this "normal" pattern is based upon the built-up experience of existing multi-family areas (see Tables 1 and 2). The boundary streets of Area A average 100 ft in width, and the "normal" gridiron system requires nearly 57 acres, or about 35% of the 160-acre gross area. Each block measures 575 ft by 200 ft.

TABLE 2.—LAND USE ALLOCATION ON EACH 160 ACRES IN RESIDENTIAL NEIGHBORHOODS

(*a*) GENERAL DATA AND POPULATION

Description	Area A	Unit A_1	Unit A_2	Unit A_3
Types of residence	Old multi-family	Rebuild Area A	Rebuild Area A	New multi-family
Percentage covered[a]	....	68	50	50
Borough	Manhattan	Manhattan	Manhattan	Brooklyn
Street system	Fig. 2	Fig. 3	Fig. 3	Fig. 3
Population provided for	46,000	38,550	33,270	33,270
Persons per:				
Residential acre	678	731	540	540
Gross acre	287	241	208	208
Persons actually housed[b]	36,620	36,620	31,600	31,600

(*b*) AREA ALLOCATION

Description	Area A Acres	Area A % of gross	Unit A_1 Acres	Unit A_1 % of gross	Unit A_2 Acres	Unit A_2 % of gross	Unit A_3 Acres	Unit A_3 % of gross
Streets in use	56.88	35.55	42.08	26.30	42.08	26.30	42.08	26.30
Parks and playgrounds	4.79	2.99	22.05	13.78	19.02	11.88	19.02	11.88
Institutions	11.25	7.03	21.00	13.12	18.14	11.32	18.14	11.32
Business	14.88	9.30	22.15	13.86	19.15	11.96	19.15	11.96
Residence	67.83	42.40	52.72	32.94	61.61	38.54	61.61	38.54
Vacant	4.37	2.73	....	....	....	....	....	....
Total	160.00	100.00	160.00	100.00	160.00	100.00	160.00	100.00

[a] Percentage of residential land covered by apartments. [b] Persons housed after allowing for vacancies.

Master Block Plan.—The master block arrangement indicated by Fig. 3 is used for Unit A_1, Unit A_2, and Unit A_3. This development plan of "super" blocks is designed to limit "through" traffic to boundary and major thoroughfares. Boundary streets of the 160-acre tract are each 140 ft wide, including a local service road. Each master block (1,200 ft by 535 ft) is surrounded by a

service roadway separated by a safety isle or planting strip from the wider through-traffic streets bounding the block. A single master block is equivalent to five ordinary blocks in the usual gridiron system, including the area devoted to four minor cross streets in the gridiron system but excluding the area needed for service roadways. The theoretical 160-acre community has eight such master blocks in two rows of four each. They are separated by three north-and-south streets, each 120 ft wide with service road, and by one east-and-west street, 100 ft wide, running through the center (see Fig. 4).

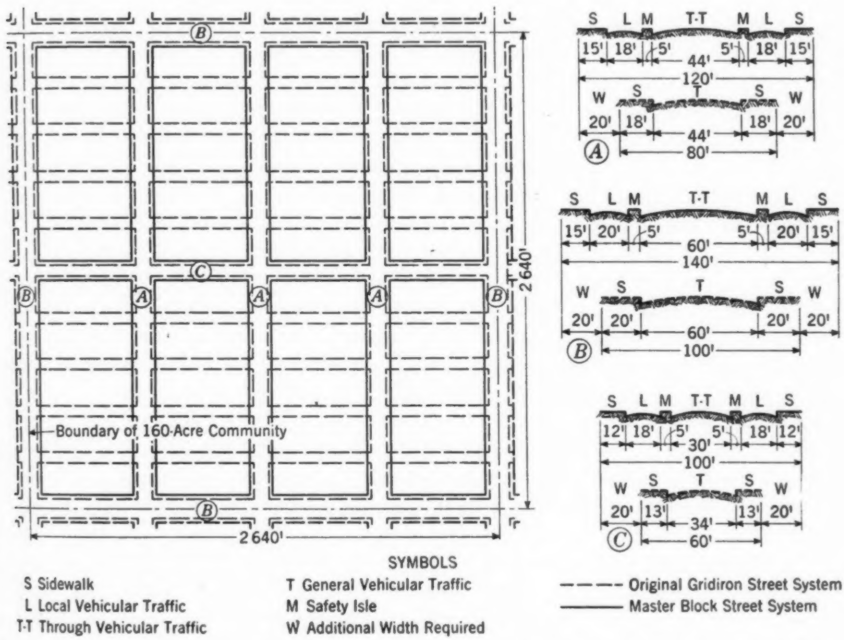

Fig. 4.—Conversion of "Normal" Gridiron Street System of Area *A* to Master Block Layout for Units A_1 and A_2

This master block plan is applicable in the rebuilding of most of the old sections that are now developed on a "normal" gridiron street system and can be easily used for any neighborhood to be built on a vacant and relatively flat site. In this design, streets account for about 42 acres, or 26.3% of the 160-acre gross area.

"Population" Factors and Standards.—With the street pattern fixed, the next step in the determination of land use involved the allocation of the remaining acreage to the parks, institutions, business, and residential uses needed by the community's inhabitants.

Reference to Table 1 will recall that there are cemeteries and industries in the large communities surveyed by the Mayor's Committee on City Planning. Within these larger communities, however, there are many sections of 160 acres that do not contain either a cemetery or any significant industrial uses. For

the purpose of this study, therefore, such uses are excluded from all of the communities.

Uses in Terms of Population.—The extent of public services and private facilities needed in the neighborhood units is necessarily determined by the total number of people that can be housed in each community. In the case of an "old" development, it is reasonable to express land use and service needs in terms of the population that could live in the area if all available dwellings were occupied. Otherwise, any measurement of adequacy is based on a population figure which fluctuates with the changing occupancy of dwellings, and a facility deemed adequate for 80% occupancy is found insufficient when 90% of the dwelling units are occupied. In this study, therefore, any land use or facility depending upon the population factor is predicated on 100% occupancy of available family quarters. Real Property Inventory reports show that few of the "old" residential districts are fully occupied, but the newer and more modern housing developments usually have a "waiting list" of families that desire to move in as soon as a vacancy occurs.

It is believed that some of the buildings have become entirely vacant since 1934 and are boarded up or demolished; but subsequent new construction and alterations have probably added some family quarters in the existing multi-family areas. However, there are no data available to indicate the exact number of family quarters now existing in Area *A*. Consequently, the conditions revealed by the 1934 Real Property Inventory are used to estimate the population resulting from 100% occupancy of family quarters in Area *A*.

Built-Up Experience in Area A.—The analysis of selected existing multi-family areas, surveyed by the Mayor's Committee on City Planning, reveals that there would be 678 persons per residential acre if all family quarters were occupied. In Area *A*, the 678 persons per net acre are housed in apartments averaging 5.3 stories in height and covering 70% of the residential land.

Full occupancy of the family quarters provided in Area *A* determines the total population and extent of land used for residence, parks, institutions, and local business. Table 2 shows the number of acres and the proportion of gross land area used for each major purpose as well as the population provided for under built-up experience in the 160-acre specimen community, Area *A*. The acres per 1,000 persons of park, institution, and local business found in Area *A* are given in Table 1.

"Minimum Standards" in Units.—The extent of land required for each major purpose in the neighborhood units is predicated on the unit formula and the minimum standards for parks, institutions, local business, and housing, as follows:

Park and Playground.—Standards believed feasible in New York City, which were developed by the Mayor's Committee on City Planning, call for a minimum of one acre of park and playground per 1,750 persons. This allows for a playground, a small quiet park, and active recreation areas but excludes large parks.

Institutions.—Schools, fire stations, libraries, and other public and private buildings of a local character, as well as churches, are included under "Institutions." Because of the varying spheres of influence of such services, some

of which do not coincide with the 160-acre tract under consideration, the land allocated to institutions in each unit is based upon the average built-up experience of eleven residential communities studied by the Mayor's Committee on City Planning. This average is just more than one half of an acre per 1,000 persons, and is believed to be generally adequate.

Business.—Local business includes retail shops, garages, gas stations, laundries, cleaners, barbers, theaters, banks, amusements, local offices, and similar services that would be needed by the people of a self-contained residential community.

In this study the business standard is fixed at 50 ft per 100 people. With lots averaging 100 ft deep this is equivalent to one acre of business per 870 population in each neighborhood unit. Table 1 shows that about 0.3 acre per 1,000 people is used for commercial structures in the existing multi-family areas of the city. The remaining business frontage usually required to serve such communities is found on the ground floor of apartment buildings. Consequently, in Units A_1 and A_2 (the alternative schemes for rebuilding Area A), and in Unit A_3, it is assumed that one half of the required business frontage is on the ground floor of multi-family buildings and the remainder in commercial structures.

Residential Requirements.—In rebuilding Area A to rehouse the present population of 36,620 persons, provision is made in Unit A_1 for a normal vacancy of 5%. On this basis the new apartments must accommodate 38,550 persons. Allowing 242 sq ft of gross floor area per person, and buildings averaging six stories high, these apartments would cover approximately 68% of the net residential land. If land coverage is held to 50%, the buildings would average 8.15 stories in height. In either case there would be 731 persons per net residential acre compared with a potential density of 678 in Area A. This greater density in Unit A_1 results from the relatively larger land areas that are devoted to park, institutional, and business purposes under minimum standards, which reduce the area of land available for residence (compare Area A and Unit A_1 in Table 2). Although these other facilities may be considered adequate, the building bulk of apartments in Unit A_1 is believed to exceed a reasonable standard in overcrowding the land and is therefore considered obsolete.

It is assumed that the housing bulk resulting from 50% land coverage and six-story apartment buildings is the maximum development desirable under reasonable standards. At 242 sq ft of gross floor area per person, these apartments would accommodate 540 people on each residential acre. When properly designed, it is believed that such a development could offer sufficient amenities to maintain a desirable residential character despite the high density. Accordingly, this average building bulk is the housing "standard" in rebuilt Unit A_2 and in new Unit A_3.

Application of Factors and Standards.—Using the aforementioned factors and "standards," the requirements of the population for residence, park, institution, and business were determined for each of the neighborhood units. The resulting population and the land area allocated to these major uses are

given in Table 2. Inasmuch as the neighborhood units are fully developed, there is no vacant land.

Public Improvements and Services

This investigation attempts to ascertain the nature and extent of public improvements and services now provided in Area *A*. The land uses and degree of development in Area *A* are derived from the built-up experience of the larger communities of which this theoretical 160-acre area is a representative sample. Likewise, the character and extent of public improvements and services in Area *A* are determined from the built-up experience in the larger residential community of East Harlem. The average conditions revealed by public records to exist in East Harlem, when applied to Area *A*, indicate the public facilities now serving this theoretical 160-acre community.

The public services required to serve the neighborhood units adequately are determined on the basis of "minimum service standards," which are discussed herein. Practically all of the public facilities required to serve Unit A_3, which is situated on vacant land, must be entirely new improvements.

In the rebuilding of Area *A* on a master block plan, it is also necessary for the city to provide the adequate and up-to-date facilities needed under "minimum service standards" by the population rehoused in each of the alternative neighborhood schemes, Units A_1 and A_2. Therefore, it is assumed that: (1) Any public facility existing in Area *A* that is not adequate as measured by minimum service standards must be augmented in Units A_1 and A_2 to the full extent of the indicated deficiency; and (2) all existing public services in Area *A* are utilized in so far as needed, but those retained must be replaced in Units A_1 and A_2 to the full extent of the indicated obsolescence.

Improvements and Services Studied.—Only those permanent improvements and facilities supplied by the city that may be directly charged to a local community on the basis of services provided are included in this study. The needs for water supply reservoirs, courts, jails, hospitals, museum buildings, and other general services are not determined by the type of residential development, and they are influenced only indirectly by the geographic location of residential communities. In comparison with local neighborhood facilities, such improvements are city-wide services for which the needs are determined by the entire population and area of the city or a borough. In this study the public services discussed are considered essential to the proper functioning of self-contained residential communities.

Street Utilities.—Sidewalks, curbs, street paving, sewers, water mains, and street lighting are termed "street utilities" in this study. The installation of these utilities in New York City depends almost entirely upon the type of street plan and the various construction practices in each borough of the city rather than upon the number of people to be served.

The borough standards, quantities, and unit costs of street utilities are given in the Appendix. The "minimum service standards" for street utilities in the neighborhood units coincide with current practice in the boroughs, but the quantities needed, or course, are determined by the master block layout.

Population Facilities.—Population facilities include parks and playgrounds, elementary and high schools, branch libraries, district health buildings, fire and police stations, sanitation services, sewage and refuse disposal, and rapid transit. With the exception of rapid transit, which is located in public streets, population facilities are built upon a part of the "developed area" that is classified as "Institutions" in Tables 1 and 2.

The extent to which these public improvements and services are provided in Area *A* and are needed in the neighborhood units is determined by the population to be served. This does not imply that all such services existing in Area *A* are adequate to meet the needs of modern urban living, but rather that the basic requirements for these facilities are not dependent upon the land or "area factor" alone. For example, schools are provided to accommodate a given number of children, and the "area served" depends upon the size of the school and the density of population in the surrounding development. Other parallels can be drawn but the relationship between population and schools illustrates the general nature of "population facilities."

Practically all of these population facilities have been provided in the larger residential communities surveyed by the Mayor's Committee on City Planning. In general, small parks, schools, and sanitation services are entirely local in character, but the other population facilities may have sufficient capacity to take care of a greater number of people than are accommodated in a typical 160-acre community. Existing facilities with an "area of service" greater than 160 acres may not be located within the boundaries of a 160-acre district, and the extent of service available to each community must be based, therefore, on borough or city-wide measurements in relation to the needs of the population of each of the theoretical communities.

The public improvements and services in the neighborhood units that are dependent upon the population factor are identical to the population facilities supplied to Area *A*, and the same procedures are used to determine the extent of each service needed. The "minimum service standards" for parks and playgrounds, elementary schools, libraries, refuse disposal, and rapid transit service in the neighborhood units differ from current borough practices. City standards and departmental programs for the remaining population facilities are considered adequate for the units. The methods of determining needs, and the "standards" and charges against each community for population facilities, are outlined in the Appendix.

Obsolescence in Area *A*

In rebuilding Area *A* under the "standards" adopted for this study, it would be necessary to expand existing public services that are inadequate and to replace any depreciated facility to the full extent of indicated obsolescence.

Street Utilities.—Fig. 4 suggests the extent of alteration and rearrangement of "street utilities" involved in the change from the normal gridiron street system of Area *A* to the master block plan of Units A_1 and A_2. The changes, quantities, and costs involved are given in the Appendix.

Population Facilities.—Many of the existing population facilities serving Area *A* must be replaced in whole or in part when this old district is rebuilt

on the master block plan of Units A_1 and A_2. The obsolescence of population facilities serving Area *A* generally is predicated on the present age and useful life of these improvements.

The present age of facilities is obtained from public records, with the exception of libraries, fire stations, police precincts, and sanitation garages and section houses. It was assumed that the age of these facilities is equal to the average age of existing school buildings in East Harlem. The first school in this section, still standing in 1941, was built in 1867, but the average age of all schools in East Harlem is forty-three years. It seems reasonable to suppose that the average age of the aforementioned facilities in East Harlem will approximate the average age of schools.

The useful life of most population facilities is derived from the replacement experience of the appropriate public agencies. Whenever a facility reaches the limit of its useful life, replacement is assumed necessary for the satisfactory continuance of such service. The Appendix contains detailed information on the present obsolescence of public services in Area *A*.

Improvement Costs and Annual Expenses

The reported costs of existing improvements in old Area *A* are used when available. In other instances the estimates are derived from current contract prices reported by city agencies. The cost estimates for improvements in neighborhood units are based upon current practice and contract costs in the appropriate borough, or upon the estimated costs of proposed improvements. Land acquisition is estimated at 1.3 times the average assessed value per acre in each community, but all street land is assumed to be dedicated without cost to the city. The cost estimates for each kind of improvement or service are summarized in Table 3(*a*) by community.

The unit costs used in the estimates of annual expenses are derived from the most recent information available from the operating agencies. The per capita expenses, where used in this investigation, are generally obtained from reported total annual expense and the estimated population actually living in the city. The "actual" population living in each community, rather than the "demand" or total population that could be accommodated with 100% occupancy of dwelling units, is used, therefore, in determining annual expenses that are based on per capita costs.

The estimates of annual expenses indicate the expenditures of the city for operation and maintenance of public services provided in each community. No attempt is made to set up a "balance sheet" of annual expenses and of city revenue derived from a community. The estimates do not in any sense represent the amounts of taxes levied or collected in each community. Annual expenses in any of these residential areas may exceed the taxes paid to the city by the particular community, but the expenses must be met from general city revenues in any event. The estimated yearly expenses charged to each community are summarized in Table 3(*b*), and the procedures in this work are outlined in the Appendix.

Meaning of Estimates.—The estimated costs of permanent improvements represent the present investment of the city in the public facilities now found

in Area *A* without any allowance for depreciation or for replacement of the original improvements. The estimates of annual expenses represent the costs to the city for operation and maintenance of existing public services, some at less than capacity because of the relatively low occupancy ratio in dwellings of Area *A*.

TABLE 3.—ESTIMATED TOTAL COSTS TO THE CITY

No.	Description	Area *A*	Unit A_1	Unit A_2	Unit A_3
	(*a*) FOR PERMANENT IMPROVEMENTS				
1	Sidewalk and curb	$ 341,730	$ 145,960	$ 145,960	$ 72,872
2	Street paving	467,626	334,740	334,740	373,146
3	Sewers, etc.	1,070,700	324,450	324,450	310,220
4	Water mains, etc.	151,800	43,660	43,660	133,420
5	Street lighting	....	23,410	23,410	23,410
6	Parks and playgrounds	1,767,550	1,045,850	133,140	383,140
7	Elementary schools	1,740,000	1,672,000	1,365,000	1,936,000
8	High school	2,210,000	450,000	388,500	1,597,500
9	Public library	150,900	69,000	59,500	84,250
10	Health building	50,400	....	....	76,600
11	Fire station, etc.	196,600	75,200	65,900	84,100
12	Police precinct	104,250	69,100	39,400	41,600
13	Sanitation	101,890	61,200	52,800	47,676
14	Sewage disposal	987,000	....	....	431,000
15	Refuse disposal	70,000	28,650	21,650	65,000
16	Rapid transit	1,655,000	1,626,800	783,400	7,080,600
17	Total	$11,065,446	$5,970,020	$3,781,510	$12,740,534
	(*b*) FOR ANNUAL OPERATION AND MAINTENANCE EXPENSES				
18	Street cleaning	$ 96,430	$ 77,063	$ 77,063	$ 56,826
19	Sewer maintenance	2,063	905	905	874
20	Water mains	6,188	2,652	2,652	2,694
21	Street lighting	6,200	1,468	1,468	1,468
22	Parks and playgrounds	2,520	11,575	9,986	9,986
23	Elementary schools	559,000	470,000	405,700	405,700
24	High school	292,560	245,100	211,630	211,630
25	Public library	31,750	26,985	23,275	23,275
26	Health building	2,820	2,150	1,860	2,160
27	Fire protection	153,500	153,500	132,400	99,860
28	Police protection	252,750	252,680	218,000	101,760
29	Refuse removal	79,758	79,758	68,825	63,895
30	Sewage disposal	23,550	19,720	17,050	17,730
31	Refuse disposal	19,987	18,392	15,872	13,248
32	Debt service on rapid transit	80,900	160,400	119,200	346,000
33	Total annual expenses	$ 1,609,976	$1,522,348	$1,305,886	$ 1,357,106

The costs of public improvements in the neighborhood Unit A_3 represent the investment that the city must make to provide new public services that are adequate under minimum service standards. The estimated costs of permanent improvements in the rebuilt communities, Units A_1 and A_2, represent the city expenditures necessary to make existing deficient services adequate and to replace the obsolete part of the present plant that is retained in rebuilding the old apartment Area *A*. The estimates of annual expenses indicate the costs to the city for operation and maintenance of public services in the neighborhood units with 95% of the available housing occupied.

Total and Per Capita Costs.—Rapid transit is not the same kind of local improvement as elementary schools and similar neighborhood facilities. However, transit has exerted a tremendous influence upon the direction and extent of residential development. Since 1900 about 85% of New York's population has lived within one-half mile of rapid transit lines. Urban life depends upon mobility of people as well as goods. Rapid transit, therefore, is considered essential to the life of residential communities in New York City. Rather than permit the transit factor to complicate these findings, the estimated costs will be examined, first with transit charges and second excluding transit charges.

The findings will be considered on the basis of "costs per person housed" in each area. The "costs per person housed" are based upon the actual population living in each area after allowing for vacancies.

With Rapid Transit Charges.—Total improvement costs in new Unit A_3 are 1.15 times the costs in old Area A, and the costs per person housed in Unit A_3 are 1.33 times the per capita costs in Area A (see Table 4(a)).

TABLE 4.—Estimated Total and per Capita Costs

Area and units	(a) With Rapid Transit				(b) Without Rapid Transit			
	Improvement Costs		Annual Expenses		Improvement Costs		Annual Expenses	
	Total	Per person	Total	Per person	Total	Per person	Total	Per person
A	\$11,065,446	\$302.16	\$1,609,976	\$43.96	\$9,410,446	\$256.97	\$1,529,076	\$41.75
A_1	5,970,020	163.02	1,522,348	41.57	4,343,220	118.60	1,361,948	37.19
A_2	3,781,510	119.66	1,305,886	41.32	2,998,110	94.87	1,186,686	37.55
A_3	12,740,534	403.18	1,357,106	42.94	5,659,934	179.11	1,011,106	31.99

Per capita improvement costs for new Unit A_3 are much larger than the costs involved in rebuilding old Area A; costs per person housed in Unit A_3 are 3.3 times the cost in rebuilt Unit A_2 and 2.4 times the per capita costs in rebuilt Unit A_1. This is significant because the rebuilt Units A_1 and A_2 provide additional services to meet present deficiencies of old Area A, and because the deteriorated parts of existing facilities are replaced to the full extent of the indicated obsolescence. It is also interesting to note that the city needs to spend less than \$120 per person to house a population of 31,600 in Unit A_2, compared to \$163 per person to house 36,620 people in Unit A_1.

Annual expenses per person housed do not vary as much between the several communities as improvement costs. The range is from \$41.32 in Unit A_2 to nearly \$44 in Area A. The higher total and per capita expenses in Area A suggest that public services in deteriorated old sections are not utilized efficiently. The costs per person housed in new Unit A_3 are \$42.94, as against \$43.96 in Area A and \$41.57 in rebuilt Unit A_1 (see Table 4(a)).

Excluding Transit Charges.—If rapid transit should become an entirely self-sustaining service, the costs to the city for improvements in residential developments would be reduced accordingly. The exclusion of transit charges reduces the remaining improvement costs in new Unit A_3 to about 60% of

those in old Area A; but the costs in Unit A_3 are 1.3 times the costs in rebuilt Unit A_1 and nearly 1.9 times the costs for improvements in Unit A_2 (see Table 4(*b*)).

In terms of persons housed, the costs are lower in the rebuilt units than in the new development. In new Unit A_3 the improvement costs per person housed are 1.5 times the costs in rebuilt Unit A_1, and about 1.9 times more per person than need be spent in rehousing an equivalent population in rebuilt Unit A_2.

Annual operating expenses are lower in new Unit A_3 than in other communities when transit charges are deducted. Per capita service expenses are generally higher in Manhattan, and in many instances a larger measure of service is provided than in Brooklyn. The expenses per person housed in new Unit A_3 are about 75% of the expenses in Area A, and approximately 83% of expenses in Units A_1 and A_2.

Rehabilitation Versus Decentralization

The surprisingly low per capita cost for the rebuilt Unit A_2 is particularly interesting. The conditions existing in Area A are indicative of the deficiencies of older residential districts in central parts of the city. The average age of this development exceeds forty years; some of the public facilities and private structures are more than seventy years old.

Certain public services are inadequate as measured by the standards adopted for this study, and many existing public improvements are nearing a point of deterioration that will shortly necessitate their replacement. A variety of circumstances have contributed to the high vacancy in apartments, among others being: (1) The heavy vehicular traffic passing through the district on local streets; (2) an influx of commercial and industrial activities into residential blocks; (3) too great a density of buildings and people on the land; and (4) a hesitancy of owners to modernize old housing. Private owners do not wish to increase their investment in old buildings because of the uncertain outlook for individual properties and the hope of putting the land to a more intensive use. This hope has not materialized, but modernization is difficult because many properties have not been amortized over the years of productive use. These and other circumstances make Area A an unstable district, and the lack of bold correctives further contributes to this unhealthy condition. It is only natural that people should move from districts like Area A whenever they can afford to locate in newer, outlying sections that offer a more desirable residential environment.

When population shifts from older districts to newer developments, the city must provide additional street utilities and population facilities in the new sections to meet the resulting demands. There are enough people left in the older districts, however, to require the maintenance of public services. Consequently, the city is in reality forced to duplicate existing facilities in the newer developments and at the same time to operate the old plant below capacity.

The older residential districts cannot be abandoned entirely and left to stagnate; nor are the people willing to remain in these undesirable neighborhoods so long as modern private developments and cheap transportation

enable them to live more pleasantly in outlying communities. What can be done to remedy this apparent impasse? Under present tax structures the city cannot continue this inefficient and expensive process unless it is to face bankruptcy or to experience a far greater population growth than the more optimistic prognosticators now foresee.

This investigation suggests that, in so far as city expenditures are concerned, it would be economical and desirable to rehabilitate deteriorated residential districts completely on the neighborhood unit scheme. A brief examination of the current obsolescence of public services will reveal the extent of investments to be salvaged in this process and the cost of rehabilitation as against decentralization.

Public Facilities in Area A.—Table 3 shows that the costs to the city, for modernization of existing public facilities in Area A on the master block scheme of the neighborhood Unit A_1, amount to about $6,000,000, or $163 per person housed. This value does not indicate the exact obsolescence of existing public facilities because of the changes in street utilities necessitated by the conversion of the present gridiron street system of Area A into the master block plan. The extension of population facilities required by minimum service standards also calls for a greater expenditure than is necessary merely to replace the obsolete part of the existing plant. A calculation of the costs of replacing obsolete public facilities in Area A, without changing the present gridiron street plan, shows that a total outlay of about $4,900,000 would be needed. Of this total, replacing obsolete street utilities would require $850,000 and merely replacing obsolete population facilities would involve expenditures of more than $4,000,000. The replacing of obsolete facilities without altering the gridiron street plan of Area A, or supplying adequate parks, is herein termed operation "Re-Area A."

Assuming that the $11,065,446 cost to the city for improvements now existing in Area A is the original public investment, the estimated expense of replacing deteriorated parts is 44.3% of this original cost. In other words, it appears that public facilities in Area A are already 44% obsolete and must be replaced shortly in order to maintain city services for the 36,620 persons living in this community. This of course excludes replacing such new improvements as the sewage disposal plant and a local health building.

Replacing Facilities with Adequate Parks.—When the city does replace obsolete facilities in old Area A, it would be preferable to provide the additional park acreage required under minimum service standards to serve the present population. The Appendix shows that the transformation of the gridiron street system into the master block plan makes it possible to obtain new park acreage by utilizing the land available from interior streets, which are discontinued. In the case of Unit A_1, the required total park acreage exceeds the sum of existing park area plus leftover street area by only 2.5 acres. On this basis the city needs to purchase 2.5 acres of additional land to meet the park requirements of Unit A_1.

By rehabilitating public improvements in old Area A without changing the existing gridiron street system to the master block plan, the city would not be able to discontinue any local streets. As a result, it would be necessary to

purchase more than 17 acres of land which, with the 4.8 acres now in parks, would provide the 22 acres of parks and playgrounds needed to serve, adequately, the present population of old Area A. This operation can be designated as "New Area A."

With land acquisition at 1.3 times the average assessed value and development costs at $7,000 per acre, parks cost more than $6,400,000 in this rehabilitation operation for New Area A under minimum standards. The replacement of obsolete street utilities and population facilities, other than parks, requires the same expenditure as in Re-Area A—namely, about $4,900,000. The total cost of up-to-date public facilities in New Area A on the old gridiron street plan exceeds $11,300,000, or $308 per person housed.

Cost Ratios Per Person.—It is significant to find that the rebuilt Unit A_2 is less costly in respect to public services than "Re-Area A," "New Area A," or the new Unit A_3. Because of this finding the per person cost ratios in Table 5 are based on the cost in Unit A_2 as 100.

TABLE 5.—CITY EXPENDITURES PER PERSON

(With Transit Charges)

Community	Improvement Costs		Annual Expenses	
	Per person	Ratio	Per person	Ratio
Old Area A[a]	$ 302	252	$43.96	106
Re-Area A[a]	134	112	42.15	102
New Area A[a]	308	258	42.30	102
Unit A_1[a]	163	136	41.57	101
Unit A_2[b]	119	100	41.32	100
Unit A_3[b]	403	337	42.94	104

[a] Population housed is 36,620 persons.
[b] Population housed is 31,600 persons.

The replacement of obsolete public facilities, without altering the gridiron street plan of Area A, would cost the city 12% more with present park acreage and nearly 2.6 times more with adequate park area than the rebuilding of old Area A on the master block scheme of Unit A_2. The rebuilding of Area A on the master block plan of Unit A_1 would cost only slightly more than merely replacing obsolete facilities for Re-Area A, but less than for New Area A.

The replacing of obsolete facilities for Re-Area A and New Area A would not, however, produce the social and economic results believed possible with the neighborhood unit communities on a master block plan. Such replacement of public facilities would tend to "freeze" the present character of private development on the unsatisfactory gridiron street plan, and to postpone the necessarily bold public and private corrective measures for at least another fifty or seventy-five years. By that time it is probable that the social and economic conditions of this community will become quite chaotic, unless present trends could somehow be arrested.

The advantages of the neighborhood unit schemes, having already been enumerated, will not be repeated. It is surprising to find, however, that such sound types of redevelopment as the neighborhood Units A_1 and A_2 also have cost advantages over the replacement of obsolete facilities, both for permanent improvements and annual operation. The entirely new neighborhood Unit A_3 is much more expensive than the several types of rehabilitated communities. When one recalls that, to a considerable extent, the public facilities in this

new unit would duplicate city services now provided in other sections of the city that are already built up, the apparently favorable ratio for operation in Unit A_3 is reduced accordingly.

These findings are not entirely conclusive, but they indicate that large-scale rehabilitation of old central apartment districts is economically feasible and preferable to new developments in outlying sections in so far as city expenditures are concerned. When one considers that all residents pay local taxes, which are obviously increased through duplication of city services, the case of rehabilitation versus decentralization becomes a question of public concern. It would seem, therefore, that essential studies of the private costs involved, and of the procedures to be used in large-scale rehabilitation of entire neighborhoods, should be undertaken immediately under a joint private and public sponsorship.

ACKNOWLEDGMENTS

The original investigations, from which this paper is drawn, were made during 1939 for the Department of City Planning of the City of New York to ascertain the relative costs of public services in various types of residential developments. Commissioner R. G. Tugwell (who heads the Department and is Chairman of the City Planning Commission) has been kind enough to release the material for this paper prior to publication of the full report on the original study.

APPENDIX

The extent of public services existing in Area A, the standards used to determine the needs for improvements in the neighborhood units, the obsolescence of facilities now provided in Area A, and the cost estimates are outlined herein.

The reported costs of existing improvements in old Area A are used when available. In other instances the estimates are derived from current contract prices reported by city agencies. The cost estimates for improvements in neighborhood units are based upon current practice and contract costs in the appropriate borough, or upon the estimated costs of proposed improvements. Land acquisition is estimated at 1.3 times the average assessed value per acre in each community, but all street land is assumed to be dedicated without cost to the city.

Street Utilities.—The quality of street utilities is fixed by construction standards in the various boroughs, and the quantity is determined by the street plan and character of development. The quantities and unit costs (based on recently reported contract prices) are summarized in Table 6.

Sidewalks and Curbs (Item No. 1, Table 6).—Sidewalk standards in Manhattan call for 7 in. of cinders, 4 in. of concrete, and 1 in. of cement with steel-faced concrete curbs. The Brooklyn standard is 6 in. of cinders and 5 in. of concrete. Straight curbs are concrete in Brooklyn, but granite is used for corners and 3 ft beyond the corner curve.

Street Paving (Item No. 2, Table 6).—One half the cost of paving the through traffic roadway of boundary streets is charged to each community; but the service roadway adjacent to boundary streets in the master block plan, as well as other paving within the community, is charged in full to the neighborhood units.

TABLE 6.—STREET UTILITIES

No.	Item	Area A		Units A_1 and A_2		Unit A_3	
		Quantity	Unit cost	Quantity	Unit cost	Quantity	Unit cost
1	Sidewalk (sq ft)	962,192	$ 0.20	409,040	$ 0.20	282,040	$ 0.20
	Curb (lin ft)	66,352	2.25	28,512	2.25	27,712	0.50
	Curb (lin ft)	......	......	......	......	192	2.50
	Curb (lin ft)	......	......	......	......	608	3.50
2	Street paving (sq yd)	34,800	3.50	8,081	3.50	80,800	2.55
	Street paving (sq yd)	122,200	2.83	61,524	2.83	61,478	2.20
	Street paving (lin ft)[a]	......	......	58,592[a]	2.25	57,696[a]	0.50
	Street paving (lin ft)[a]	......	......	......	......	384[a]	2.50
	Street paving (lin ft)[a]	......	......	......	......	512[a]	4.00
3	Sewers (lin ft)	35,690	30.00	15,450[b]	30.00	29,110	5.50
4	Water mains (lin ft)—20 in.	2,640	9.00	2,640[c]	9.00	2,640	9.00
	Water mains (lin ft)—12 in.	5,280	5.00	5,280[c]	5.00	5,280	5.00
	Water mains (lin ft)— 8 in.	29,040	3.50	7,920[d]	3.50	23,760	3.50
5	Street lighting—units	......	......	176	132.50	176	132.50

[a] Curbs for safety islands in master block layout. [b] 70% of length shown obsolete and replaced. [c] 50% of length shown obsolete and replaced. [d] 66% of length shown obsolete and replaced.

Paving standards are as follows: In Manhattan heavily traveled streets have 3 in. of asphalt on a 9-in. concrete base; other streets are paved with 3 in. of asphalt on a 6-in. concrete base. In Brooklyn heavily traveled streets have an 8-in. concrete base; otherwise the practice is the same as that for Manhattan.

In changing Area A to the master block layout, new sidewalks and curbs are required for all of the local traffic streets and new curbs for one side of the safety islands. Service roadways will have to be newly paved to the full widths and lengths indicated in Fig. 4. It is assumed that the surfaces of existing roadways retained in the master block scheme are in good condition but that curb replacement in connection with safety islands requires repaving of 10% of the road surface retained in the master block plan. Any other incidental repaving is considered a maintenance item rather than a permanent improvement.

Sewers (Item No. 3, Table 6).—There are no trunk sewers in the Manhattan communities, Area A and Units A_1 and A_2; but trunk sewer costs for Unit A_3 are estimated on a flat rate of 3 cents per sq ft of "buildable" area. Catch basins, inlets, and manholes are included in the costs since sewers serve as combination storm-water and sanitary drains. One half of the cost of sewers in boundary streets is charged against each community.

Standards in Manhattan call for sewers averaging 4 ft by 2.33 ft. In Brooklyn the average is 18 in., but, due to the long blocks, sewers in Unit A_3 are assumed to average 24 in. and are placed in service roadways.

The average age of sewers in Area A is seventy years and their useful life one hundred years, so that 70% of the sewers in existing streets that are retained in the public streets of the master block plan require replacement in either Unit A_1 or A_2. New surface drains are also required for the service roadways in the master block schemes.

Water Mains (Item No. 4, Table 6).—The costs of water mains include valves, fire hydrants, and manholes but exclude all house connections, which are made at the expense of the property owner. One half the cost of water pipes in the boundary streets is charged to each community, except for Unit A_3, where current practice would require the original installation of water mains in each of the service roadways of the master block plan, rather than a single pipe in the bed of boundary and interior streets.

According to records of the New York Department of Water Supply, Gas and Electricity, the average age of water mains in East Harlem is fifty years. The useful life of 12-in. and 20-in. lines is about one hundred years and for 8-in. pipe, seventy-five years. On this basis, one half the 12-in. and 20-in. mains and two thirds of the 8-in. pipes in Area A are obsolete and that portion retained in either Unit A_1 or A_2 would have to be replaced. Obsolete fire hydrants must also be replaced, and new and old hydrants relocated to conform to the master block plan.

Street Lighting (Item No. 5, Table 6).—Under current standards and prices lighting equipment for the neighborhood units costs $93.36 per light. The installation is estimated at $39.15 per light.

The city formerly rented street lighting equipment from utility companies; hence there is no city outlay for this equipment in Area A. Under present policy the city purchases and installs street lighting equipment. Both the new and rebuilt communities are therefore charged in full for this equipment as noted in Table 6.

Population Facilities.—The character of development and the size of the population that can be accommodated in a community create well-defined demands for parks and playgrounds, elementary and high schools, branch libraries, local health buildings, fire stations and police precincts, sanitation garages and section houses, sewage and refuse disposal plants, and rapid transit. These public services are termed "population facilities."

The obsolescence of existing facilities in Area A is determined by the present age and useful life of these improvements. The present age of most facilities is obtained from public records, but the age of libraries, fire stations, police precincts, and sanitation garages and section houses is assumed equal to the average age of existing schools in East Harlem, which is forty-three years. The useful life of these facilities is considered to be sixty years; hence they are 71.6% obsolete.

Parks and Playgrounds.—Table 2 shows that public parks and playgrounds account for 4.79 acres, which is equivalent to one acre for each 7,100 persons that can be housed in the community. The park land required in the neighborhood units is based on standards used by the staff of the Mayor's Committee on City Planning, which call for one acre for each 1,750 persons.

Closing certain interior streets in converting the gridiron street plan of Area A into the master block layout of Units A_1 and A_2 makes about 14.8 acres of land available for other purposes (see Fig. 4). Inasmuch as this land is now in the bed of publicly owned streets, it is assumed to be a city asset, which could be sold or traded for building purposes in exchange for equivalent acreage in appropriate locations for additional parks at no cost to the city.

In Unit A_1 the 4.79 acres (see Table 7) of existing parks plus 14.8 acres from discontinued streets would produce 19.59 acres of parks. The purchase of 2.46 additional acres would bring park acreage to the 22.05 acres required in Unit A_1. For Unit A_2 the existing park acreage plus the area of discontinued streets would provide the 19.02 acres needed for parks and would leave about 0.5 acre that could be used for other public purposes.

TABLE 7.—PARK NEEDS

Unit	Area, in acres	Land cost	Development ($7,000 per acre)
A	4.79	$1,734,000	$ 33,550
A_1	22.05	891,500	154,350
A_2	19.02	....	133,140
A_3	19.02	250,000	133,140

The extensive reconstruction involved in transforming Area A to a master block plan would necessitate the redevelopment of parks and the provision of new park equipment in either Unit A_1 or A_2. The existing park equipment in Area A, therefore, is assumed unusable in Unit A_1 or A_2.

Elementary Schools (Table 8(*a*)).—The number of children from each unit that may be expected to register in public elementary schools is estimated at 9.7% of the total population. The elementary school standard limits class-rooms to thirty-five students, and the school needs of each unit are predicated on this average value.

TABLE 8.—EDUCATIONAL NEEDS

Area and units	(*a*) ELEMENTARY SCHOOLS			(*b*) HIGH SCHOOL			(*c*) PUBLIC LIBRARY			
	Student registration	Needed classrooms	Rooms replaced	Registration (seats)	Percentage of seats replaced	Unit cost (dollars)	Persons per branch	Ratio of use	Replacement ratio	Cost of branch (dollars)
A	4,462	127.4	....	1,840	....	1,200	50,750	90.6	....	166,600
A_1	3,740	106.8	79.6	1,542	24.3	1,200	50,000	77.1	71.6	89,600
A_2	3,227	92.2	65.0	1,331	24.3	1,200	50,000	66.5	71.6	89,600
A_3	3,227	92.2	....	1,331	....	1,200	50,000	66.5	....	126,520

Records of the Board of Education of New York City show that 79% of the classrooms available in East Harlem are in buildings that were erected prior to 1901. These buildings generally cover more than three quarters of plots averaging only 31,200 sq ft in area. It is assumed, therefore, that about 80% of the 127 classrooms available to Area A are obsolete so that only 27 classrooms in buildings less than forty years old are considered usable in Unit A_1 or A_2.

Unit A_1 needs about 107 classrooms, of which 27 are supplied by existing schools, and nearly 80 must be provided in new buildings to replace obsolete classrooms. Unit A_2 requires about 93 classrooms, and, by utilizing the 27 now usable in Area A, new schools containing only 65 classrooms need be built to meet the requirements of Unit A_2.

Since fewer classrooms are needed for Units A_1 and A_2, it will not be necessary to acquire more land for schools. If the new playgrounds required under minimum standards were located adjacent to school sites, both old and new buildings in Unit A_1 or A_2 would have more light, air, and play space than is now available in Area A.

Reports of the Board of Education indicate that the accumulated cost of land, equipment, and building for existing elementary schools in East Harlem average $13,650 per classroom. Elementary school costs in all of the neighborhood units are based on $21,000 per classroom, a city-wide average for schools constructed during recent years.

High Schools.—Approximately 4% of the population of each unit is expected to register in public high school. The standard requires the provision of one seat per student in an up-to-date school building.

On the premise that buildings more than forty years old are obsolete, the Manhattan high school plant measured in classrooms is 24.3% depreciated. This obsolescence factor is applied against the number of old seats needed in Units A_1 and A_2 to determine necessary replacement. The cost per seat for land, building, and equipment (see Table 8(*b*)) is a general average for high schools recently constructed.

Public Library.—The standard used by the Mayor's Committee on City Planning forms the basis for determining public library needs in the units.

The obsolescence of the branch library serving Area A is estimated at 71.6%. In either Unit A_1 or A_2 the need for library service is less than that required by Area A. It is assumed that the obsolete part of this existing branch must be replaced to the full extent of the indicated depreciation.

As evidenced by capital budget and program requests, branch libraries cost about $125,000 each for construction and equipment. This sum forms the basis for cost estimates in all communities, exclusive of plots 50 by 100 ft, which are assumed to cost 1.3 times the average assessed value. The charge to each community is determined by the "ratio of use" (see Table 8(*c*)), which, of course, is population expressed as a percentage of the capacity of a branch library.

Health Building.—The Health Department program for health education, preventive medicine, and local health service aims at the eventual provision of health center buildings throughout the city. The capacity of each health building under this program forms the "standard," and costs are obtained from the same source. The "ratio of use" (Table 9(*a*)) determines the charge against each community (see also Table 3(*a*)). The district health center now serving Area A is adequate for Units A_1 and A_2, and this building is so new that obsolescence is negligible.

Sanitation.—The present distribution of sanitation garages and section houses in each borough is assumed a satisfactory standard for all communities.

In Manhattan there is one sanitation truck for each 1,540 persons and one section house for each 32,300 residents. In Brooklyn the averages are 2,540 persons and 44,350 persons respectively. Data from the Department of Sanitation indicate that trucks require about 6,300 cu ft of garage space, which, at 35 cents per cu ft, amounts to $2,205 per truck. Section houses for the storage of street cleaning and sanitation equipment average $25,000 each.

TABLE 9.—HEALTH AND SANITATION

Area and units	(*a*) Health Building			(*b*) Sanitation Buildings			(*c*) Sewage Disposal			(*d*) Incinerators		
	Persons per building (thousands)	Ratio of use	Cost of building (thousand dollars)	Garages (dollars)	Section houses (dollars)	Replacement percentages	Persons per plant (thousands)	Ratio of use	Plant cost (thousand dollars)	Daily collection (cu yd)	Ratio burned (%)	Capacity (cu yd)
A	227.4	20.2	50.4	1.44	0.76	....	1,270	3.62	27,255	471.5	44.4	209.5
A_1	227.4	....	....	1.44	0.76	71.6	1,270	....	....	395.3	57	225.5
A_2	227.4	....	....	1.44	0.76	71.6	1,270	....	....	341.2	57	195
A_3	250.0	13.3	76.6	0.87	0.56	....	400	8.32	5,176	341.2	57	195

The per capita costs, estimated on the basis of current distribution of these facilities in each borough, are shown in Table 9(*b*). The total charges to each community are derived from the population and appropriate per capita value. In Area A depreciation is estimated at 71.6% and the charges in Table 3(*a*) for Units A_1 and A_2 are to cover replacements of obsolete plant only.

Sewage Disposal.—Sewage disposal is assumed essential in all of the communities, and the program for plants and treatment methods proposed by the Department of Public Works is taken as the standard. Costs are also derived from this source. Units A_1 and A_2 are served by the new Wards Island plant; Unit A_3 would be served by the proposed 26th Ward plant in Brooklyn.

The Wards Island disposal plant is practically new, and its capacity is adequate to serve either Unit A_1 or A_2. Hence, no replacement nor enlargement of existing sewage disposal facilities is involved in rebuilding Area A on the master block plan. Area A and Unit A_3 are charged according to the ratio of use (see Table 9(*c*)) of the rated capacity of the plants serving their population.

Refuse Disposal.—It is assumed desirable to incinerate all combustible material and to use only ashes and other inorganic refuse for land fill. Estimates made for the Mayor's Committee on City Planning show that 57% of the refuse is combustible material and the remainder is inorganic matter that can be disposed of by land fill. About 45% of the refuse from Area A is now incinerated (see Table 9(*d*)). It is assumed that land acquisition is not involved in disposing of the remaining refuse by the land-fill method.

The charges for refuse disposal are based on the use of incinerators of 750-ton daily capacity costing $1,500,000. Combustible material from each community is figured at approximately 600 cu yd per ton.

Estimated daily collection of all refuse is based on 3.2 cu yd per capita annually at 313 working days. The existing plant serving Area A handles 209.5

cu yd and is assumed 33% obsolete. An additional capacity of 16 cu yd is needed for the 225.5 cu yd to be incinerated in Unit A_1. New plant costs chargeable to Unit A_1 are $5,350 and replacement of obsolete part, $23,300. The 195-cu-yd capacity needed in Unit A_2 is 15 cu yd less than the existing plant serving Area A; this excess can be used for another community, and Unit A_2 is charged only for replacement of the obsolete part of the capacity of the old incinerator needed to serve this rebuilt community.

Fire Station.—The present distribution of fire stations in each borough is assumed adequate for the protection of residential neighborhoods. The existing facilities serving Area A are assumed 71.6% obsolete (see Table 10(*a*)) and to require replacement to this extent.

TABLE 10.—FIRE AND POLICE BUILDINGS

Area and units	(*a*) FIRE STATION NEEDS				(*b*) POLICE PRECINCT BUILDINGS			
	Persons per station	Ratio of use	Replacement ratio (%)	Cost per station (dollars)	Persons per building	Ratio of use	Replacement ratio (%)	Cost per building (dollars)
A	30,400	151.0	....	116,600	68,000	67.6	....	154,100
A_1	30,400	126.3	71.6	53,600	68,000	56.7	71.6	122,200[a]
A_2	30,400	109.3	71.6	53,600	68,000	48.9	71.6	80,600
A_3	37,400	89.0	....	76,520	92,400	36.0	....	115,540

[a] Additional land included in total replacement cost.

The total cost of stations is estimated at $75,000 per building plus 5,000 sq ft of land at 1.3 times the average assessed value of each community. The charge to each community, of course, is based upon the "ratio of use" shown in Table 10(*a*).

The fire alarm telegraph system now uses underground conduits and alarm boxes at approximately every alternate street intersection. On the normal gridiron layout, this calls for twenty alarm boxes per community. The master block plan, however, needs only sixteen alarm boxes per unit. Installation costs, at $1,000 per box, are $20,000 for the gridiron street plan and $16,000 for the master block plan. One half of the twenty existing boxes in Area A are assumed obsolete, but four of the boxes not needed in Unit A_1 or A_2 have a salvage value of $200 each, which reduces the replacement cost to $7,200 in each of these units.

Police Precinct.—The standards for police protection for all communities are also based upon the present distribution of precinct buildings in each borough. The obsolescence of police stations serving Area A is the same as fire stations—namely, 71.6% (see Table 10(*b*)). Replacement to this extent is necessary for Units A_1 or A_2. Most of the old police precincts occupy plots 50 ft wide, but garages for motor equipment adjacent to new stations require lots 100 ft wide. In Unit A_1 the additional land must be acquired. After providing for park needs in Unit A_2, however, sufficient land would be left over from discontinued interior streets to allow for the larger police station lot.

The costs charged to each community (see also Table 3(*a*)) are based on the respective ratios of use, with buildings at $112,500 each and land at 1.3 times the average assessed value.

Rapid Transit.—This study assumes that adequate transit service is essential to the proper functioning of the neighborhood unit communities. The measure of adequate transit service during the morning rush hours (see Table 11) is predicated on "reasonable" carloadings, as defined by the Transit Commission, and sufficient capacity on lines to transport the passengers contributed by each unit with out exceeding this "reasonable" load.

TABLE 11.—Morning Rush-Hour Passengers

Area and units	Percentage of population	Total load	I.R.T. subway	Independent subway	Elevated lines
A	14.4[a]	5,178	3,791	.....	1,387
A_1	14.4[b]	5,450	4,140	1,310	.....
A_2	14.4[b]	4,705	4,140	565	.....
A_3	18.8[b]	6,255	.....	6,255	.....

[a] Ratio of actual population. [b] Ratio of population at full occupancy of housing.

The "reasonable" carload (about 3 sq ft per standing passenger with all seats filled) does not represent the most desirable or convenient transportation, but it is a practical basis for measuring line capacity. Besides carloading, two other factors are involved in determining the capacity of transit lines serving each unit: (1) The number of cars per train, and (2) the headway between trains during the morning rush hours. Adequate transit service, therefore, must provide sufficient capacity to carry the passengers contributed by each unit, as well as by the remaining half-mile zone served by each line, without exceeding "reasonable" carloadings during the morning rush hours on any line into central business districts. In 1935 the Interborough Rapid Transit Company (I.R.T.) was carrying loads during the morning rush hours that are estimated to be 1.2 times greater than the reasonable capacity of the line serving East Harlem. As a result, the actual number of persons living in Area A is used to determine rush-hour loads. In the neighborhood units, the population that could be housed at full occupancy of dwellings determines the loads and the capacity required.

Table 11 shows the estimated rush-hour passengers contributed by each community and the number using each rapid transit line. Of the total passengers contributed by Area A, only 83% of those using the I.R.T. subway are assumed within the capacity of this line because of the present overcrowding. With certain remedial measures it is believed possible to increase the capacity of this line by 1.3 times during the morning rush hours. This would increase the present 3,165-passenger reasonable capacity of the I.R.T. subway available to Area A to 4,140 rush-hour passengers.

In rebuilding Area A it is supposed that the obsolete elevated lines serving East Harlem will be demolished and that remedial measures will be undertaken on the I.R.T. subway. Thus, all traffic from Units A_1 and A_2 must use subway lines, including a new subway to replace the Elevated Railway.

Unit A_1 will contribute 5,450 rush-hour transit passengers, of which 4,140 are expected to use the I.R.T. subway, and 1,310 passengers will have to be

accommodated on the new subway line that will be needed after demolition of the Elevated Railway. Unit A_2 is expected to contribute 4,700 passengers during the morning rush hours. With the reasonable capacity available after remedial measures, the I.R.T. can serve 4,140 of these people, and the new subway line will accommodate the remaining 565 passengers from Unit A_2.

The Elevated line in East Harlem was built at private expense and cannot be charged against Area A. On the basis of total city contributions to the I.R.T. subway construction, it is estimated that the city's capital outlay averages $437 per passenger carried during rush hours in 1935. According to studies of the Mayor's Committee on City Planning, remedial measures would increase the capacity of the I.R.T. by 1.3 times at a cost of $147.50 for each passenger that could be carried over and above the present reasonable capacity of this line. Assuming a total of $815,000,000 as the cost of the Independent (including sections under construction), and the rated capacity used in the Mayor's Committee Rapid Transit Study, the city will have spent $1,132 for each passenger that this system can carry into work centers during the morning rush hours. These estimates of capital costs to the city per passenger carried during the morning rush hours on each line serving the several communities determine rapid transit charges summarized in Table 3(*a*).

Estimates of Annual Expenses

This section of the Appendix outlines the methods used in estimating the annual expenses for operating and maintaining the public services provided in each community.

Except as otherwise noted, unit costs are based upon the most recent information available from the operating agencies. The per capita costs are generally derived from the total annual expense and the estimated population actually living in the city. Therefore, the "actual" population living in the several communities, rather than the "demand" or potential population that would result from 100% occupancy of residential quarters, is used in determining annual expenses that are based on per capita costs. The estimated yearly expenses charged to each community are summarized in Table 3(*b*) by item.

Street Cleaning.—Annual costs of street cleaning average 27.3 cents per sq yd in Manhattan and 13.1 cents in Brooklyn. Snow removal averaged 20.3 cents per sq yd over a five-year period. Computed for the roadway area, there are 157,000 sq yd of roadway to be cleaned in Area A and 142,520 sq yd in each unit. On the basis of full street width there are 263,884 sq yd to be cleaned of snow in Area A and 187,969 sq yd in each unit.

Sewer Maintenance.—The unit cost of cleaning sewers in Manhattan during 1938 was $203.50 per mile and $8.60 per catch basin. Since data were not available for Brooklyn, the unit cost is the same as Queens—namely, $102.50 per mile of sewer and $8.60 per catch basin.

There are 6.76 miles of sewers and 80 basin manholes in Area A, but only 2.92 miles of sewers and 36 basin manholes in Units A_1 and A_2. Because pipe is placed in service roadways, Unit A_3 has 5.52 miles of sewers and there are 36 basin manholes.

Water Mains.—The annual expenses per mile of pipe for water-main maintenance include 70% of the motor vehicle operations cost for mains in existing streets, regardless of pipe size. The age of mains is not considered a factor because unconsolidated fill tends to cause new mains to break more frequently than old pipes. The annual maintenance expense in Area A, and Units A_1 and A_2, is computed at the Manhattan average of $884 per mile of pipe. The cost in Unit A_3 in Brooklyn is based on $449 per mile of pipe per year. Pipe length is given in Table 6.

Street Lighting.—In Area A the annual rental of street lighting equipment from the utility companies is estimated to average $25 per light. In the neighborhood units the city would install and maintain its own equipment. Recent operating experience indicates that the annual expense would be $8.34 per light.

Parks and Playgrounds.—An examination of the Comptroller's Report indicates that appropriations for the operation and maintenance of parks and playgrounds amounted to more than $8,800,000 during 1938. This appropriation represents an average cost of about $525 per acre of park, including all facilities under the jurisdiction of the Park Department. This unit cost is used in estimates of annual expenses in all communities.

Elementary Schools.—The average cost per classroom for physical operation and maintenance of elementary and junior high schools as reported for 1937 is $387 in East Harlem (Area A); but the city average approximates $400 per classroom, which value forms the basis for estimates in all neighborhood units.

The cost of instruction, excluding general administration, averages about $4,000 per classroom for the entire city. Similar data are not available by school district and, therefore, instruction costs for all communities are based on the city average.

High School.—The annual expenses for instruction and high school building maintenance, shown by reports to approximate $159 per student registered, are the basis for the estimates given in Table 3(*b*).

Public Library.—Requests for public libraries submitted in connection with the capital budget indicate that the annual operation and maintenance costs average $35,000 for the type of branch that serves residential districts. With this value as a base, costs are allocated to each community on its "ratio of use" of library facilities.

Health Building.—Each community is charged with its proportionate share of the annual expense of maintaining and operating health center buildings, as given in the Health Department program submitted to the City Planning Commission. The expense in the East Harlem district (Area A, and Units A_1 and A_2) is given as $13,970 per yr, and in the district serving Unit A_3 the total cost is $16,210.

Fire Protection.—Estimates of yearly expenses for fire protection are based on the following per capita charges:

Borough	Dollars per year
Manhattan	4.19
Brooklyn	3.16

These values are derived from reported costs for operation, including equipment, telegraph, buildings, and personnel in the smaller stations, which are assumed typical for residential districts.

Police Protection.—The annual charges for police personnel, motor and other equipment, and building maintenance average $3.22 per person living in Brooklyn and $6.90 per person living north of 80th Street in Manhattan.

Refuse Removal.—The annual maintenance of sanitation garages and section houses, as well as the costs of collecting refuse, are included under "refuse removal" expenses. These expenses average $2.17 per person in Manhattan and $2.02 in Brooklyn. Application of these figures to the population of each community produces the estimates summarized in Table 3(*b*).

Sewage Disposal.—The costs of sewage treatment given for the various plants in the "Tentative Plan for Sewage Disposal" prepared by the Department of Public Works and submitted to the City Planning Commission are used to determine the total operating expenses for each plant. The estimated operating expenses for the Wards Island plant are $650,400 per yr, and for the 26th Ward plant, $213,100 per yr. Each community is charged with its share of the annual expenses in proportion to the "use ratio" of plant capacity.

Refuse Disposal.—The cost per cubic yard of material incinerated is about 20 cents in Manhattan and 18 cents in Brooklyn (see Table 12). The disposal

TABLE 12.—REFUSE DISPOSAL EXPENSES

Area and units	Annual (cu yd)	Incinerated		Land Fill		Dumped	
		%	Cost[a]	%	Cost[a]	%	Cost[a]
A	117,477	44.4	20	4	10	51.6	15
A_1	117,477	57	20	43	10	....	..
A_2	101,373	57	20	43	10	....	..
A_3	101,373	57	18.1	43	6.4	....	..

[a] Cents per cubic yard.

of ashes and other non-combustible material is assumed at about 10 cents per cu yd in Manhattan and approximately 6 cents in Brooklyn. Table 12 shows the total annual collection of refuse in square yards, the disposition by community, and the unit cost (see Table 3(*b*) for the total annual expenses).

Debt Service on Rapid Transit.—This study considers the capital costs of rapid transit a proper charge against communities in proportion to the line capacity used. The annual costs of operation and maintenance of the lines, however, are paid out of operating revenues. Under existing operating conditions these expenses are not met by the city and are not charged against the communities. The city does contribute substantial sums to service its outstanding rapid transit debt. Accordingly, debt service on that part of the subway improvement costs allocated to each community is charged as an annual expense in Table 3(*b*).

The capital cost shown in Table 3(*a*) is assumed to be financed by fifty-year corporate stock requiring annual payments of 4% interest and instalments to a sinking fund earning 3% for amortization of principle.

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PAPERS

METHOD OF PREDICTING THE RUNOFF FROM RAINFALL

By Ray K. Linsley, Jr.,[1] and William C. Ackermann,[2] Juniors, Am. Soc. C. E.

Synopsis

The problem of estimating runoff accurately and quickly from reported rainfall is the most difficult phase of river forecasting. The writers have analyzed hydrological and meteorological records of the Valley River Basin in North Carolina to develop a rational method of predicting runoff based on average rainfall and evaporation from a standard land pan.

Description of the Basin

The Valley River above the U. S. Geological Survey gaging station at Tomotla, N. C., drains an elliptical basin of 104 sq miles. Tomotla is near the lower end of the river, which follows roughly the long axis of the ellipse. Topographically the basin consists of a gently rolling valley floor bounded on the north by the Snowbird Mountains and on the south by the Valley River Mountains. Elevations within the basin range from 1,600 to 1,900 ft above sea level on the valley floor and from 3,000 to 4,500 ft above sea level along the surrounding rim. The channel slopes of the river average about 30 ft per mile in the upper half of its course and about 15 ft per mile in the lower reaches.

The rock underlying the Valley River Basin is metamorphic, consisting mainly of gneisses and mica schists. The structure is rather complicated as the rocks are extensively faulted. A thin belt of marble underlies almost the entire basin. There is an extensive local alluvial fill from 20 to 30 ft deep over part of the basin that results in sustained ground-water flows. There is no evidence of loss of water from the Valley Basin by deep seepage.

The City of Andrews with a population of about 2,000 is located near the center of the basin. The remainder of the area is rather sparsely settled. The

Note.—Written comments are invited for immediate publication; to insure publication the last discussion should be submitted by **October 15, 1941.**

[1] Junior Hydrologic Engr., U. S. Weather Bureau, Washington, D. C.; formerly Junior Hydr. Engr., TVA, Knoxville, Tenn.

[2] Junior Hydr. Engr., TVA, Knoxville, Tenn.

valley floor, comprising about 60% of the total area, is dominantly cleared and about one half of this is in cultivation. The remaining area is forested, with mixed hardwoods and long-leaf pine at intermediate levels and hardwoods at the higher elevations.

Data Used

Since appreciably different runoff characteristics exist in the various seasons of the year, it is essential to segregate data by seasons before attempting an analysis. In this study only the flood season, which begins about the middle of December and extends to the middle of April, is considered. This period falls within the dormant season and coincides with the period of general cyclonic rains. The limits of this season for a particular year can be determined by inspection of the temperature and stream-flow records. The study is based on the three flood seasons of 1934–1935 to 1936–1937.

Average rainfall was computed by the weighted-area method, using the records at Andrews, Hyatt Creek, Tatham Gap, and Old Road Gap in North Carolina. Time of rainfall was determined by inspection from the various recording gages (Fig. 1) and from the reports of the observers. Records of

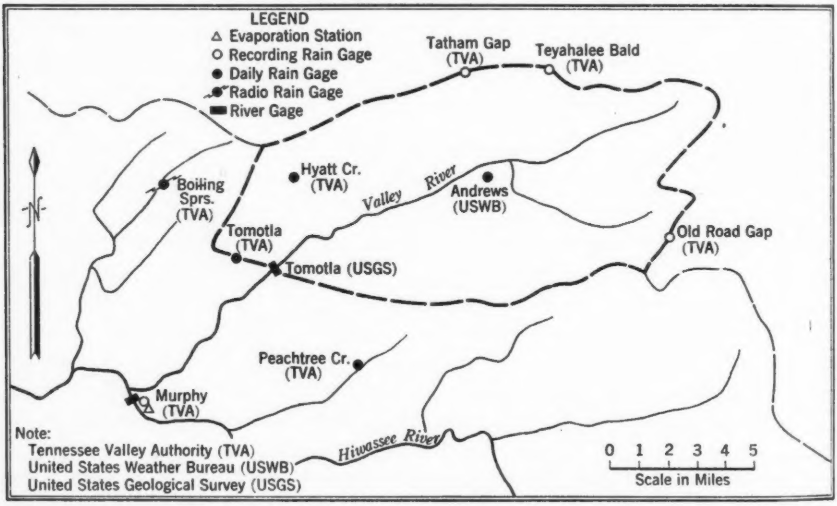

Fig. 1.—Map of Valley River in North Carolina

evaporation and temperature were obtained from the Class *A* evaporation station maintained by the Tennessee Valley Authority (TVA) at Murphy, N. C. A continuous stream-flow hydrograph was developed from the recorder charts of the rated U. S. Geological Survey recording stream gage at Tomotla.

Theory

Rainfall occurring over the natural river basin is assumed to be disposed of as follows:

(*1*) *Surface Loss.*—Surface loss is that part of the rain which is intercepted by the vegetal cover and natural or artificial retention basins from which it eventually evaporates and is thus prevented from entering the stream channels. This loss has been found to vary with the rainfall, being approximately constant for rains of the same amount. In other words, the greater the depth of rain the greater will be the quantity lost at the surface until a practical maximum is reached at some extremely high rainfall. Observation indicates that the full value of the potential surface loss is again available within a few hours after the end of a storm; therefore, whenever rains are separated widely enough to be analyzed independently, the full loss may be assumed for each rain. There is some indication that the amount of the surface loss is affected by the intensity of rainfall; but the complexity of the problem prevents a solution for the laws governing the variation. Storms occurring during flood season, which are considered herein, rarely reach extreme rates of rainfall and little error is introduced by the use of average values.

(*2*) *Field-Moisture Loss.*—Field-moisture loss is that part of the rain which is absorbed in the upper layers of the soil where it remains until removed by evaporation or transpiration. The potential field-moisture loss at any time is equal to the net amount of moisture taken from the soil by evaporation and transpiration since the last time the soil was saturated. The writers have used the accumulated evaporation from a standard evaporation pan as an index of field-moisture deficiency. It has been demonstrated that the moisture loss from the soil is greater during the first few days immediately after a rain than during the succeeding days, but the complexity of the problem precludes the determination of a variable coefficient for converting the pan evaporation to evaporation from the soil. There is a practical maximum that the field-moisture deficiency cannot exceed, and this value can be determined. Inasmuch as the storms of the flood season come at relatively short intervals, the use of an average coefficient introduces little error and reduces, considerably, the work involved in the use of the method.

(*3*) *Surface Runoff.*—Surface runoff is that part of the rain which travels across the ground surface to the nearest stream channel and thence to the main river system. The subsurface flow that travels through the upper layers of the soil to the stream channel is included with the surface runoff.

(*4*) *Ground-Water Accretion.*—The ground-water accretion is that part of the rainfall which percolates through the soil to the ground-water level. The resultant rise in the ground-water level causes an increased flow from the ground-water storage to the stream channel, until an amount equal to the accretion has drained into the stream channel.

Analysis of the Hydrograph

Stream-Flow Separation Charts.—The total loss for each storm studied was computed by subtracting measured runoff from observed average rainfall. Continuous discharge hydrographs were plotted to a scale 2,000 cu ft per sec to the inch vertically and two days to the inch horizontally (Fig. 2). The average rainfall was plotted in block form on the same sheet, together with

notes regarding snow on the ground or sub-freezing temperatures. These hydrographs were analyzed and the runoff measured by storms.

To aid in the complete separation of the hydrographs by storms, the following curves and relations were developed:

Ground-Water Recession Curve.—Fig. 3(*a*) is a ground-water recession curve plotted as the relation between ground-water flow at a given instant and the flow twenty-four hours later.[3] The curve is a composite of a large number of

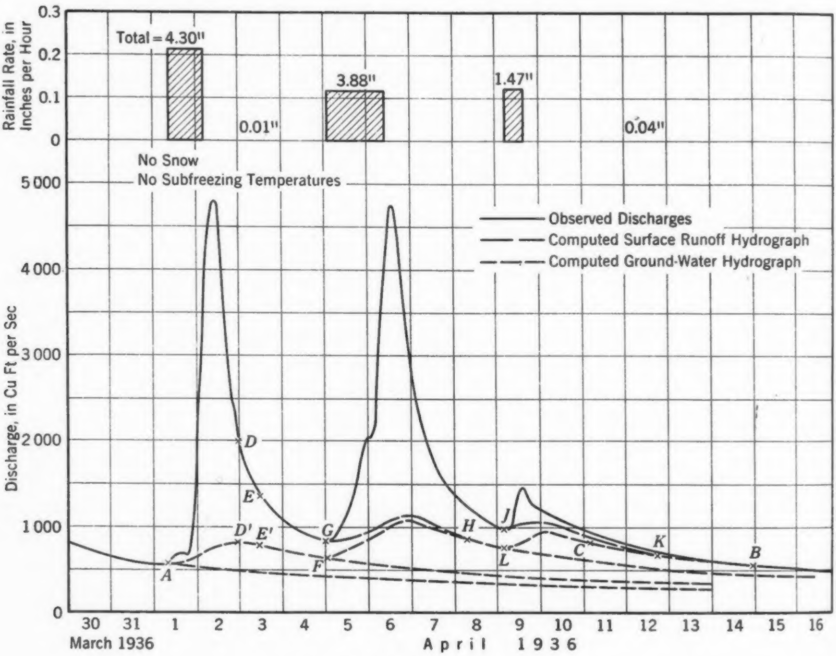

FIG. 2.—TYPICAL HYDROGRAPH

points from selected data for five years of record. The relation plots as a straight line on logarithmic paper and, for the range shown as a dashed line, represents an extension beyond the limit of these data. Care must be taken that the data selected for this curve are free of surface runoff. The duration of surface runoff may be taken as the length of the unit graph base or may be computed.[4]

Surface-Runoff Recession Curve.—Fig. 3(*b*) is a surface-runoff recession curve in a form similar to the ground-water recession curve. In this case the time increment was taken as twelve hours because of the higher rates of change involved. Data for this curve were obtained by drawing the ground-water hydrograph under the receding limbs of selected storm hydrographs by using

[3] "Some Channel-Storage Studies and Their Application to the Determination of Infiltration," by W. B. Langbein, *Transactions*, Am. Geophysical Union, 1938, Pt. 1, p. 435.

[4] "Synthetic Unit-Graphs," by F. F. Snyder, *loc. cit.*, Pt. 1, p. 447.

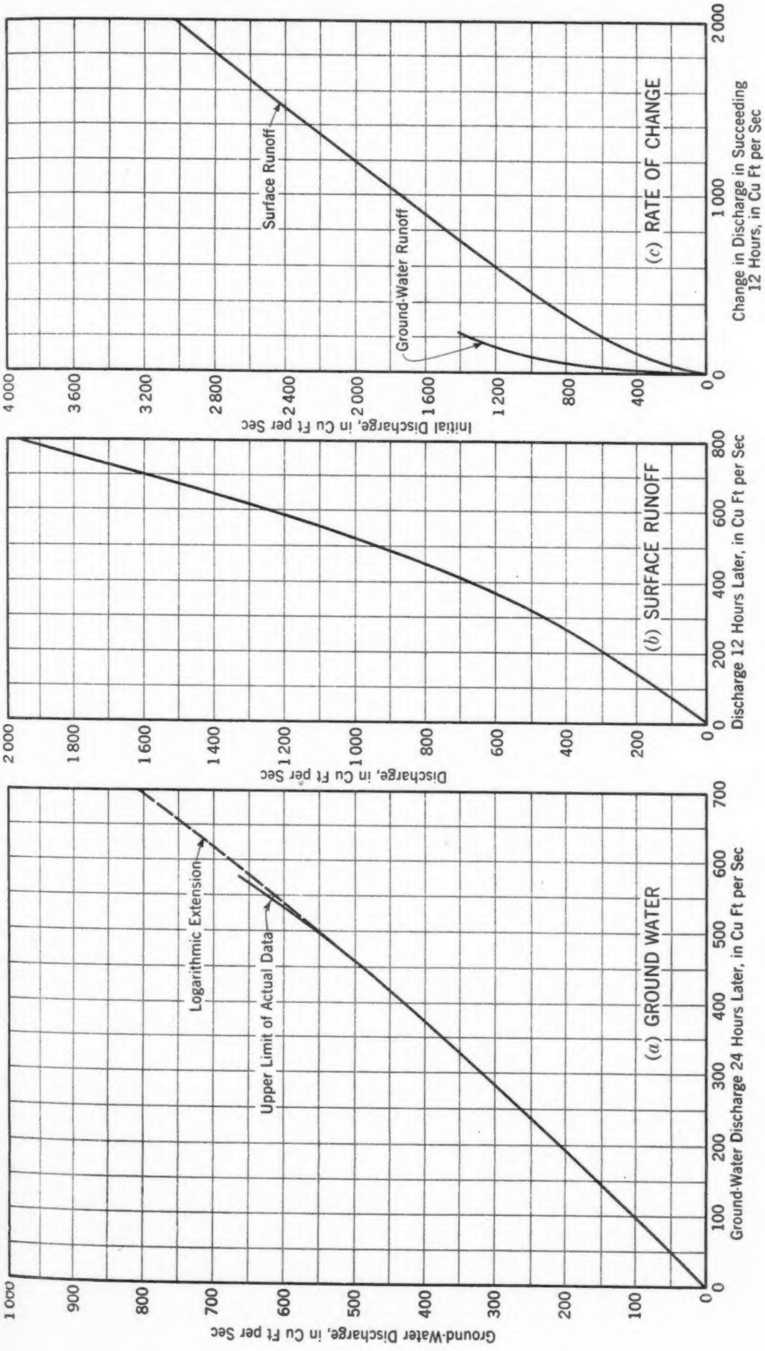

FIG. 3.—RECESSION CURVES

the ground-water recession curve in Fig. 3(*a*). Ordinates between the ground-water discharge hydrograph and the total discharge hydrograph at 12-hr intervals then furnish the data for plotting the surface-runoff recession curve in Fig. 3(*b*).

Rate of Change in Flow Curves.—In Fig. 3(*c*) the ground-water recession curve and the surface-runoff recession curve are plotted as initial flow against the change in flow for the succeeding twelve hours. This curve is purely a tool to be used in the process of stream-flow separation.

Ground-Water Volume Curves.—A study of the ground-water recession curve and stream-flow records indicated that a period of about two months was required for the ground-water flow to recede to 50 cu ft per sec from an initial flow of 500 cu ft per sec. After a flow of about 50 cu ft per sec is reached, the flow change becomes so slight that it may be neglected. For this reason the lower recession of ground-water flow existing prior to a storm was extended downward to 50 cu ft per sec and then held steady until it intersected the recession of the current ground-water hydrograph. The area between these two curves then represents the volume of ground-water runoff from the current rain.

In order to reduce the labor required in measuring the volume of runoff between the upper and lower recessions for each individual storm, the ground-water volume curves, Fig. 4, were developed. The volumes between a number of pairs of recessions were computed in order to determine the family of curves

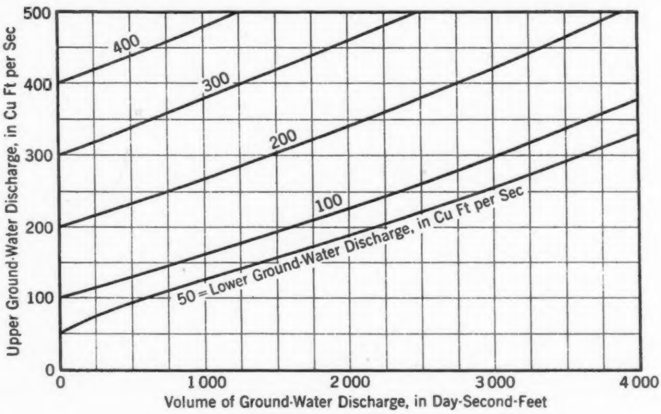

FIG. 4.—GROUND-WATER VOLUME CURVES

shown. Since the lines are practically straight, three points are ample to fix each curve. By entering these curves with the values of the upper and lower recessions at some convenient time after the peak of the ground-water runoff, the volume of subsequent ground-water runoff may be read directly. The total volume under the ground-water hydrograph then may be determined by measuring the remaining area under the hydrograph and adding it to the partial volume determined from the ground-water curves.

Method of Separation.—If the hydrograph of Fig. 2 is assumed to result from the rains of April 1, 5, and 9, the problem is to determine the surface and

ground-water runoff for each rain. The ground-water recession that would have occurred if there had been no rain may be drawn by applying the ground-water recession curve (Fig. 3(*a*)), starting at point A. In a like manner the ground-water recession curve may be applied starting at point B, where the shape of the hydrograph indicates that the flow is entirely from ground water, and carried backward until it departs from the stream-flow hydrograph and reaches a point C somewhere near the estimated point of inflection of the falling limb of the total ground-water hydrograph. The remaining part of the ground-water hydrograph between A and C must be computed by another method.

The change in flow for the 12-hr period $D\,E$ on the stream-flow hydrograph is the sum of the change in flow $D'\,E'$ of the ground-water recession and the concurrent change in the surface-runoff recession. If the ground-water flow at point D' is assumed, an approximate value for the change in ground-water discharge in the succeeding twelve hours may be obtained from Fig. 3(*c*). If this value is subtracted from the observed change in discharge, $D\,E$, the result is the change in surface runoff in twelve hours. By re-entering Fig. 3(*c*) with this rate of change of the surface runoff, the initial surface discharge, $D\,D'$ may be determined. The value of ground-water flow at point D', computed by subtraction, should agree with the assumed value. If this value is far different from the preliminary estimate, a recomputation is indicated. In this manner that part of the total ground-water hydrograph under the observed stream-flow recessions may be computed. Since the ground-water discharge comes to a peak considerably after the peak surface runoff, this method permits the computation of the peak ground-water flow. With this flow known, the rising limb of the ground-water hydrograph may be drawn in arbitrarily without introducing any appreciable error. The receding limbs of the ground-water hydrographs for the storms of April 1 and 5 may be drawn in with the ground-water recession curve starting at points F and L respectively. The computed ground-water flows for the storm of April 9 will join the recession near point C and the ground-water separation is complete.

Starting with the surface-runoff ordinate $G\,F$, the ordinates of the surface-runoff recession for the storm of April 1 may be computed from the surface-runoff recession curve (Fig. 3(*b*)) and added to the total ground-water discharge to give the recession $G\,H$. In a similar manner the recession $J\,K$ for the storm of April 5 can be computed. The volume of ground water may be computed as outlined under the discussion of the ground-water volume curves. The volume of surface runoff may be measured on the hydrograph.

This method of stream-flow separation is based on the time characteristics of the stream flow. This is the classification of stream flow in common use today.

Supplemental Relations.—If selected isolated storms are attacked first with the preceding method of separation, certain data may be developed that will be helpful in the solution of the more complicated cases. The writers have developed several relations in the course of this study that can be used for further separation work or to predict the ground-water hydrograph in forecasting work. These relations are:

Total Runoff Versus Ground-Water Runoff.—Fig. 5 shows the relation between total runoff and ground-water runoff for the storms included in this study. A curve similar to this has been presented by other writers, but as yet no explanation or logical basis for the relation has come to the writers' attention. An attempt was made to determine an infiltration rate or rate of ground-water accretion in inches per hour, but the results varied so widely as to lead to the belief that no consistent rate existed. This belief was further strengthened when the relation shown in Fig. 5 was developed.

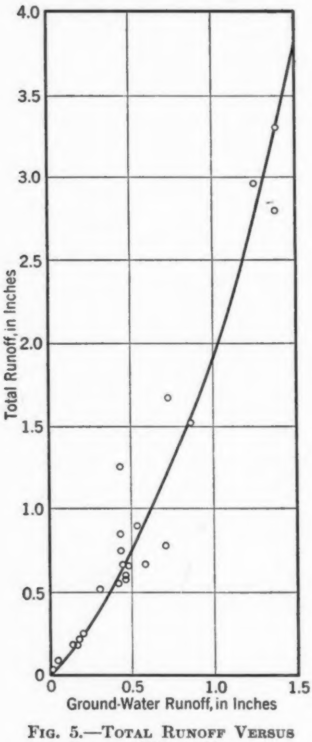

FIG. 5.—TOTAL RUNOFF VERSUS GROUND-WATER FLOW

Ground-Water Runoff Versus Net Peak Ground-Water Flow.—Fig. 6 presents the relation between the volume of ground-water runoff and the net peak height of the ground-water hydrograph.

Duration of Rainfall Versus "Time to Ground-Water Peak."—The relation between the duration of rainfall and time-to-peak ground-water discharge is shown in Fig. 7, as indicated by the data for the Valley River. "Time to peak" was computed from the beginning of rainfall.

ANALYSIS OF RAINFALL LOSSES

Method.—The analysis of stream flow having been completed, it was possible to compute the total runoff from each individual storm and to determine the total rainfall lost. As outlined under the heading "Theory," this total loss is to be separated into surface loss and field-moisture loss. The problem of dividing this total loss into two parts, each governed by different factors and with all the governing relations unknown, does not lend itself to a direct mathematical solution. Therefore, a more philosophical method was used.

Based on the writers' observations in forecasting work and their knowledge of the area under consideration, preliminary assumptions were made for the variation between rainfall and surface loss. First assumptions for the relation between pan evaporation and soil evaporation were taken from standard texts on hydrology. Using these assumptions, the total losses for several entire flood periods were studied and the indicated adjustments were made to the basic assumptions so that the computed losses approximated the measured total losses for the periods.

With these revised assumptions the study of individual storms was begun. Immediately after a rain of several inches, or a series of lesser rains totaling several inches in a few days, the accumulated pan evaporation between successive rains was computed and multiplied by the assumed ratio between pan and

soil evaporation. Adding to this value an estimated surface loss based on the amount of rainfall gave a computed value of the theoretical total loss for the storm. By comparing this computed value with the observed loss, in the light of all the available data, further readjustments of the assumed relations were made and certain other governing rules were established until, by successive approximations, relations that seemed to give the most consistent results were developed. The final relations developed were as follows:

Surface Loss.—As outlined herein under "Theory" the surface loss was found to be proportional to the amount of rainfall. Fig. 6 shows the relation developed for the Valley River Basin. This curve represents the final result of successive approximations which in combination with the final relations for field-moisture loss gives the lowest standard error of estimate for the data studied. The points plotted on the curve represent, for each storm studied, the difference between observed total loss and estimated field moisture loss as computed from the relations outlined under "Analysis of Rainfall Losses: Field Moisture Loss."

Field-Moisture Loss.—As stated under the heading "Theory" the writers have used the pan evaporation as an index of the field-moisture deficiency.

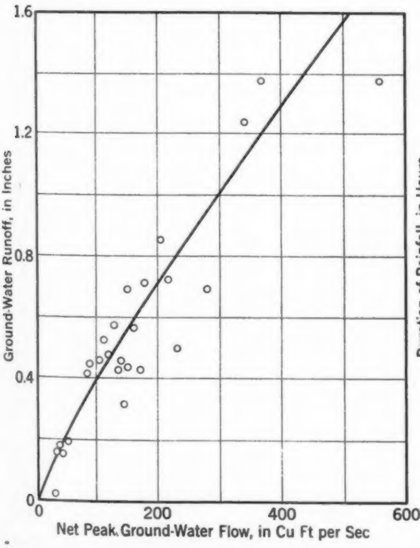

FIG. 6.—GROUND-WATER RUNOFF VERSUS NET PEAK GROUND-WATER FLOW

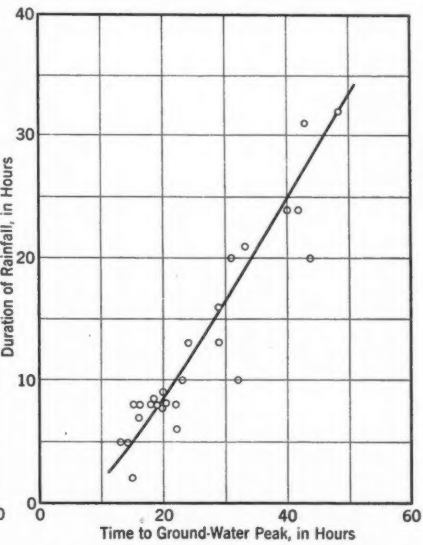

FIG. 7.—DURATION OF RAINFALL VERSUS TIME TO GROUND-WATER PEAK

It was found that the field-moisture deficiency at any time was equal to 0.9 times the total pan evaporation since the ground was last saturated, less any additions made to the field moisture by intervening rains. Studies of storms occurring after protracted dry spells showed that the total observed loss less surface loss averaged very close to 0.8 in. This value was taken as the maximum possible value of field moisture deficiency.

In the course of the study it was observed that for storms of short duration the observed loss less the estimated surface loss was often less than the available field-moisture deficiency despite the fact that there was an adequate volume of rainfall to satisfy the available deficiency. A further study of these cases showed that the difference between observed loss and estimated surface loss could be expressed very closely by 0.05 times the duration of the rain in hours.

TABLE 1.—TABULATION OF DATA AND COMPUTATION OF LOSS AND RUNOFF
(Quantities Expressed in Inches Except Duration, Which Is in Hours)

	DATA						FIELD-MOISTURE LOSS						
Date	Rainfall (1)	Surface runoff (2)	Ground-water runoff (3)	Duration of rainfall (4)	Measured total loss (5)	0.9 × accumulated evaporation (6)	Residual deficiency (7)	Total deficiency (8)	Maximum possible absorption (9)	Estimated absorption (10)	Surface loss (11)	Computed total loss (12)	Computed total runoff (13)
2-9-35	0.96	0.05	0.20	8	0.71	0.68	0	0.68	0.40	0.40	0.30	0.70	0.26
2-13-35	1.19	0.14	0.42	4	0.63	0.14	0.28	0.42	0.20	0.20	0.34	0.54	0.65
2-14-35	1.66	0.37	0.53	8	0.76	0.01	0.22	0.23	0.40	0.23	0.42	0.65	1.01
2-22-35	0.42	0.01	0	2	0.41	0.44	0	0.44	0.10	0.10	0.21	0.31	0.11
2-25-35	1.19	0.14	0.46	7	0.59	0.22	0.34	0.56	0.35	0.35	0.34	0.69	0.50
3-5-35	1.19	0.09	0.58	5	0.52	0.34	0.21	0.55	0.25	0.25	0.34	0.59	0.60
3-6-35	1.01	0.21	0.45	6	0.35	0.06	0.30	0.36	0.30	0.30	0.31	0.61	0.40
3-10-35	0.77	0.03	0.19	8	0.55	0.35	0.06	0.41	0.40	0.40	0.27	0.67	0.10
3-11-35	0.36	0	0.18	13	0.18	0	0.01	0.01	0.65	0.01	0.19	0.20	0.16
3-12-35	2.41	0.94	0.73	13	0.74	0	0	0	0.65	0	0.54	0.54	1.87
1-2-36	3.15	1.02	0.70	24	1.43	....	....	0.80[a]	1.20	0.80	0.65	1.45	1.70
1-4-36	1.15	0.32	0.43	9	0.40	0	0	0	0.45	0	0.33	0.33	0.82
1-6-36	1.22	0.43	0.43	8	0.36	0.03	0	0.03	0.40	0.03	0.34	0.37	0.85
1-8-36	1.82	0.82	0.44	8	0.56	0.07	0	0.07	0.40	0.07	0.45	0.52	1.30
1-9-36	0.70	0.20	0.32	2	0.18	0.02	0	0.02	0.10	0.02	0.26	0.28	0.42
1-15-36	0.68	0.01	0.02	8	0.65	0.24	0	0.24	0.40	0.24	0.25	0.49	0.19
1-18-36	4.41	1.93	1.38	24	1.10	0.22	0	0.22	1.20	0.22	0.82	1.04	3.37
2-17-36	1.06	0.13	0.46	8	0.47	0.05	0	0.05	0.40	0.05	0.32	0.37	0.69
4-1-36	4.30	1.71	1.24	20	1.35	0.58	0	0.58	1.00	0.58	0.81	1.39	2.91
4-5-36	3.88	1.42	1.38	32	1.08	0.34	0	0.34	1.60	0.34	0.75	1.09	2.79
4-9-36	1.47	0.09	0.70	10	0.68	0.33	0	0.33	0.50	0.33	0.39	0.69	0.78
12-6-36	1.93	0.14	0.57	16	1.22	....	....	0.80[a]	0.80	0.80	0.46	1.26	0.67
12-18-36	2.19	0.22	0.73	21	1.24	....	....	0.80[a]	1.05	0.80	0.51	1.31	0.88
1-1-37	3.35	1.30	0.50	27	1.55	....	....	0.80[a]	1.35	0.80	0.68	1.48	1.87
1-5-37	0.53	0.03	0.16	8	0.34	0.20	0	0.20	0.40	0.20	0.23	0.43	0.10
1-6-37	0.33	0.01	0.17	5	0.15	0.01	0	0.01	0.25	0.01	0.19	0.20	0.13
2-6-37	1.34	0.19	0.48	20	0.67	0.29	0	0.29	1.00	0.29	0.36	0.65	0.69
2-9-37	1.90	0.66	0.86	10	0.38	0.07	0	0.07	0.50	0.07	0.46	0.53	1.37

[a] Maximum possible deficiency.

Application of Method.—Table 1 presents the data used in the study and a computation of the losses and runoff for these storms. Cols. 1 to 5 contain observed data obtained from rainfall records and the analysis of the hydrograph. The accumulated evaporation between storms multiplied by 0.9 is entered in Col. 6. The residual field-moisture deficiency existing at the end of the previous rain is given in Col. 7. The sum of these two values gives the total field-moisture deficiency at the beginning of the current storm in Col. 8. The maximum possible absorption by the field moisture in Col. 9 is equal to 0.05 in. multiplied by the duration in Col. 4. The estimated absorption in Col. 10 is equal to the total deficiency except where this is greater than the maximum

possible absorption, in which case the latter value is used. The surface loss in Col. 11 is determined from the relation in Fig. 8 by entering with the rainfall for the storm shown in Col. 1. The sum of Cols. 10 and 11 gives the computed total loss in Col. 12 that may be compared with the actual loss in Col. 5. Total runoff in Col. 13 is obtained by subtracting the computed total loss from the rainfall in Col. 1.

Conclusions

(1) The methods of stream-flow separation presented herein are satisfactory for the Valley River. By reducing the degree of personal judgment required in stream-flow analysis to a minimum, consistent results were obtained from all data studied.

(2) Accumulated evaporation from a standard evaporation pan is an index of field-moisture deficiency.

(3) Surface loss varies approximately with the amount of rainfall and appears to approach a practical maximum for high rainfalls.

(4) The methods of estimating runoff presented in this paper represent a distinct refinement over rainfall-runoff relations in which such third variables as initial flow, ground-water flow, or days to the last rain are used as criteria of runoff.

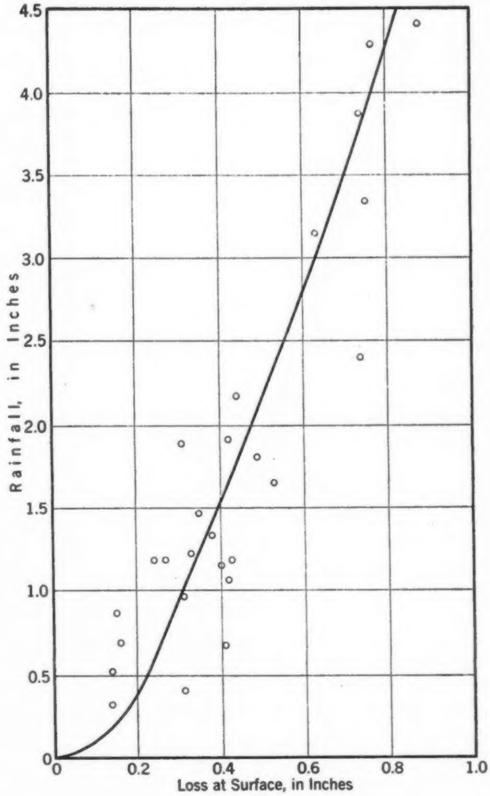

Fig. 8.—Surface Loss Curve

(5) Further study of a parallel nature is indicated in order to determine how these relations vary for other types of basins and for other seasons of the year. Perhaps further study on other basins will furnish data that will permit a greater refinement of the method. However, the average difference between measured and computed losses for the storms studied herein is only + 0.01 in., whereas the standard error of estimate is 0.10 in., which represents rather satisfactory accuracy for this problem.

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PAPERS

AN INVESTIGATION OF PLATE GIRDER WEB SPLICES

By J. M. Garrelts,[1] Assoc. M. Am. Soc. C. E., and I. E. Madsen,[2] Jun. Am. Soc. C. E.

Synopsis

In the design of plate girders several types of web splices are in current use. Probably the most common is the type with one plate on either side of the web extending the full depth between the toes of the flange angles. This splice was investigated by the writers both for the condition of uniform rivet spacing and for that of nonuniform rivet spacing. The six-plate splice, with three plates on each side of the web between the toes of the flange angles, two for moment and one for shear, was also investigated. The other type considered was that in which the web is spliced for its full depth between the toes of the flange angles for shear and strap plates over the flange angles to splice for moment. In many cases fill plates are required under the strap plates. In most cases the cost of the splice increases with the number of pieces to be handled.

Apparently the design of such splices has been based largely on assumptions which have never been checked by tests, and even the relative merit of these various designs has been unknown. This paper reports a series of tests designed to secure information concerning the behavior of plate girder web splices under load and to determine the relative values of four types of web splice design commonly used in modern practice.

Introduction

Four plate girders, each having a depth of $24\frac{1}{4}$ in. and a length of 24 ft and identical except for the types of web splices used, were subjected to test. The procedure was divided into two main parts: (1) All girders were first tested at working stresses (that is, at loads producing stresses below the elastic limit, thus affording a basis for the comparison of their behavior in the elastic range); and

Note.—Written comments are invited for immediate publication; to insure publication the last discussion should be submitted by **October 15, 1941.**

[1] Associate Prof., Civ. Eng., Columbia Univ., New York, N. Y.

[2] Asst. Research Engr., Fritz Eng. Laboratory, Lehigh Univ., Bethlehem, Pa.

(2) the tests of each girder were continued to failure, in order to reveal possible variations under increased loading.

In the first phase, or elastic range tests, strain rosette readings were taken along the center line of the splice and on two parallel lines, one on each side of the splice except for G_4 where two parallel lines were used on each side of the splice. The strains were measured on three intersecting gage lines at each strain rosette with tensometers using 1-in. gage lengths. Normal and shear stresses were computed for a vertical cross section through each rosette point using the measured strains. The longitudinal stresses in the flange angles and cover plates were determined from additional strain gage measurements on these elements using tensometers with 0.5-in. gage lengths. The variations in direct and shear stress intensities over the full depth cross section of each girder are given by curves drawn to represent the intensity of stress at any point. The variations along the girder of the longitudinal stresses in the toes

TABLE 1.— Total Measured Moment and Shear on a Cross Section

Location[a]	Bending Moment, in Kip-In.[b]				Shear,[c] in Kips[b]	
	Applied	Measured	In the 20-In. Depth Between Flanges			
			Measured	Theoretical	Measured	Theoretical
Girder G_1						
Line *A*	1,110	1,059	148.66	125.4	12.20	13.84
Line *B*[d]	1,190	1,135	144.74	......	12.45	......
Line *C*	1,273	1,280	160.0	144.0	13.40	13.84
Girder G_2						
Line *A*	1,110	1,076	152	125.4	11.3	13.84
Line *B*[d]	1,190	1,138	140	......	12.0	......
Line *C*	1,273	1,289	171	144.0	12.9	13.84
Girder G_3						
Line *A*	1,080	1,067	173.3	122.0	13.0	13.84
Line *B*[d]	1,190	1,120	174.3	......	14.5	......
Line *C*	1,300	1,284	167.7	147.0	13.8	13.84
Girder G_4						
Line *A*	1,070	1,031	184.51	121.0	12.6	13.84
Line *B*	1,140	1,066	145.51	129.0	10.3	......
Line *C*[d]	1,192	1,121	162.67	......	10.38	......
Line *D*	1,243	1,192	152.48	140.4	11.0	......
Line *E*	1,320	1,312	172.60	149.1	11.45	13.84

[a] See Fig. 1. [b] 1 kip = 1 "kilo-pound" = 1,000 lb. [c] Applied shear = 16 kips in each case. [d] Splice.

of the vertical legs of the flange angles are also given by curves drawn to represent those stresses.

During the second phase, or ultimate load tests, 10-in. strain gage measurements were taken, from which the relative rotation and the relative vertical

displacement at the splice were computed. The stresses at the top and bottom of the splice plates and in the adjacent toes of the flange angles were determined from 2-in. strain gage measurements.

Although the results are given in terms of stresses, these have been computed on the basis of a constant proportionality factor of stress to strain. The values given for stresses above the yield point represent strain rather than stress.

Deflections were measured by the stretched wire and scale method. A load-deflection curve for one girder is included in this report. The curves for the other girders were similar.

The total measured moment and shear on a cross section were computed by a process of summation, using the stress curves drawn to represent the measured values. The results for these values are given in Table 1 for the gage lines shown in Fig. 1.

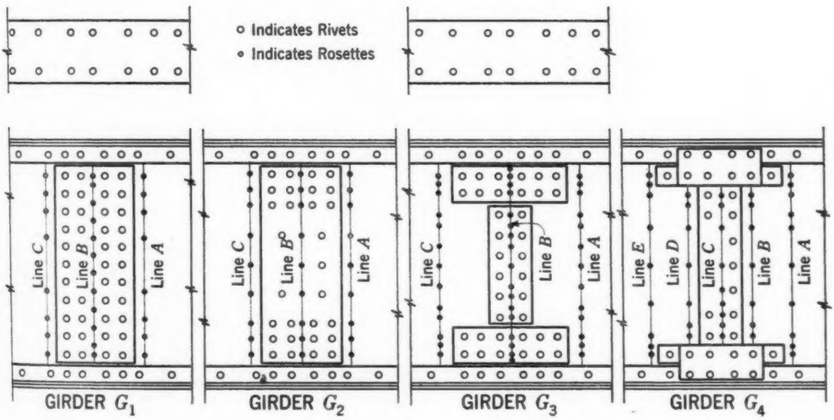


Fig. 1.—Gage Lines for Location of Strain Rosettes

Description of Test Specimens

The dimensions of the test girders were selected by making them proportional to those of girders used in practice and as large as could be handled in the laboratory.

The four test girders were alike, except for the material and arrangement at the splices. A typical test girder, with the splices omitted, is shown in Figs. 2(*a*) and 2(*b*). The main material consisted of the 24-in. by $\frac{3}{16}$-in. web plate, four angles 3 in. by 2 in. by $\frac{3}{16}$ in., two cover plates $6\frac{1}{4}$ in. by $\frac{3}{16}$ in., and two cover plates at $6\frac{1}{4}$ in. by $\frac{1}{8}$ in. Rivets 0.5 in. in diameter were used. The girders were fabricated with a depth of $24\frac{1}{4}$ in. back to back of flange angles and a span of 24 ft center to center of bearings. The relative position of each of the splices in a girder is specified by referring to them as 1, 2, or 3 as shown in Fig. 2(*b*).

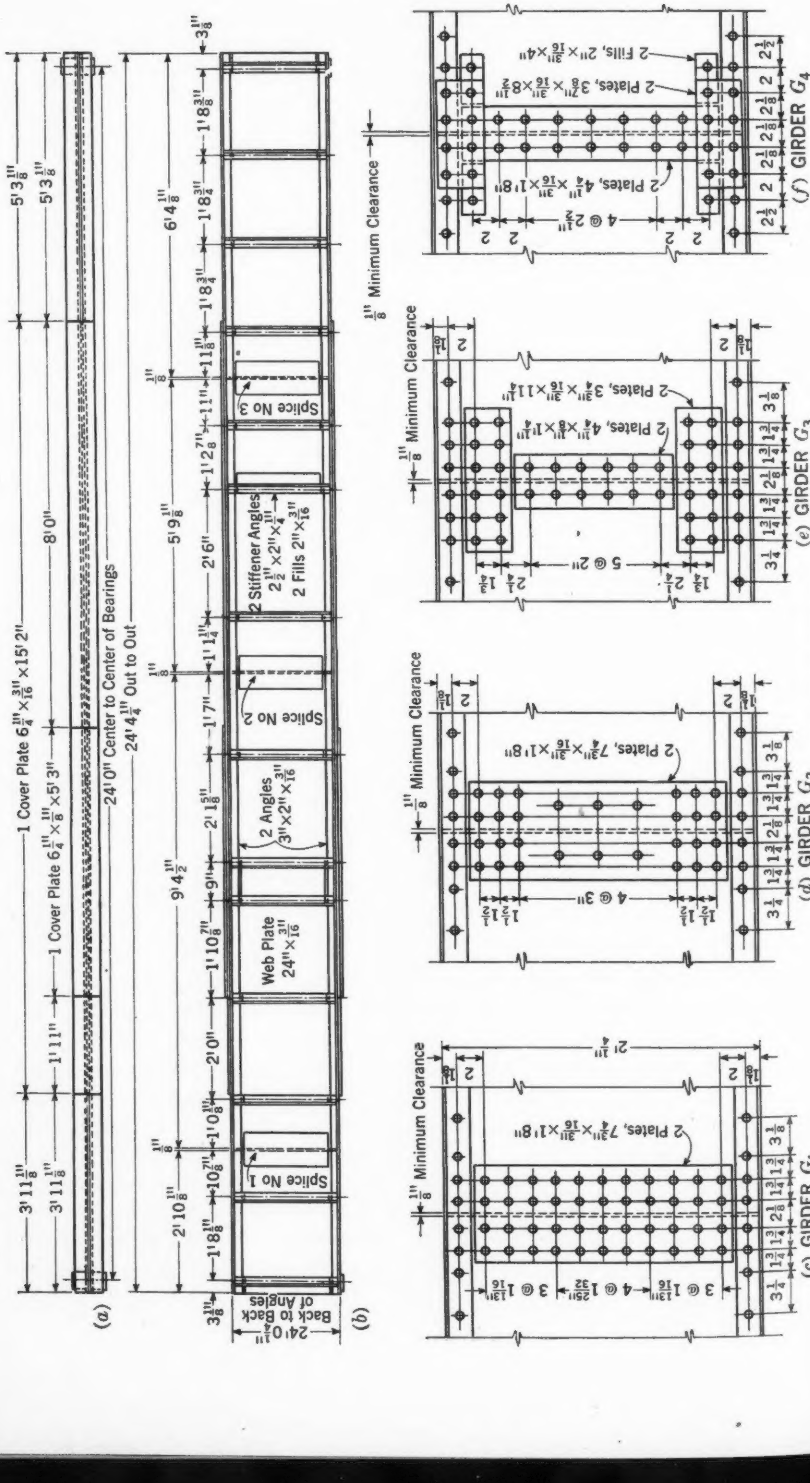


FIG. 2.—DETAILS OF TEST GIRDERS

The splice materials and the rivet arrangement for each type of splice are shown in Figs. 2(*c*), 2(*d*), 2(*e*), and 2(*f*). Each of the four different types of splice arrangement is identified by the same notation as that which is used to designate the girders; that is, G_1, G_2, G_3, and G_4.

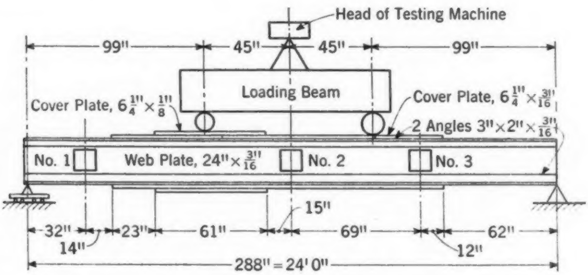


Fig. 3.—Loading Arrangement

The preliminary test program at the time the girders were fabricated called for the application of a single concentrated load at a third point of each girder. The cover plates were designed to suit this loading condition, although the

TABLE 2.—Physical Tests on Steel in Girder

Description	Flange angles	Web plates	3/16-in. cover plates	⅛-in. cover plates
Yield point (lb per sq in.)	43,225	32,950	31,300	33,825
Ultimate strength (lb per sq in.)	61,300	46,300	44,575	48,700
Modulus of elasticity (kips per sq in.)	29,400	29,100	29,300	29,700
Percentage elongation in two inches	46	51	55	49
Percentage reduction of area	69	71	71	69

method of loading the girders was later changed to a symmetrical arrangement.

Tensile flats were made of coupons taken from the angles, cover plates, and web plates of each of the test girders. These specimens were tested in tension to determine the yield point, ultimate strength, ductility and modulus of elasticity. The yield point stress was determined by the "drop of the beam" method. The results of the tests are summarized in Table 2.

Description of Tests

The loading arrangement used for each girder is shown in Fig. 3. The loads were applied to the girders by a 600-kip, hydraulically-operated, universal testing machine, through rollers located near the third points (1 kip = 1 "kilo-pound" = 1,000 lb). End reactions were provided by a knife-edge support at one end of the girder and a roller nest at the other end.

In the first phase of the program, strain readings were taken at 1-kip and 33-kip loads. The difference between these strain readings corresponds to a load of 32 kips which is necessary to produce a theoretical maximum extreme fiber stress of 17.6 kips per sq in. In this elastic range, a line of strain rosettes

was used along the center line of splice 3, and in addition parallel lines of rosettes were taken on the web plate adjacent to the splice plates (see Fig. 1).

Longitudinal strains were measured on vertical legs of the flange angles and on both the inside and outside faces of horizontal elements of the flanges. The strains on the horizontal parts of the flanges were taken along seven gage lines on the outside faces and six gage lines on the inside faces and the average of these values was used for computing the stresses on any face.

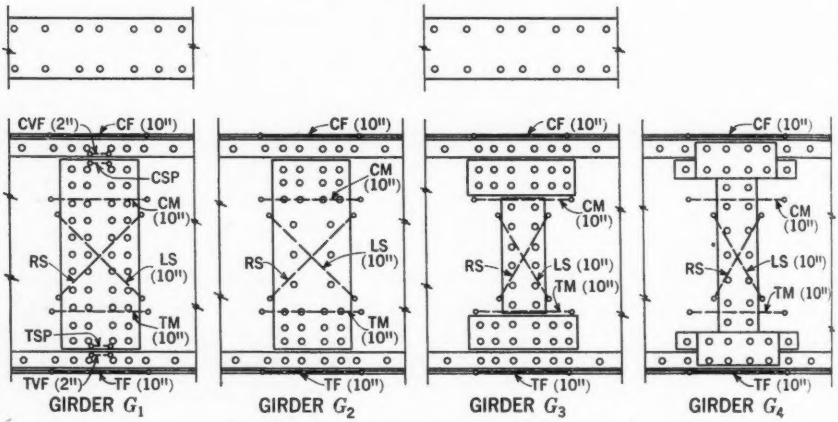


FIG. 4.—GAGE STATIONS FOR STRAIN READINGS

In the second phase or ultimate-load tests, strain-gage readings were taken at the following loads:

Girder	Initial test loads (kips)	Final increments of increase to failure (kips)
G_4	0, 1, 33, 36, 39, etc.	3
G_3	0, 11, 22, 33, 37, 41, 45, etc.	4
G_1 and G_2	0, 33, 37, 41, 45, etc.	4

Strain readings were taken at the stations shown in Fig. 4, in which:

CF = Compression flange—outstanding leg . . . . 10-in. gage length
CVF = Compression flange—vertical leg . . . . 2-in. gage length
CSP = Compression edge—splice plate . . . . 2-in. gage length
CM = Compression rotation . . . . 10-in. gage length
RS = Right shear strain . . . . 10-in. gage length
TSP = Tension edge—splice plate . . . . 2-in. gage length
LS = Left shear strain . . . . 10-in. gage length
TM = Tension rotation . . . . 10-in. gage length
TVF = Tension flange—vertical leg . . . . 2-in. gage length
TF = Tension flange—outstanding leg . . . . 10-in. gage length

RESULTS OF TESTS

First Phase (*Elastic Range Tests*).—Normal and shear stresses along lines A, B, and C for splice 3 of girder G_1, computed from strain rosette readings, are

shown in Fig. 5. Longitudinal stresses for the toes of the vertical legs of the flange angles are also shown. The curves for both measured and theoretical stresses represent the values for a 32-kip load on the girder. The gross moment of inertia of the girder section was used for calculating theoretical stresses.

Curves representing the stresses for girder G_2 are shown in Fig. 6. Stresses for girder G_3 are shown by the curves of Fig. 7. The strain readings for the flange-stress curves were taken at cross sections through the rivets and at cross sections half-way between the rivets. Although these stresses are computed from average strains they show a definite zigzag effect due to the rivets.

The stresses for girder G_4 are shown by the curves of Fig. 8 for five cross sections *A*, *B*, *C*, *D*, and *E*. Longitudinal stresses in the toes of the flange angles, where exposed, and along a corresponding line in the plates covering the angles, are also shown.

The values of the applied moments and shears, and the measured moments and shears for each of the lines of rosettes at and near splice 3, are given in Table 1. The values of the measured moments and shears were computed by numerical summation using the stresses determined from the strain readings.

Second Phase (*Ultimate Load Tests*).—From strain readings on gage lines RS and LS, splice 1 (Fig. 4), the relative vertical or shear displacements were computed and the values are given for all four girders by the curves of Fig. 9.

The relative angular displacements at splice 2, for the four girders, are given by the curves of Fig. 10. This displacement is obtained by dividing the sum of the strains on gage lines TM and CM by the 12-in. distance between these lines.

The average tension and compression flange stresses computed from strains measured on 10-in. gage lengths along lines TF and CF on both sides of the girder at splice 2 are given by the curves of Fig. 11.

TABLE 3.—Results of Breaking Tests (Maximum Loads and Moments)

Girder G_1			Girder G_2			Girder G_3			Girder G_4		
Forces, in Kips		Moment (kip-in.)	Forces, in Kips		Moment (kip-in.)	Forces, in Kips		Moment (kip-in.)	Forces, in Kips		Moment (kip-in.)
Load	Shear		Load	Shear		Load	Shear		Load	Shear	
63.5	31.75	3,140	63.5	31.75	3,140	65.3	32.65	3,230	63.0	31.5	3,120

TABLE 4.—Summary of Design Computations

Description	Web plate	G_1	G_2	G_3	G_4
For rivet group Σd^2 (in.^{2})	....	727	719	895	960
Plate moment, in kip-in.	322.0	366.0	366.0	332.0	797.0
Plate shear, in kips	62.4	97.5	97.5	76.3	135.3
Rivet moment, in kip-in. (no shear)	....	322.0	319.0	391.0	353.0
Rivet shear, in kips (no moment)	....	88.0	60.0	72.0	60.0

Deflection curves for girder G_1 at each of the test loads are shown in Fig. 12. The corresponding curves for the other girders were similar and have not been

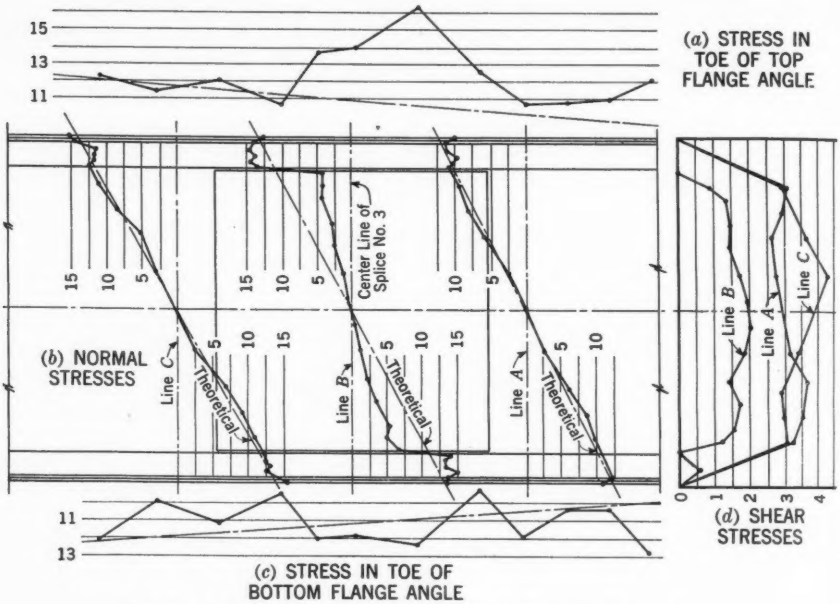


FIG. 5.—STRESSES AT SPLICE 3, GIRDER G_1, IN KIPS PER SQUARE INCH

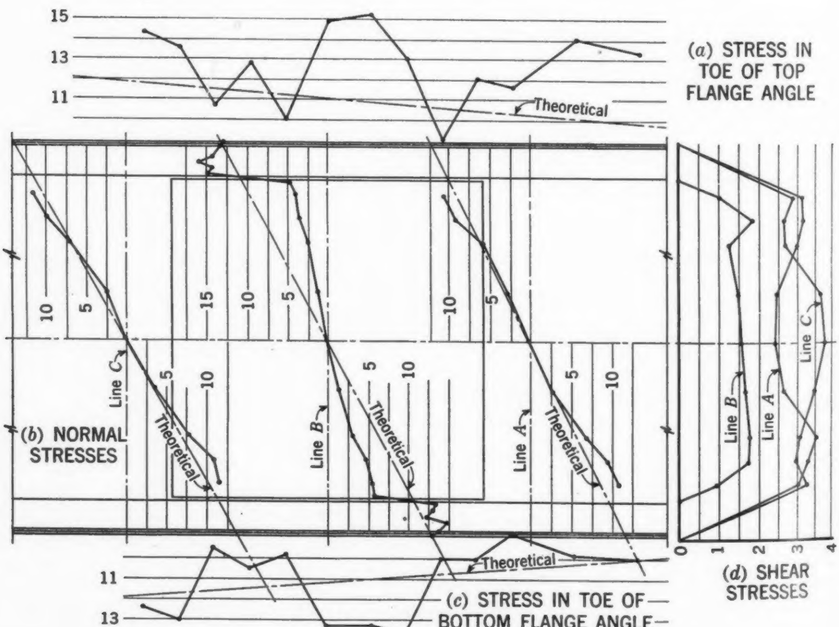


FIG. 6.—STRESSES AT SPLICE 3, GIRDER G_2, IN KIPS PER SQUARE INCH

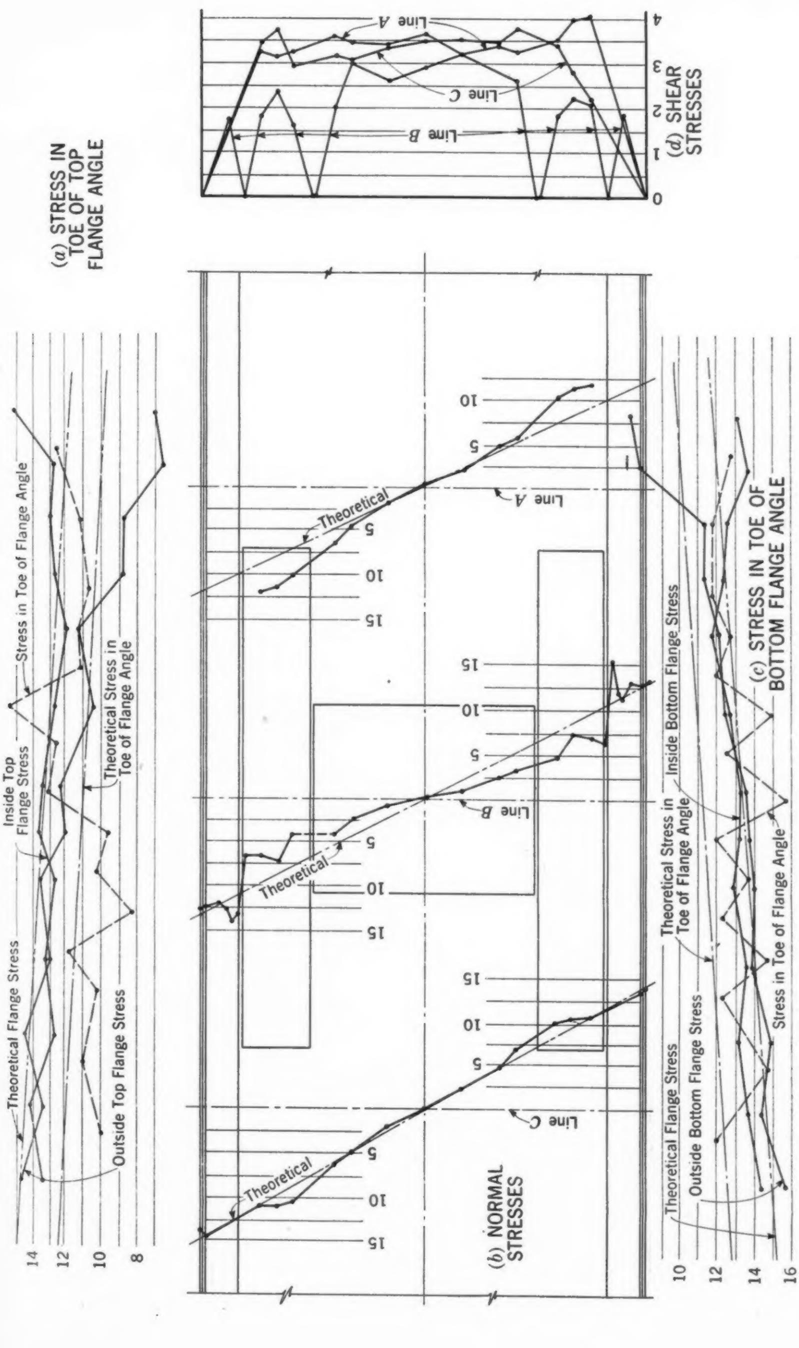


Fig. 7.—Stresses at Splice 3, Girder G_3, in Kips per Square Inch

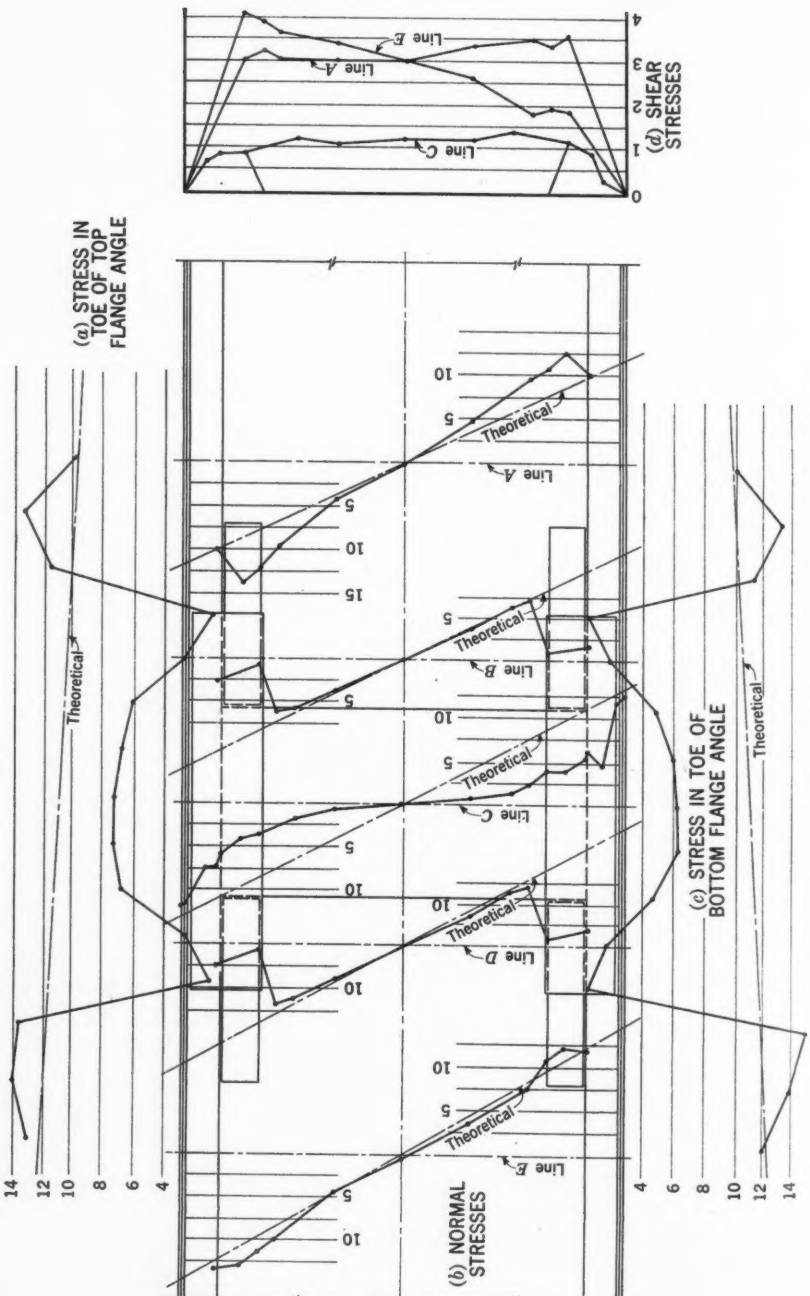


FIG. 8.—STRESSES AT SPLICE 3, GIRDER G_4, IN KIPS PER SQUARE INCH

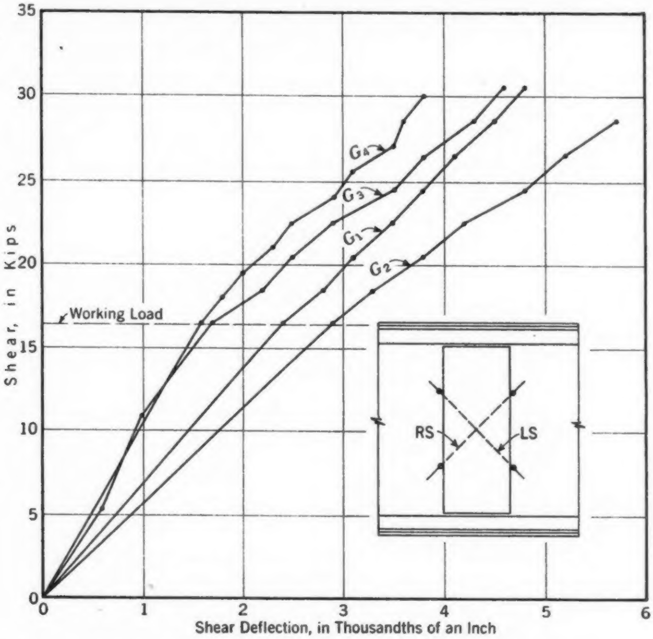


FIG. 9.—SPLICE 1; SHEAR VERSUS RELATIVE SHEAR DISPLACEMENT

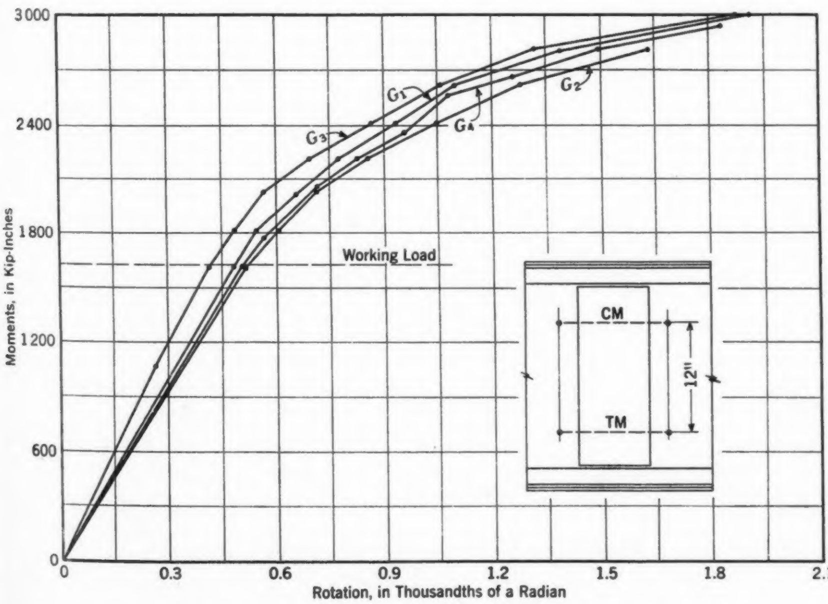


FIG. 10.—SPLICE 3; MOMENT VERSUS RELATIVE ROTATIONS

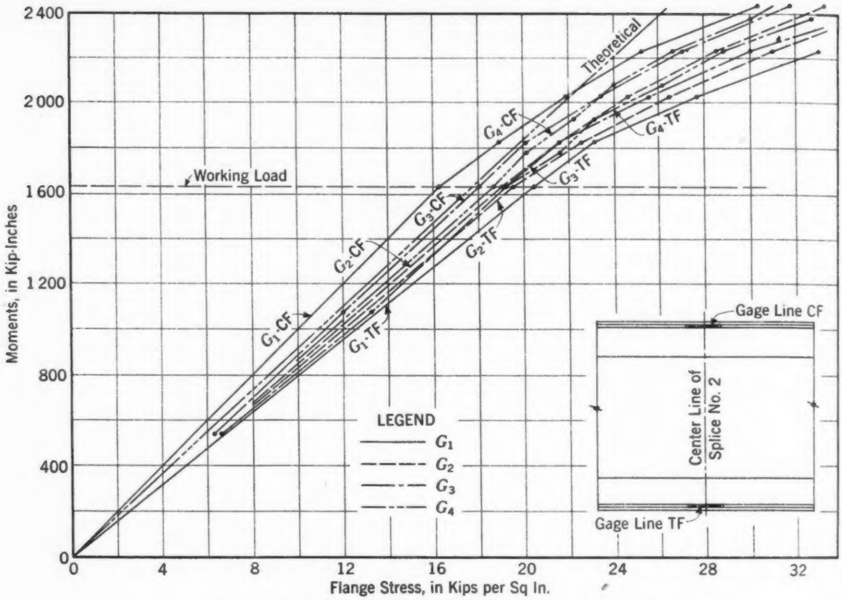


FIG. 11.—SPLICE 2; MOMENT VERSUS FLANGE STRESS

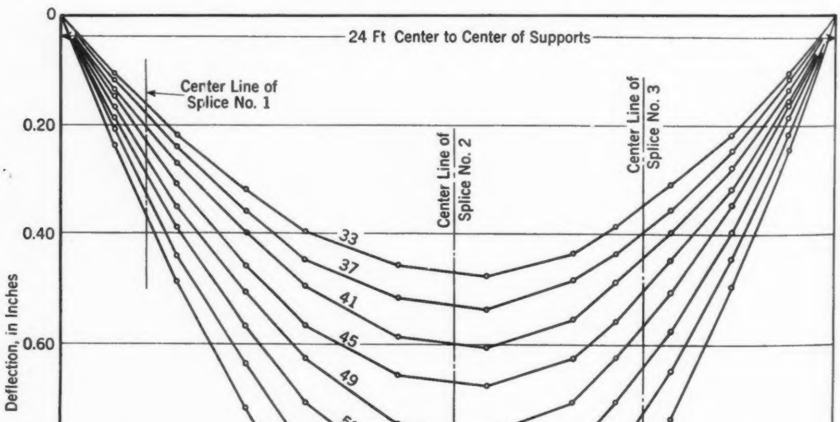


FIG. 12.—GIRDER G_1; DEFLECTION CURVES

included. The curves of load versus maximum deflection in Fig. 13 show a similar behavior for all four girders.

Stresses in the top and bottom edges of the splice plates of girder G_1 and in the edges of the adjacent flange angles are shown, for splice 2, in Fig. 14.

The ultimate loads for the girders are given in Table 3. In each case, failure was by buckling in the top flange, at splice 2 (see Fig. 15).

Discussion of Results

A summary of the design values for the four types of splices tested is given in Table 4. The design stresses and loads are as follows:

Unit stress	Kips per square inch
Bending in extreme fiber	18
Shear on gross section	13
Allowable stresses for rivets—	
Shear	15
Bearing (double shear rivets)	40
Maximum stress in plates at a distance d from the neutral axis	$18 \times \frac{d}{12.3}$
Working load, P, to produce a maximum extreme fiber stress of 18 kips per sq in.	32.8
Maximum moment at working load in kip-in.	(1,620)

The maximum stress in the rivet of any group due to bending is taken at the maximum bearing value. The theoretical moments and shears in the 20-in. depth of the web plates between the flange angles and the values computed from the measured strains are also included in Table 1.

An examination of the curves for direct stress reveals that at the center line of splice the stress is not directly proportional to the distance from the neutral axis. This variation is partly dependent on the relative thicknesses of the web plate and splice plates, on the length of splice plates parallel to the center line of girders and on the number of rivets in the splice plates.

Computations show that the moments carried by the splice plates on girders G_1, G_2, and G_3 are equal to the moments in the 20-in. depth of the web plate between the flange angles (Table 1). The stress due to moment in the web plate under the flange angles is taken by the flanges at the cut section. This increase in the flange stress is effective over a short length and is greater in the vertical legs of the flange angles than in the horizontal parts of the flanges.

The direct stresses along lines A and C adjacent to the spliced section of girder G_1 are essentially in agreement with the theoretical values. The curves representing the direct stresses on lines A and C for girders G_2 and G_3 and lines A and E for girder G_4 show some variation from the theoretical values. The lines representing the theoretical stresses are assumed to be unaffected by the adjacent splice.

From the data of Table 1, it is seen that the measured shear in the splice plates along the center line of splice is less than the total shear on the section. This suggests that the flange angles carry a greater part of the shear than is

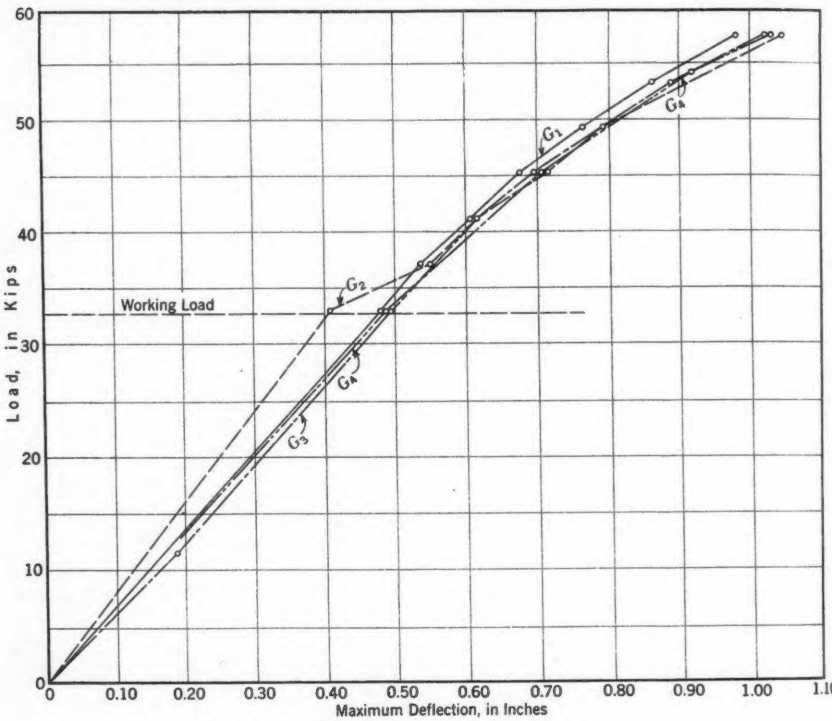


FIG. 13.—LOAD VERSUS MAXIMUM DEFLECTION

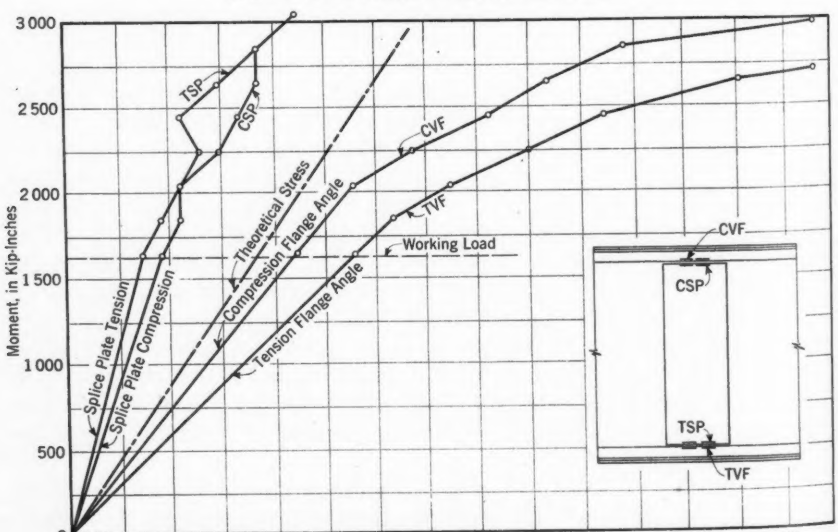


FIG. 14.—GIRDER 1, SPLICE 2; FLANGE STRESSES (2-IN. GAGE LENGTHS)

usually assigned to them. The stress curves for the toes of the flange angles show that the angles are subjected to a secondary bending stress at the cut section due to this shear. The well-known parabolic curve of shear distribution results when the loads producing the shear are farther apart than the depth of the beam. Since the loads taken by the splice plates are applied through the rivets, the shear distribution in the splice plates is affected by this method of loading the plates. There is also some disturbance in the shear curves for the lines outside the splice plates due to the adjacent splices.

The relative shear-displacement curves of Fig. 9 show a slightly different behavior for the girders. The curve for G_1 is essentially a straight line. The girders had all been subjected to the working load several times so that only the part of the curves above that load represents the action under the first application of load. The displacement at the splice on girder G_2 is about 80% greater than that of G_4 at the working load and that of G_1 is about 50% greater at that load.

The relative rotation curves of Fig. 10 show that the four splices act in a similar manner in the region of pure moment. At the working load the variation between the degree of rotation for G_3 which was minimum and G_2, the maximum, was about 25%.

The flange-stress curves of Fig. 11 show a similar behavior for the flanges of all girders. At the working load and above, the flange stresses are higher than the theoretical value based on the gross section.

The deflection curves of Fig. 12 do not indicate a noticeable weakness in any of the splices. The curves of Fig. 13 break away from the straight line at a load of about 40 kips, which is consistent with the action shown by the curves of Figs. 10 and 11.

The curves of Fig. 14 show that the stresses developed at the inside edges of the flange angles are larger than the theoretical, and the stresses in the adjacent splice plates are less than the theoretical, although both elements are essentially the same distance from the neutral axis. The curves indicate that there is a difference in these stresses for all loads and that the angles and the splice plates are acting independently. This difference is affected by the local bending in the angles and by the stress transfer through the rivets.

Behavior of the different splices was similar in character but different in degree. The action of splice G_4 was somewhat involved, but the values obtained indicate that the flange splice plates did not have enough rivets to develop the full action of the plates.

Distribution of stresses recorded during the breaking tests is consistent with that obtained for the elastic range. The buckling of the compression flanges occurred near the splice and in the region carrying an overstress due to the splicing action of the flange. Although failure did not occur in the splices, the relative actions of the four types of splices can be observed. For a girder of the type tested, there can be no failure of the splice due to moment until after one of the flanges has yielded.

The load tests on the girders indicate the following conclusions:

(1) For all of the splices tested, each component part of any splice carried only the stress in that part of the web plate beneath that element of the splice.

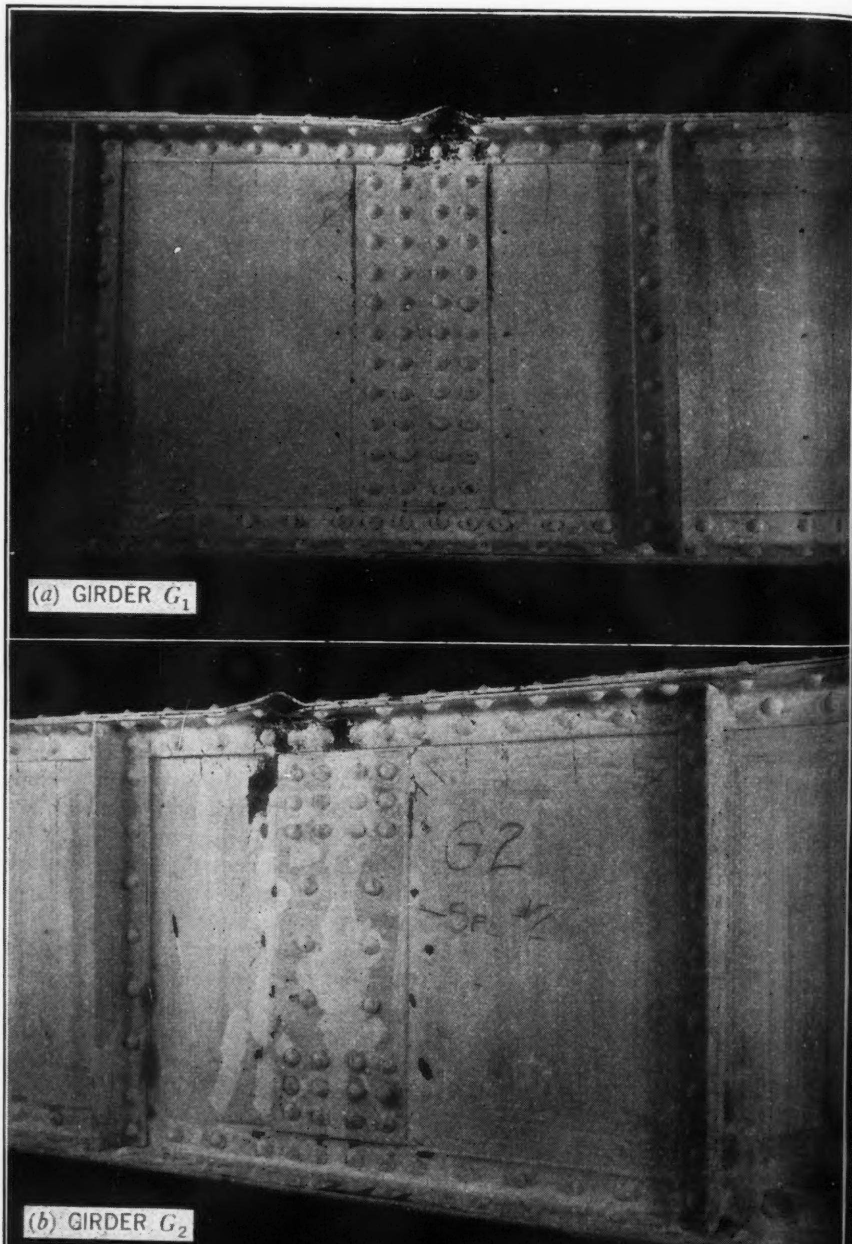
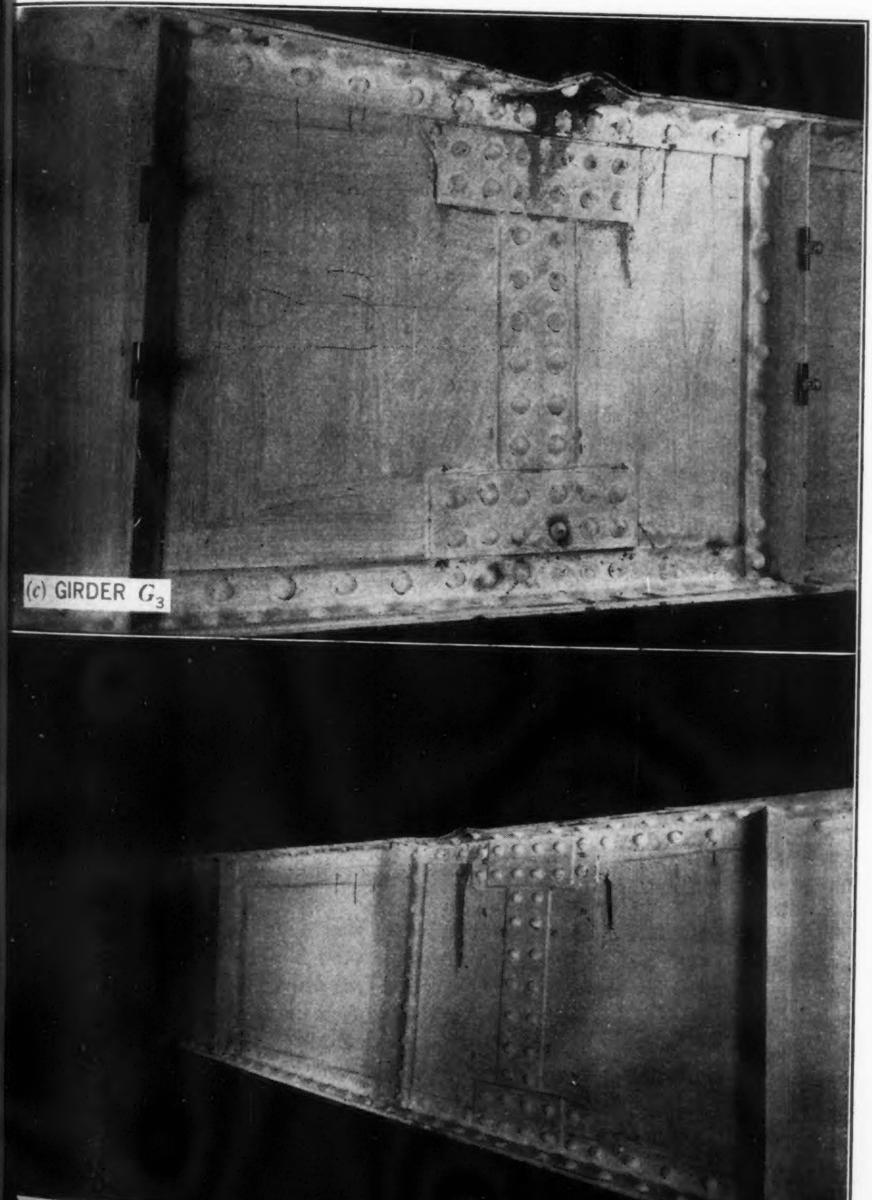


FIG. 15.—VIEWS OF TES



SPECIMENS AT FAILURE

(2) That part of the web-plate stress, not carried by the splice plates, produces an increased stress in the flanges.

(3) The splices on girders G_1, G_2, and G_3 carried only the moment in the 20-in. depth of web plate between the flange angles.

(4) At a spliced section, the component parts of the splice do not act together as a unit with the girder unless the splice is designed so that the strains in the elements of the splice are consistent with those of the girder.

(5) A splice that is weak in shear-carrying capacity will cause secondary bending stresses in the flange angles due to the shear carried by the angles.

(6) The behavior of all splices within the working load was satisfactory.

(7) The length of splice material along the girder apparently affected the action of the splices. The longer splice material of girder G_3 restrained the angular rotation at the splice. It is believed that splices of G_1 or G_2 type would have been at least as effective as G_3 in this respect if they had been extended one line of rivets on either side.

(8) A variation in the type of web splice used, between those tested, does not appreciably affect the ultimate strength of the girder if failure does not occur in the elements of the splice itself.

(9) Excess stress in the flanges, produced by the splicing action of the flanges, probably influenced the buckling of the compression flange and caused that flange to buckle at a slightly lower load than it would have for an unspliced girder.

(10) The lack of shear resistance at horizontal planes where the splice plates were discontinuous did not measurably affect the behavior of the splice.

Acknowledgment

The tests reported herein were planned by the American Institute of Steel Construction in cooperation with the Research Laboratories of the Department of Civil Engineering, Columbia University, New York, N. Y. Preliminary studies were made by Bruce Johnston, Assoc. M. Am. Soc. C. E., in 1938 while he was at Columbia University. The girders were designed and fabricated by the Bethlehem Steel Company.

F. H. Frankland, M. Am. Soc. C. E., and other members of the Committee on Technical Research of the Institute cooperated in the development of the program and offered valuable suggestions and criticisms as the work progressed.

The conduct of the tests was under the supervision of Prof. W. J. Krefeld, M. Am. Soc. C. E., director of the Engineering Materials' Laboratories, Columbia University, who also gave freely of his time and advice in the analysis and interpretation of results.

The authors also desire to express their appreciation for the encouragement and cooperation of J. K. Finch, M. Am. Soc. C. E., head of the Civil Engineering Department.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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PAPERS

A DIRECT METHOD OF FLOOD ROUTING

By C. O. Wisler,[1] M. Am. Soc. C. E., and E. F. Brater,[2] Jun. Am. Soc. C. E.

Synopsis

A method of flood routing is described herein, the successful use of which depends only upon the availability of dependable stream-flow records during a typical flood at various points on the main stream or on the tributaries whose flow is to be routed downstream. No cross sections of stream channel or velocities of flow are required. Nor are discharge records on all of the tributaries needed. A hydrograph of inflow from the unmeasured area is directly computed. This flow and that at each of the upstream stations is then routed downstream.

These routed flows show the extent to which each of the upper tributaries contribute to the flood peak at each downstream point. A check on the accuracy of the results is provided by adding the routed flows and comparing the resulting hydrograph with the actual records.

The entire procedure is based upon the storage equation and upon the principle that, for all high stages, there is a straight-line relationship between the volume of storage contained in any reach of river channel and the sum of the inflow rate at the upper end and the outflow rate at the lower end of that reach. Except perhaps for unusual channel conditions, this relationship holds true.

Introduction

In large drainage basins, floods usually wreak their greatest havoc at some downstream point. At any instant the total flow at this point comprises waters contributed by many different tributaries. The percentage of the total flow occurring at any instant that is contributed by any single tributary is constantly changing throughout the flood period. Perhaps the maximum flow from no tributary reaches this point at the same time that the total combined flow

Note.—Written comments are invited for immediate publication; to insure publication the last discussion should be submitted by **October 15, 1941.**

[1] Prof., Hydr. Eng., Univ. of Michigan, Ann Arbor, Mich.

[2] Asst. Prof., Civ. Eng., Univ. of Michigan, Ann Arbor, Mich.

becomes the maximum. The flood waters from some of the tributaries may have passed this point long before the peak stage occurs, whereas the waters from some of the more remote tributaries do not arrive until after that stage has passed.

In all studies in which it is proposed to secure improved conditions by reducing the flood peak through the use of reservoirs, by reforestation, or by means other than local protective measures, such as levees and flood walls, it is essential to know to what extent each of the several tributaries contributes to the resulting flood peak. Without this information no intelligent planning of flood control is possible. With a hope of reducing the downstream flood peak, vast expenditures might be made on a certain tributary, the net result of which would actually be negligible or even damaging instead of beneficial.

The writers' interest in this problem was aroused by the Potomac River flood-control studies of the U. S. Department of Agriculture. These studies are being made as an inter-bureau endeavor by the Forest Service, Soil Conservation Service, and Bureau of Agricultural Economics under the chairmanship of the Appalachian Forest Experiment Station at Asheville, N. C. In connection with that work this method of flood routing was developed and successfully applied to the Shenandoah River, a tributary of the Potomac, and later to the Monongahela River in West Virginia.

It is believed that the clearest presentation of this method of flood routing can be made by means of an illustrative example. Because of the excellent discharge records that are available and because of a complication resulting from a subsequent rain that fell during the recession period of the hydrograph, the March, 1936, flood on the Monongahela has been selected for a detailed presentation.

The Monongahela River rises in the north central part of West Virginia, and flows in a general northerly direction through Morgantown, W. Va., which is a few miles south of the Pennsylvania boundary (see Fig. 1). The drainage area above Morgantown is 2,670 sq miles. The principal tributary is the Tygart River which empties into the Monongahela approximately thirty miles upstream from Morgantown. The drainage area above Fetterman, W. Va. (later a part of Grafton, W. Va.), which is about twenty miles above the mouth of the Tygart, is 1,340 sq miles. Discharge records taken at 2-hr intervals at Morgantown and Fetterman are published in U. S. Geological Survey *Water Supply Paper No. 800*.

The first operation described in the paper is the derivation of the hydrograph of inflow into the intervening area between Fetterman and Morgantown. This inflow and the measured discharge at Fetterman are then routed separately to Morgantown. The procedure for determining the effect at Morgantown of flood-control measures on the Tygart is discussed, and a method of determining corresponding effects at other points on the stream is suggested. Finally, the results of two more applications of the method are briefly presented.

Determination of Unmeasured Inflow

The first step in the solution of this problem lies in the determination of the hydrograph of channel inflow from the intervening area. The intervening

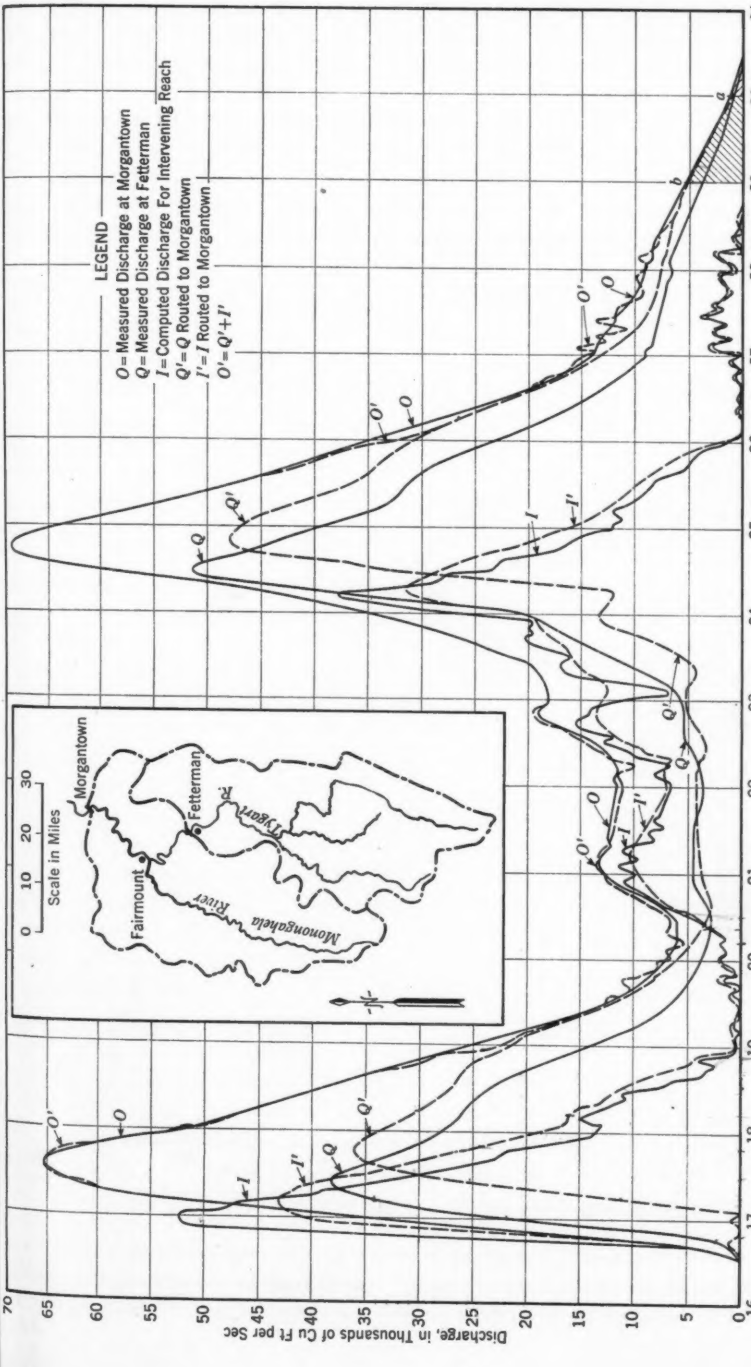

FIG. 1.—HYDROGRAPHS OF SURFACE RUNOFF, WATERSHED OF THE MONONGAHELA RIVER

area here includes all of the drainage area above Morgantown exclusive of the area above Fetterman. These rates of inflow are determined by applying the storage equation to the entire river channel above Morgantown exclusive of the channel above Fetterman for successive 2-hr intervals. In so doing, let Q_0, O_0, and S_0 represent, respectively, the discharge at Fetterman, the outflow at Morgantown, and the storage in the intervening reach, all at the beginning of the period considered. Then let Q_1, O_1, and S_1 represent, respectively, the same quantities at the end of the period. Also let I represent the average inflow from the intervening area during the same period. The resulting storage equation is

$$S_0 + 2\left(\frac{Q_0 + Q_1}{2}\right) + 2\,I - 2\left(\frac{O_0 + O_1}{2}\right) = S_1 \qquad (1)$$

If the periods considered were any other than 2-hr periods, Eq. 1 would have to be changed accordingly. The equation may be written in the form

$$I = \frac{S_1 + O_0 + O_1 - (S_0 + Q_0 + Q_1)}{2} \qquad (2)$$

in which all quantities on the right-hand side of the equation are known except S_1 and it can be determined from a curve of relation between S and $(Q + O)$ shown in Fig. 2.

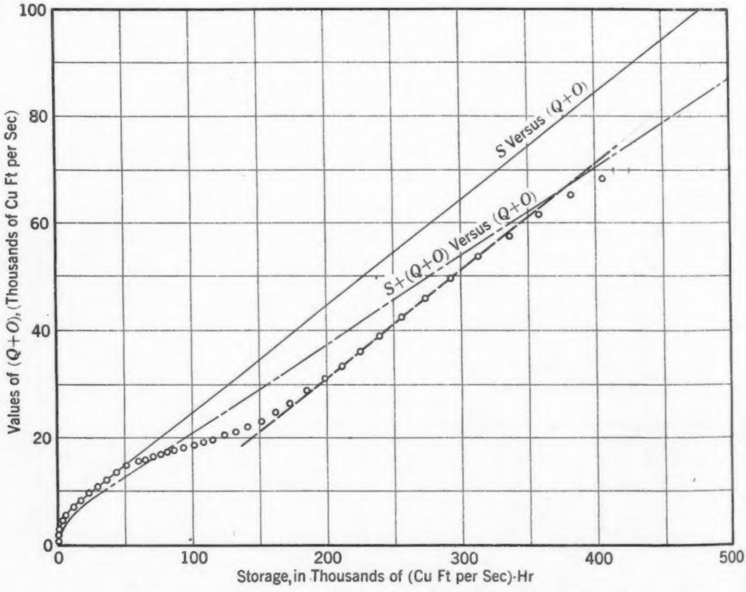

FIG. 2.—RELATION BETWEEN $(Q + O)$ AND THE STORAGE IN THE INTERVENING REACH

The derivation and characteristics of the curve of relationship between S and $(Q + O)$ will be discussed in some detail. The storage in the intervening reach is obtained by making use of a principle that was apparently first utilized by R. E.

Horton,[3] M. Am. Soc. C. E.—namely, that at any time after the point of contraflexure occurs on the recession side of a hydrograph of surface runoff, the total volume of channel storage is equal to the total area beneath the hydrograph subsequent to that time. For instance, in Fig. 1, the total volume of channel storage above Morgantown at noon on the 30th is equal to the area of the shaded part beneath the hydrograph marked O and to the right of a. Also the total channel storage at noon on the 29th is represented by the shaded area to the right of b, etc. In a similar manner, the total channel storage occurring at any instant in the Tygart River above Fetterman can be determined from the hydrograph Q, Fig. 1. By deducting the channel storage above Fetterman occurring at any instant from the simultaneous storage above Morgantown, the volume of channel storage contained in the intervening reach is obtained.

For any given reach of river channel, there is a definite relationship between the volume of channel storage contained therein at any instant and the sum of the simultaneous discharge rates at the two ends of the reach. Such a relationship exists because Q is proportional to the river stage at the upper end of the reach, O is proportional to the stage at the lower end, and therefore $(Q + O)$ is proportional to the average stage which in turn depends upon the volume stored. When these two quantities are plotted on ordinary cross-section paper, the graph is a straight line, except for the lower stages. This result is most fortunate because if this relationship did not plot as a straight line, it would be extremely difficult to extend the curve into the higher stages in such a manner as to eliminate the effects of overland flow. It could probably be done only by trial and error and would be a long and laborious process.

In the foregoing, the fact was stated that Mr. Horton's method of determining the volume of channel storage from the area beneath the hydrograph is applicable only back to the point of contraflexure. It is believed that overland flow ceases at or near this point. This concept is substantiated by the experience of the writers that, in every application made to date (1941), this straight-line relationship holds true only for values derived to the right of the point of contraflexure. If the derivation is extended much beyond that point, the storage values all fall to the right of the straight line, indicating that some overland flow is being included as channel storage. As an illustration, it may be noted that in Fig. 2 the points fall to the right of the straight line for values of $(Q + O)$ greater than 58,000 cu ft per sec. It may be seen from Fig. 1 that $(Q + O)$ is equal to 58,000 at noon on March 26, which is approximately at the point of contraflexure of the curves of Q and O.

To obtain the hydrographs of surface runoff at Morgantown and at Fetterman (see graphs O and Q, Fig. 1) it was necessary to deduct the ground-water flows from the total discharges. This was done by drawing, on the hydrographs of total flow, straight lines connecting the points at which surface runoff appeared to start and end. Although this procedure is perhaps not accurate, it is sufficiently correct for this purpose.

[3] "Surface Runoff Phenomena," by Robert E. Horton, *Publication 101*, Horton Hydrological Lab., Voorheesville, N. Y., 1935, p. 35.

In Fig. 2 the points indicated by small circles were obtained by plotting the value of $(Q + O)$ at any instant as shown in Fig. 1 against the area between the curves Q and O subsequent to that instant. For example, at 2:00 p.m. on the 29th, $O = 4{,}800$, $Q = 3{,}540$, and $S = 18{,}840$ (cu ft per sec)-hr, the latter value representing the area between the graphs O and Q (Fig. 1) after 2:00 p.m. on the 29th. At noon on the 29th, $O = 5{,}200$, $Q = 3{,}720$, and therefore

$$S = 18{,}840 + 2\left(\frac{5{,}200 - 3{,}720 + 4{,}800 - 3{,}540}{2}\right) = 21{,}580 \text{ (cu ft per sec)-hr.}$$

In this same manner the other points indicated by circles in Fig. 2 were obtained.

It should be observed that for values of $(Q + O)$ between 8,000 and 15,000, and also between 30,000 and 55,000, these points fall on two straight lines and furthermore that these lines are parallel. Now the value of $(Q + O)$ was 30,000 at 4:00 a.m. on the 27th and 15,000 at 6:00 p.m. on the 28th. During the intervening period channel inflow was taking place as a result of a subsequent rain that fell during the recession period. This channel inflow from the intervening area amounts to about 65,000 (cu ft per sec)-hr and is represented by the area under the hydrograph I during this period. Its net effect is to move the S versus $(Q + O)$-curve horizontally to the right a distance representing 65,000 (cu ft per sec)-hr. Had it not been for this subsequent channel inflow, the curve of relation between S and $(Q + O)$ would have extended beyond the value of $(Q + O) = 15{,}000$ cu ft per sec as shown by the solid line in Fig. 2. This solid line, therefore, is taken as the curve of relation between S and $(Q + O)$. Since both Q and O are known, it is possible to determine the values of S corresponding to any value of $(Q + O)$ and thus solve equation 2 for I. The graph of I (Fig. 1) was obtained in this manner.

A check on the accuracy of these computations can be made by deducting the total discharge at Fetterman from the total outflow at Morgantown and comparing this quantity with the total computed inflow from the intervening area. In this case the total outflow at Morgantown is equal to 3,894,200 (cu ft per sec)-hr, and the total discharge at Fetterman is 2,326,900 (cu ft per sec)-hr. The difference between these two quantities is 1,567,300 (cu ft per sec)-hr as compared with a total computed inflow of 1,566,900 (cu ft per sec)-hr.

Such a remarkably close agreement should not be expected in all cases. Nevertheless, the maximum error experienced by the writers to date has amounted to about 6%. In such cases all computed inflows from the intervening area may be corrected by a constant percentage in order to make them equal to the difference between Q and O. If the error should be considered to be excessive, however, it might be better to reconstruct the S versus $(Q + O)$-curve by trial until the error virtually disappears. This latter procedure would have to be followed if the stations were separated by such a great distance that this curve would perhaps deviate from a straight line.

Routing Q and I Through to Morgantown

The next step in the solution of this problem is the determination of the hydrographs at Morgantown resulting from (1) the foregoing discharge rates

at Fetterman, and (2) the inflows from the intervening area. In other words, when do the peak flows from these two sources reach Morgantown? What are their magnitudes? How are the total flows distributed at that point? In answering these questions, the intervening channel is considered to be a storage reservoir through which the flow from each source is routed separately and independently of the other. A check upon the accuracy of the results is then obtained by combining these two graphs of routed flows at Morgantown and comparing the resulting hydrograph with the actual recorded hydrograph at that point.

First, route the Fetterman discharge. For this purpose, the inflow from the intervening area, I, will be ignored and the storage formula (Eq. 1) may be written as follows:

$$S_0 + Q_0 + Q_1 - O_0 = S_1 + O_1 \qquad (3)$$

All of the quantities on the left-hand side are known, thus giving the sum of S_1 and O_1 but not their individual values. These may be obtained by adding Q_1 to each side of the equation, thus giving Eq. 4,

$$S_0 + Q_0 + 2\,Q_1 - O_0 = S_1 + (Q_1 + O_1) \qquad (4)$$

which can be solved by means of a curve of relation between $(Q + O)$ and $S + (Q + O)$, as shown in Fig. 2. Since Q is known, O can be obtained, and finally S. These values of S_1, Q_1, and O_1 become the values of S_0, Q_0, and O_0 for the next period and the computations become continuous.

It should be observed that O here takes on a significance quite different from that which it possessed previously. Heretofore it represented the total recorded outflow at Morgantown, whereas it now represents the computed outflow at Morgantown resulting from the flow Q at Fetterman. In Fig. 1 this quantity is designated as Q' and the same symbol would be used here were it not for the confusion that would result from the use of Fig. 2.

In routing I through to Morgantown, since Q is now ignored completely, Eq. 1 may be written

$$S_0 + 2\,I - O_0 = S_1 + O_1 \qquad (5)$$

In this equation O again takes on a new significance, now representing the computed outflow at Morgantown resulting from the inflow from the intervening area. In the solution of Eq. 5 all the quantities on the left-hand side are known, and therefore $S_1 + O_1$ is known. Referring now to Fig. 2, since Q is ignored, the dash-dot curve becomes one of O versus $(S + O)$, and from it O can be determined, and finally S. These values of O_1 and S_1 then become the values of O_0 and S_0 for the next period and the procedure is repeated. These outflows at Morgantown resulting from the inflows from the intervening area are shown graphically in Fig. 1 by the dotted line marked I'.

By combining the graphs I' and Q', Fig. 1, the outflow graph, O', at Morgantown is obtained. The agreement of this graph with the actual hydrograph O is quite remarkable, especially throughout the higher stages during which the greatest need exists for accurate and dependable results.

Reduction of Flood Peak at Morgantown

After the flood of March, 1936, a dam and storage reservoir were constructed on the Tygart River near Fetterman. The effectiveness with which the storage thus provided may be expected to reduce the flood peak at Morgantown or at any other downstream point depends upon the rules that are formulated governing the operation of that storage. The reservoir outflows resulting from any such proposed set of rules can be computed and this revised Q can then be routed through and the reduction benefits determined at points downstream.

On the other hand, if it is proposed to reduce the flood flows at the upper station by terracing, by change in land use, or by some other means, then a revised hydrograph of Q at that point must necessarily be based on judgment.

After the revised Fetterman hydrograph has been routed through to Morgantown, this revised graph of Q' can be combined with I' to give a revised graph of O'. Making use of the stage-discharge relationship curve this revised O' hydrograph can be converted into a stage graph which, upon comparison with the original records, will show the amount of reduction in stage at Morgantown resulting from the flood control improvements made above Fetterman. A similar procedure can be followed for determining the amount of stage reduction that can be effected at Morgantown from any contemplated improvements in the intervening area.

Reduction of Peak at Other Points

One is not always so fortunate as to have discharge records available at all damage centers where flood reduction benefits are to be appraised. So frequently are such records missing that it seems desirable to summarize, briefly, a procedure that may be followed in such cases. Suppose, for instance, that the damage center is at Fairmont, West Virginia, which is just below the junction of the Tygart and the Monongahela. The reduction at this point may be found as follows:

(1) Determine the inflow from the intervening area between Fetterman and Morgantown and route this flow and the Q at Fetterman through to Morgantown in the manner already described.

(2) Measure the intervening area above Fairmont and find the percentage that this area is of the total intervening area above Morgantown. Then apply this percentage to the various inflows already computed (shown graphically as I, Fig. 1) to obtain a hydrograph of I for the intervening area above Fairmont.

(3) Construct a stage-discharge relationship curve for Fairmont based upon any discharge measurements that may be obtainable during normal and low stages and upon high water-marks for floods whose discharges are known at Fetterman and at Morgantown. If the approximate stage at Fairmont corresponding to zero discharge can be determined, a satisfactory stage-discharge curve should be obtained by plotting the foregoing data on logarithmic paper.

(4) A curve must now be derived showing the relation between channel storage in the intervening reach above Fairmont and the sum of the discharges at Fetterman and at Fairmont. For any value of $(Q + O)$ the total channel

storage above Fairmont can be closely approximated by prorating the total storage above Morgantown in the ratio of the drainage area above Fairmont to that above Morgantown. For the ordinates of this curve Q remains the same as before, but O must also be reduced in the ratio of these two drainage areas. An auxiliary curve showing the relation between $(Q + O)$ and $S + (Q + O)$ should now be plotted. With the aid of this curve and the storage equation, the outflows at Fairmont can be found in a manner previously described.

From this point, the procedure for the determination of the effect of a storage reservoir or other proposed improvement, either above Fetterman or in the intervening area, is the same as has already been described.

Applications to Shenandoah River

In order to test this method of flood routing for different types of stream pattern and shapes of drainage basin, two other routings were made, both on the Shenandoah River in Virginia. A comparison of the watersheds in Figs. 1 and 3, together with Table 1, shows these differences of conditions quite clearly.

For these Shenandoah tests a flood was used that occurred in March, 1936, at the same time as the preceding one used on the Monongahela. The discharge records are also obtained from *Water Supply Paper No. 800.*

In determing the inflow occurring at any instant from the intervening area above Lynnwood the sum of the channel storages existing simultaneously above the stations on North River at Burketown, Middle River near Grottoes, and South River at Harriston was deducted from the total channel storage above Lynnwood. This storage was plotted against the sum of the outflow at Lynnwood and the discharges at the upper three stations. These three discharages were then routed simultaneously through to Lynnwood although it is believed that the same result would have been obtained had each been routed separately and then combined later. The Lynnwood discharge, and also that from the intervening area below Lynnwood, were then routed to Front Royal.

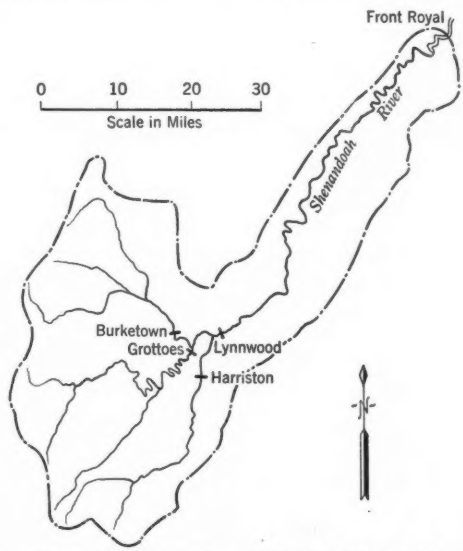

Fig. 3.—Watershed of the Shenandoah River, Above Front Royal, Va.

The results of these two routings are shown in Fig. 4. At Lynnwood the sum of the routed flows agrees almost perfectly with the actual records. At Front Royal, however, the agreement is not as good, although it can hardly be classed as poor.

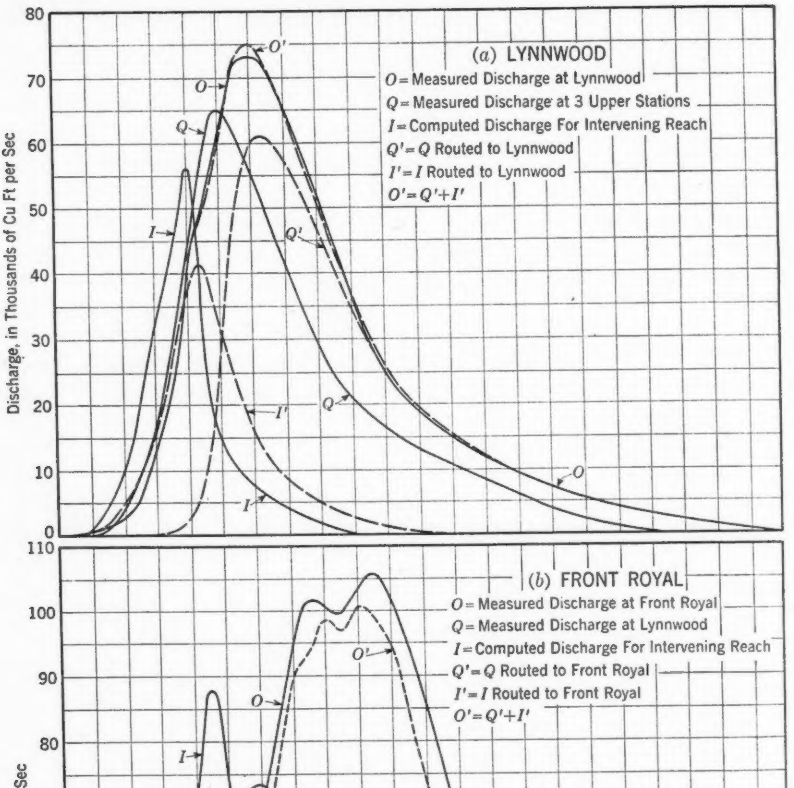

FIG. 4.—HYDROGRAPHS OF SURFACE RUNOFF

TABLE 1.—WATERSHED CHARACTERISTICS

Lower station	Area, in sq miles	Upper station	Area, in sq miles	INTERVENING: Distance, in miles	INTERVENING: Area, in sq miles
Morgantown, W. Va.	2,670	Fetterman, W. Va.	1,340	50	1,330[a]
		Burketown, Va.	381	15	
Lynnwood, Va.	1,076	Grottoes, Va.	360	10	113[b]
		Harriston, Va.	222	10	
Front Royal, Va.	1,638	Lynnwood, Va.	1,076	70	562[c]

[a] Long and nearly parallel with upper basin. [b] Wide, short, and entirely downstream from upper basins. [c] Long, narrow, and entirely downstream from upper basin.

The probable reason for the failure to obtain better results at Front Royal may be found in the following statement:[4]

> "Stage-discharge relation.—(At Front Royal) Defined by current-meter measurements below 25,000 second-feet; extended to peak stage by velocity-area study near control section, slope-area determination, and comparison of peak discharge and total run-off of flood with other records on the same or neighboring streams. Discharge may have been affected by backwater from North Fork of Shenandoah River from about 8 p.m. Mar. 17 to about 2 p.m. Mar. 18. Discharge at 8 a.m. Mar. 18 may have been about 75,000 second-feet instead of 99,900 second-feet. Volume of run-off during the flood rise would be reduced by a small percentage."

Because of the backwater to which reference is made in the foregoing notes, and also because of the fact that it was necessary to extend the discharge curve from 25,000 cu ft per sec to 113,000 cu ft per sec, it is easily conceivable that the entire disagreement between the computed graph O' and the hydrograph O, Fig. 4(*b*), may be due to faulty discharge records at Front Royal.

CONCLUSION

It is believed that, except possibly under unusual channel conditions, the curve of relationship between channel storage in the intervening reach and the sum of the discharges at the two ends of the reach is a straight line for the normal and high stages of the river. This straight line together with a supplementary curve expressing the relationship between $(Q + O)$ and $S + (Q + O)$ permits the direct determination of: (*a*) A hydrograph of channel inflow from the intervening area; and (*b*) the routed flows from the upper area and from the intervening area as they appear at the lower station. A positive check on the accuracy of both of these determinations can be made quickly and easily.

Furthermore, it is the belief of the writers that the accuracy of the results that can be obtained by this method of flood routing depends almost entirely upon the availability of: (*a*) Good stage-discharge relationship curves at various points on the stream; and (*b*) an actual flood record. With these data at hand, either the known flood or any hypothetical flood can then be routed downstream, and the benefits that may be expected from any proposed system of storage reservoirs or other plan of flood control can be determined definitely, and the benefits can be weighed against the cost.

[4] *Water Supply Paper No. 800*, U. S. Geological Survey, p. 114.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
REPORTS

LAND SURVEYS AND TITLES

SECOND PROGRESS REPORT OF THE JOINT COMMITTEE OF THE REAL PROPERTY DIVISION, AMERICAN BAR ASSOCIATION AND THE SURVEYING AND MAPPING DIVISION, AMERICAN SOCIETY OF CIVIL ENGINEERS

Introduction

The Joint Committee was appointed in January, 1937, to study the increasing difficulties of real estate transfer, and to make recommendations for betterment. The First Progress Report of this Committee was published in 1938.[1] In that Report, the Joint Committee urged the use of "State Systems of Plane Coordinates" for recording survey data, particularly in land descriptions, and recommended that each state legislature enact an enabling law defining and naming the state system. When these recommendations were published, and particularly when they were presented before the Real Property Division of the American Bar Association, it became evident that the report left certain questions unanswered from the point of view of members of the Bar Association. Section I of this report has been devoted to a discussion of these questions.

Although the Committee recommends that enabling laws be enacted, the question has been raised whether or not use can be made of the systems previous to such legislation. The Committee advocates such use: First, because although the enabling acts facilitate this use, they are not necessary to it; and second, because the advantages to be gained are too great to permit delay. Section II of this report deals with this subject.

The Committee is of the opinion that land title is improved, and transfer of title is facilitated, by adequate land descriptions in deeds. The description has two functions: (1) To identify the land so that title may be traced; and (2) to designate the position of the boundaries relative to durable landmarks so that the property lines can be marked in the field. These two functions are often fulfilled simultaneously in a description. However, one function is the province of the title examiner and the other is that of the land surveyor.

NOTE.—Written comments are invited for immediate publication; to insure publication the last discussion should be submitted by **October 15, 1941.**

[1] *Proceedings*, Am. Soc. C. E., November, 1938, p. 1879.

It is felt that this Committee, composed as it is of both lawyers and engineers, should discover, if possible, the necessary elements of a complete land description and should recommend accordingly. Section III is devoted to such recommendations.

The Committee finds that frequent sources of difficulty in retracing land boundaries are imperfect methods of officially recording survey data. Although some survey information is nearly always recorded in each deed, when the data from several deeds are compiled, they are usually found to be inconsistent. It is believed that survey information extending beyond the individual deed must be collected and given official recognition. Section IV of this report takes up this subject.

SECTION I.—QUESTIONS RAISED BY LAWYERS WITH REGARD TO STATE PLANE COORDINATE SYSTEMS

When this Committee reports to the American Society of Civil Engineers in respect to state plane coordinate systems it is able to limit itself to arguments in support of its recommendations because in so reporting it is using the language of engineers. On the other hand, when reporting to the American Bar Association, a strange language is being used and considerable explanation is necessary before entering upon any argument or recommendations. Whenever the system is discussed before lawyers the lack of understanding is apparent and its nature is rapidly developed by the questions asked. The Committee believes that it will be of potential assistance if its report includes a number of questions typical of those which arise in the minds of laymen and lawyers.

It is thought that the best test of the points needing clarification is the practical test of what has, in fact, been asked in the past. A fruitful source of information is the stenographic transcript of the discussion which followed the address of Philip Kissam, M. Am. Soc. C. E., chairman of this Committee, before the Real Property Division of the American Bar Association at San Francisco, Calif., on July 12, 1939. Substantially all of the questions herein discussed come from that source.

It has been asked whether or not the system of plane coordinates is obligatory. The answer to this question is plain if the system is considered as simply a convenient language in which to express physical facts. Those who wish to make themselves understood will use it while others are using old forms of expression which are capable of much greater misunderstandings. In other words, it will be optional with engineers to render their surveys permanent in character by resort to the system, rather than to leave them open to future uncertainty and litigation by following outmoded methods.

Lawyers repeatedly ask whether or not the system creates two methods of locating a point. Reply can be made that the system does not change the method of locating a point but offers only a new way to express that location. Whatever means of expression is adopted, the point remains the same. The system is designed only to make it possible to relocate that very point physically at any time in the future regardless of physical changes on the land itself.

The first question which occurs to the minds of legislators is whether or not an appropriation is needed. One of the major arguments made before

legislative committees is that the system can be adopted and legalized without the necessity of a new expenditure. The engineer may go even further and suggest that the adoption of the system will not only require no new appropriation but will result in substantial savings. For example, it has been found that its adoption often results in the discovery of unassessed lands which are then brought into the taxing system and made to produce revenue. Again, any system which locates corners accurately tends to save money by reason of the substantial additional expense of other methods. It is perhaps needless to add that any system which tends to locate points with complete accuracy will eliminate the substantial costs of litigation over indefinite boundaries. Still other economies will suggest themselves from time to time; but even on these points alone it can be demonstrated to legislative committees that the program is one of economy.

Other lawyers have asked whether or not the system conflicts with existing systems. They suggest the present use of such systems as the township surveys, army surveys, Public Land System, etc. The use of the plane coordinate system is not in conflict with any of these existing surveys. Its adoption would simply bring about a designation of the fixed points of the existing surveys by a new, more accurate, and more permanent method of expression.

The discussion among lawyers has shown great concern in their minds as to whether or not property lines would be changed. They have in mind surveys conflicting with previous surveys and showing either a surplus or deficiency of land. Obviously if the survey has not been properly executed it makes no difference whether the various points are expressed in the language of one system rather than in the language of another. Again the system must be regarded as simply a method of expression. If there is any defect in the location of a line already existing, use of plane coordinates will not remedy it. When the line is once established, however, the use of plane coordinates will minimize the chance of future dispute on the same subject.

Many lawyers place great reliance upon the old maxim that metes and bounds control courses and distances. They ask if the use of plane coordinates will interfere with the application of this maxim. In answering this question the engineer must first consider the fact that the maxim is not recognized by the courts to the same extent as in the past. The tendency of judges is not to be bound by fixed rules when they lead to absurdities upon the facts before them. They are likely to believe that such absurdities can better be avoided by considering the particular facts. This has led to the use of the more flexible doctrine that, where there is a dispute in the description of real property, that which is more definite, certain, and material will control. In perhaps a large majority of the cases the calls or bounds will be more definite, certain, and material than the courses and distances, and so will be given controlling effect. With the use of the plane coordinate system there will probably be a tendency to consider points fixed thereunder as assuredly more definite and certain than other means of location and quite possibly as more material. It all comes back, however, to the same proposition that the system must be considered as a means of expression.

It has been asked whether the use of the system involves a re-description of land in the states where a government "rectangular" survey has been made; that is, where the U. S. System of Surveying Public Lands has been established. Before answering this question certain facts must be emphasized. It is the opinion of the Committee that no better method is in use today for the identification of land holdings than the use of Section, Township, and Range. Such a designation frequently provides insufficient survey data. Unless recognized marks are in existence and the land is cheap, proper location is impossible. Where land has more than nominal value or where recognized markers have been lost, or both, further survey data are required and are used today even when no mention of them appears in the deed.

In some localities these data are maintained by public agencies, in others in the records of private surveyors. It is optional with the buyer whether he demands inclusion of these data in the deed in addition to the sentence identifying the land by Section, Township, and Range. The Committee recommends that the Section, Township, and Range be stated in the deed; that supplementary survey data expressed in terms of state coordinates wherever such data are in fact required shall be included in the deed either by statement or by reference to recorded plots or other official records. In the normal course of events the plane coordinate system would operate only prospectively and would be used only in surveys yet to be made. If a new survey is to be made covering land now under a government rectangular survey, the points would be located by coordinates, so that in time all of the land may be so located. The lines would remain in the same positions and the same section and township designations would be used. There is no need of going back and re-describing all of the land previously surveyed regardless of the pendency of any transaction relating to it.

It is further asked how plane coordinates would be applied to existing descriptions. This would be done only in the event of the making of a new survey, at which time one or more points thereon would be fixed by reference to the coordinates. This would not involve any change in the physical situation but would only locate a starting point for application of the courses and distances already determined.

The final question that has come to the attention of the Committee is as to the necessity of agreeing upon the location of boundaries before using the system. Such agreement would not be necessary. If there were a dispute over a boundary, the disputants naturally would employ civil engineers to lay out the boundaries claimed by each of them. These surveyors, if using plane coordinates, would each show on their maps the location of one or more points by their coordinates. The question in doubt between the parties would then have to be determined by usual methods, but when once determined the results of the decision could not be lost through physical changes in any existing monuments. It would always be possible and easy to re-locate the lost monuments and to re-establish the points and lines which had been adjudicated. As an illustrative example, it is simple to think of the west half of Massachusetts, for instance (or any part of the earth's surface), as being covered by dots like the intersections of the little squares of blue rulings on

square-ruled paper and spaced on the ground at $\frac{1}{8}$-in. intervals. Each of these dots would have its own coordinates, always remaining its own individual unchanged identifiers. The problem between two surveyors of nearby owners then is—to determine to which of the "dots" or prints identified by its coordinates the areas of their respective surveys rightly come.

In addition to these specific questions, certain additional matters have been suggested by lawyers when this subject has been discussed. It is pointed out, for example, that in rural sections the land surveyor is very often not a civil engineer in the true sense, so that the use of the system might tend to deprive such persons of work which they might ordinarily expect to receive. This objection, of course, is not consonant with well-considered policy. However, the actual use of the system is so simple that it can be used effectively by any surveyor who has the most elementary knowledge of the art, and moreover, there is nothing in the adoption of the system that prevents the use of pre-existing processes on behalf of persons who do not wish to pay the expense attendant upon having properly qualified professional men do their work for them. Lawyers also seem to fear a substantial increase in expense if a new system is introduced, but what has already been said is sufficient answer to this objection. Again, lawyers are apprehensive lest they become involved in a responsibility beyond what may reasonably be expected of them, arising out of the establishment of bounds to parcels of real estate. This again seems to be an unfounded fear because attorneys are not, under this system, taking any responsibility that they have not already assumed whenever reference is made to a map of any kind. When boundaries are actually in dispute lawyers are engaged to handle the controversy in any event, so that the burden resting upon them is no different under the plane coordinate system from that which they sustain when using any other method. The answer to these doubts and questions is, more and more, apparently to be found in increased education. As soon as lawyers begin to understand the system such questions will not arise.

At the annual meeting of the American Bar Association at San Francisco in July, 1939, the Real Property Division voted to refer the question of just what recommendations would be made, for adoption by the American Bar Association as a whole, to the council of the Real Property Section. The vote included a suggestion that this Committee provide explanatory observations in its report to the council which would help toward a clearer understanding of the purport of the report. The Committee members believe that this section of the report in large measure will constitute a compliance with such vote.

Section II.—The Adoption of State Systems of Plane Coordinates Without Resort to Legislation

Until the present time the approach to establishment, in the several states, of a plane coordinate system has been by means of legislation. This approach has been successful in New Jersey, Maryland, New York, Pennsylvania, North Carolina, Massachusetts, and perhaps other states which have not come to the attention of the Committee. The difficulties which lie in the path of such

an approach are well illustrated by the reluctance of bodies of lawyers to take action in support of the plan even after a careful explanation of it. These explanations have been made to bar associations and other groups who have listened willingly and have been anxious to learn. This field must be compared with the field present at a hearing before a legislative committee. In the latter case it happens quite often that the particular committee is made up largely of attorneys at law, but they are forced by circumstances to listen to the arguments under heavy pressure of business and are concerned largely with the political aspects of proposed legislation rather than with its intrinsic merit. The atmosphere of such hearings is one of impatience to proceed to the next subject, and no serious effort is made to understand something as technical and as new as the proposed system. When the attempt is made to obtain legislation first and practice afterward, it may be "putting the cart before the horse" from a practical standpoint. It may sometimes be the case that practice must come first and legislation after.

The situation is well illustrated by the history of this approach to a recent session of the Connecticut General Assembly. A public hearing was held upon a simple proposed bill along these lines and contained no suggestions for appropriation. There appeared at the hearing of the Judiciary Committee which had the bill before it two representatives of the State Highway Department who were in favor of the legislation in question. There was no adequate opportunity to enlighten the Committee on the real importance and import of what they were asked to do, so that a rejection of the legislation was a foregone conclusion. Before the hearing, however, the representatives of the Highway Department suggested a willingness on their part to begin the use of the system defined in the bill in all their surveys and to handle the whole matter under their current budget. Further contacts are being made in an attempt to execute this suggestion and will be pursued in the future. If these efforts are successful it will mean that large numbers of maps will begin to appear on record bearing points identified by plane coordinates. It must follow almost inevitably that questions will be asked as to the meaning of such reference and a valuable program of education will have been commenced. If such departments will undertake in connection with this work to set monuments with known coordinates, the groundwork will gradually be laid for a general use of the system. Surveyors interested in private lands adjacent to the highways will begin to use the monuments set out by the state in fixing the points of departure on their own maps. It is inconceivable that the value of such proceedings will not ultimately be appreciated even though the education of the public as a whole requires a long time. The Committee feels that it is better to encourage such use and ultimate education while awaiting and encouraging legislation.

In making this suggestion the Committee does not wish to give the impression of any pessimism on its part as to the ultimate results of the present program or to suggest that the legislative program is to be abandoned. The value of the program is not an easy one to make clear to lay minds, and it is hard to conceive of an adverse reaction by one having full apprehension of it. The important thing is to obtain practical results. The Committee wishes to

urge that the pressure made to obtain legislation be continued without abatement but that these other methods be concurrently attempted. It is quite possible that one method will be successful in some states at the same time that success is being achieved in other states by another method. It is possible that, thus far, the other methods have been ignored or neglected. The purpose of the recommendations herein is to awaken interest in active measures along both lines.

All of the states have Highway Departments, whether they be separately constituted or whether they exist as a subdivision of some other department. Not all of them will be found to have a progressive policy but certainly there will be enough forward-looking highway departments to justify an appeal to them to cooperate. There are also other departments in the various state governments in which the use of surveys is substantial, such as, for example, the Public Works Departments, Forestry Boards, Town Plan Commissions, Town Engineering Departments, etc. Of course, the campaign for the adoption of the plane coordinates system should be extended to all such departments as well as to highway departments. The Committee believes that much can be accomplished by contacts and cooperation with such agencies, concurrently with immediate approach to legislatures. In many cases it will be time enough to introduce bills to create a legal recognition of the system when the system is in actual use to a substantial extent. This will be especially true in states where the general tendency is toward extreme conservatism.

Compare this situation with some analogous circumstances: Every one knows the extent to which finger-prints are used by criminal investigation departments. It is difficult to transplant oneself, in imagination, to the days when such things were unheard of. If one were to accomplish such a feat of fancy, could one then conceive of a legislature giving serious consideration to a bill adopting the finger-print system as a means of identification? Obviously that system had to be used for years before legislators had any understanding of it. Before such understanding was achieved legislative committees would undoubtedly reject, with little or no consideration, any such bills. Engineers are in the same situation and must rely on education first, with legislation afterward. Individual engineers cannot use the system on their own initiative unless official or semi-official monuments are provided as reference points, but if so large a body of engineers as that engaged in state work of one kind or another will lay the foundation by using coordinates and setting monuments in conjunction therewith it will be easy for the engineering profession thereafter to conform to the practice and to obtain its benefits whether or not their methods are given legal recognition by law-making bodies. It is therefore the recommendation of this Committee that all appropriate measures be taken to approach this end.

SECTION III.—LAND DESCRIPTIONS

It is the opinion of the Joint Committee that, as a basic principle in land description for the purposes of deeds and other instruments, it is best that the description written into a deed or other instrument be supplemented by a plan or plat of the land which already may be on record at the office in which

the instrument is to be recorded, or which may be prepared and filed for record with the instrument or in a Land Court. This plat may serve the purposes of many deeds, as in the case of subdivision of land into lots, or it may be prepared for and serve only the deed of one tract. Even in the latter case one plan may serve with several successive transfers of the one tract. The plat should be so complete and accurate that there would be no uncertainty, at any time in the future, in restoring any of the bounds that might have been obliterated; but a plat of lower grade, even a carefully-drawn sketch with names of adjoiners and such other data as might be available, is deemed much better than none at all, provided, of course, that the verbal description and the sketch or plan, taken together, must always constitute:

(1) Certain identification of the parcel described; and

(2) Sufficient information for reproducing on the ground the boundaries of that parcel.

The plat should be incorporated in the instrument by reference, the reference to identify the plat with certainty by title, number, or other means, and by reference to the book and page at which the plat is recorded, or other customary reference to its place of official preservation. Instances are numerous where, in the interpretation of some old deed, either in title search or in survey, even a sketch of the land in question, made at the time of the drawing of the deed, undoubtedly would have helped to interpret the deed or identify the land.

The Committee realizes that there may be many occasions in the future, as in the past, when the matter of accompanying a deed by a plat will be deemed impracticable by those who are drawing the deed, and that in these cases engineers will do without it, as has been done; but it is believed that the practice that should be recommended is the inclusion of a plat.

To summarize: It is believed that the description to be written in the deed should be as brief and as general as is consistent with certainty in identification of the tract conveyed, and that there should be incorporated by reference a plat on which should be shown, as nearly as may be, complete, detailed information for retracing the boundaries of the tract and for determining its location with respect to the locations of adjoining tracts.

It is believed that wherever practicable the position of one or more corners, marked by permanent monuments, should be shown by state plane coordinates, written in appropriate places on the plat; and that, in such cases, directions of lines should be stated in terms of the azimuths used in the state plane coordinate systems.

To illustrate the foregoing recommendations the following descriptions are offered as examples.

Samples of Descriptions of Land.—These descriptions do not include rights reserved, appurtenant rights, nor encumbrances upon the premises described. They are written solely to illustrate methods of description of land areas for the dual purpose required in deeds; for example:

(1) Identification of the tract for purposes of title; and

(2) Location of the tract upon the ground, providing data for the re-establishment of the bounds if this becomes necessary because of their obliteration.

As noted in each case, some of these descriptions are regarded as satisfactory, and are submitted as recommended forms; others, not satisfactory, are included to illustrate, by way of contrast, forms of description which are sometimes found in use, but which are deemed unsuitable.

No attempt has been made to show all practicable types of description—either good or bad.

These descriptions, although made up from actual cases, do not, as here written, refer to any existing tracts.

It will be found in some or all of the sample descriptions suggested in this or subsequent reports that there are certain examples of the determination of points or lines by more than one accurately stated method without words of qualification. The Committee believes that, in the event of a conflict between two or more calls, set forth unqualifiedly in the descriptions, the courts will continue to apply well-settled legal principles in determining the dominant call, and that legal rights and liabilities will be ascertained by the same methods as heretofore. The Committee does not believe, however, that the possibility of such conflicts should be an obstacle to the use, by lawyers, of all data available and for the accuracy of which a competent civil engineer has taken the responsibility. The modern perfection of surveying methods, with the concomitant potential accuracy, makes it desirable to abandon or modify traditional reluctance to provide too much information in a deed of real estate, but rather to adopt a more progressive policy of perpetuating on the land records themselves all data which may in the future facilitate the correct location of boundary lines and the relocation of points which have been lost.

1. Sample Description, Including a Plat as Part of the Description.—

> "* * * situated in the Town of ____________, County of ____________, and State of ____________, shown (____)[2] upon a plat drawn by Fred L. Connor, Civil Engineer, dated May 1, 1929, and filed (state here the place of filing) to which plat reference is hereby made for more particular description."

It is to be noted that the use of this form of description requires in every instance the use of a plat (see Fig. 1) as an essential part thereof, which plat must be made a matter of public record.

The use of this form is not recommended when the omission from the body of the deed of words identifying the tract causes extra labor in title search (for example, when the place of storage of the plats is separate from that of the deeds). In such cases, the form Sample 2 should be used.

2. Sample Description, Including a Plat as Part of the Description.—Where further identification in the body of the deed is desired than is provided by

[2] If the parcel has a name, lot number or other designation such as "Blackacre," or "containing —— sq ft" that should be here stated. A sample plat is shown in Fig. 1. The foregoing description is to be used where only the one parcel described is shown on the plat. If the plat includes more area than that to be included in the description, there should appear in the place marked (____)[2] such a qualification as "see Lot 5 of Block 7."

Description 1, the following is suggested:

"* * * situated in the Town of __________, County of __________, State of __________, and bounded and described as follows:

Southerly by Farm Road,	123.39 ft.
Westerly by land of Arthur C. Hicks,	145.82 ft.
Northerly by land of Peter L. Prince,	62.04 ft.
Easterly by land of Peter L. Prince,	133.09 ft.

Being the same premises shown on a plat of the land hereby conveyed, drawn by Fred L. Connor, Civil Engineer, dated May 1, 1929, and filed (state here the place of filing) to which plat reference is hereby made for more particular description."

The verbal part of Description 2 furnishes positive identification of the tract for purposes of tracing title, especially if the brief description is used,

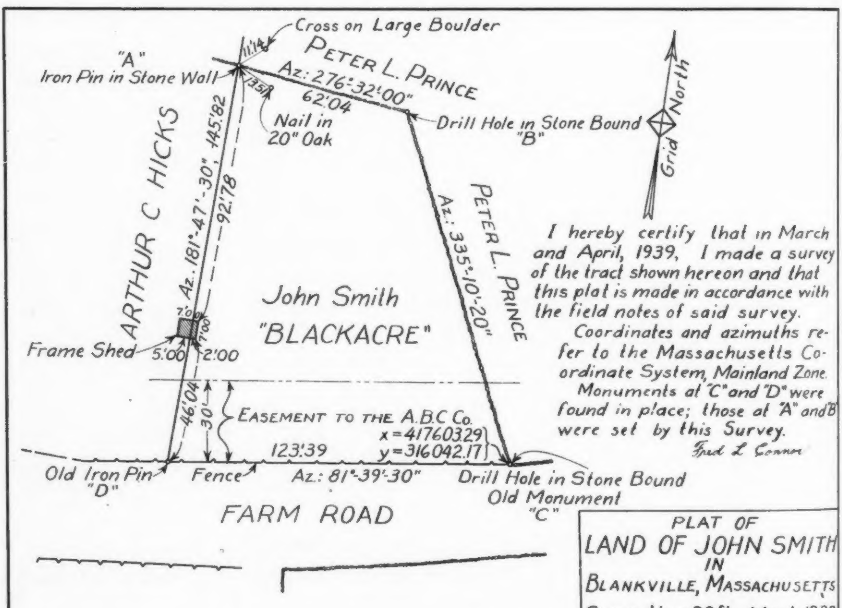

FIG. 1.—SAMPLE PLAT

verbatim, over and over in successive transfers, and, of course, much more conveniently when read with the plat at hand—as should always be the case. The plat (see Fig. 1), incorporated by reference, permits the clear showing of all data which are essential for recovering the boundaries on the ground. These data include, among other items, physical marks of boundary, a position and directions stated in terms of the state coordinate system, and names of

adjoining owners. It will be noticed that the boundary on the highway is stated first, this being for the purpose of rapid identification.

3. Sample Description by Metes and Bounds, Where No Plat is Available.—

"* * * situated in the Town of __________, County of __________, State of __________, and bounded as follows:

"Beginning at a drill hole in a stone bound which is set in the corner of a stone wall on the north line of Farm Road at the southwest corner of land of Peter L. Prince and at the southeast corner of land hereby conveyed, the coordinates of which monument referred to the Massachusetts Coordinate System, Mainland Zone, are: $x = 417{,}603.29$, $y = 316{,}042.17$.

"Thence, on an azimuth of 81° 39′ 30″, 123.39 feet along the northerly line of Farm Road to an iron pin at the southwest corner of the tract hereby conveyed;

"Thence, on an azimuth of 181° 47′ 30″, 145.82 feet along the easterly line of land of Arthur C. Hicks to an iron pin in a stone wall at the northwest corner of the tract hereby conveyed;

"Thence, on an azimuth of 276° 32′ 00″, 62.04 feet along a stone wall on the southerly line of land of Peter L. Prince to a drill hole in a stone bound in the wall at the northeast corner of land hereby conveyed;

"Thence, on an azimuth of 335° 10′ 20″, 133.09 feet along a stone wall on the westerly line of land of said Prince, to the point of beginning.

"Zero azimuth is grid south in the Massachusetts Coordinate System, Mainland Zone.

"This description was written June 1, 1939 from data secured by survey made by Fred L. Connor, Civil Engineer, in March and April, 1939.

"Together with all right, title, and interest in and to all roads and ways adjoining the above-described premises."

In Description 3 the location of the point of beginning is first definitely described, both as to physical monument and as to location with respect to adjoining land and in the Massachusetts Coordinate System. Each clause defining one course of the boundary contains the four essential elements of: (*a*) Direction, (*b*) distance, (*c*) along a physical monument if one exists, such as a wall, and along the boundary of the adjoining tract, and (*d*) physical terminus. The manner of writing the directions and distances indicates the precision with which they are stated (number of decimal places, for example). Intentionally, there is no statement of "more or less." The bounds return the description to the point of beginning.

The meaning of the directions is definitely indicated by the sentence beginning "Zero azimuth is grid south * * *"; that is, there is no uncertainty as to whether directions are referred to the true meridian, to the magnetic needle, or to the direction of the central meridian of the state coordinate system.

However, this description, even for such a simple tract of land, is long and involved, and its use is not recommended. It is susceptible to error in transcription. Moreover, it is difficult to understand and visualize until a sketch showing the major features has been made up by the reader. For these reasons, the forms shown by Descriptions 1 and 2, involving the use of a plat, should always be required.

4. Sample Description by Metes and Bounds, Incomplete and Unsatisfactory.—

> "Beginning at a point on the north line of Farm Road at the southwest corner of land now or formerly of Peter L. Prince; thence north nine degrees and fifty minutes west (N 9° 50′ W) one hundred thirty-three and nine hundredths (133.09) feet by land of said Prince; thence north sixty-eight degrees and twenty-eight minutes west (N 68° 28′ W) sixty-two and four hundredths (62.04) feet by land of said Prince; thence south sixteen degrees and forty-seven minutes west (S 16° 47′ W) one hundred forty-five and eighty-two hundredths (145.82) feet by land now or formerly of Arthur C. Hicks to Farm Road; thence easterly along Farm Road to the place of beginning.
>
> "Bearings stated are magnetic, as of May, 1939. Declination of the needle: 15° 00′ West."

This description, in a form quite commonly employed, is faulty in many respects. Among these are the following deficiencies:

The physical evidences of boundary on the ground (stone walls, stone bounds, iron pins, etc.) are not mentioned. The authenticity of these markers when they are found on the ground at some time in the future is thus left open to uncertainty and debate. Directions of lines stated in terms of bearings cannot be used directly with any considerable precision in the field. They must be translated into angles unless the very rough determinations by magnetic compass are to be used. Thus, the directions are really only relative; and the relations hold only within this one survey. This is true even though the declination is given. Usually the statement of declination is not reliable to the precision with which the bearings are stated. The statement that the third course runs "to Farm Road" raises the question, at least in some jurisdictions, whether the land conveyed runs to the center line of this road or whether it stops at the northerly boundary. Since the direction of the last course is omitted, the precision of the survey cannot be determined—the error of closure cannot be calculated. Hence no check can be applied to catch any mistake in length or direction of a course.

It would seem to be unnecessary to write numbers both in words and in numerals.

5. Description by Subdivision (Section, Quarter-Section, Etc.) of the Public Lands.—

> "The northwest quarter of the southeast quarter (NW 1/4 SE 1/4) of Section Ten (10), Township Seventy-nine North (T79N), Range Six West (R6W) of the Fifth Principal Meridian."

This description, where practicable, is convenient, easy to express, easy to understand, and entirely satisfactory.

For the sake of perpetuating the positions of the corners that control these subdivisions, the statement of their positions in terms of the state coordinate system is very desirable.

Where descriptions in public land states must be written for parcels which cannot be described as fractional parts of sections or as whole government lots the method of Description 1 or 2 should be used. Any government corners near, on, or within, the boundaries of such parcels should appear on the plat.

Except with reference to fractions of subdivisions of the public lands, no reference to fractions of lots (such as "the west half of Lot 5") should be used in descriptions, because the interpretation of such a description is open to uncertainty and debate.

Section IV.—Official Recognition of Survey Data

It is evident that the location of every land boundary affects the shape and extent of at least two land parcels, the titles to which are held by different individuals. The records primarily relied upon for determining the locations of land boundaries are found in the deeds to the various parcels. It is evident, therefore, that of necessity some means must be devised for correlating the information contained in different deeds whenever the necessity arises of marking boundaries on the ground. In order to make proper recommendations in this connection, the Joint Committee has undertaken a survey of the various states to determine what methods have already been put in practice to compile correlation data officially.

The following questionnaire was sent to officials in the various states and in the District of Columbia:

1. Are there any laws or regulations in your state relative to the recording of plans? (If so, a copy of the same would be appreciated.)
2. Is there any law or regulation in your state providing that, to entitle a plan to record, it must be approved and signed by a licensed engineer?
3. Is there any system in your state providing for a bureau for recording the title or location of boundaries?
4. Is there any central bureau or department set up for the filing of data necessary for the establishment of boundary locations, which data are made publicly available?
5. Is there any central bureau or department where such data might logically be filed?
6. Are there any laws or regulations in your state defining or prescribing rules and methods for the making of land surveys?
7. Is there any system legally established in your state to tie up surveys to geodetic points and make such geodetic system govern in the absence of other monuments?
8. Is there any law in your state requiring the registration of engineers (land surveyors)?

The answers are compiled in Table 1.

Summary.—(*a*) Eighteen states reported they had some laws regarding the recording of plans.

(*b*) Nine states reported that there was a law providing for the approval of the plan by a licensed engineer before being admitted to record.

(*c*) Five states only reported that they had any system for the recording of the title or a location of boundaries.

(*d*) Three states only had any central bureau or department for the recording and dissemination of information.

(*e*) Only thirteen states had any provision or department for the handling of such records.

(*f*) Nine states have rules for the making of land surveys.

(*g*) Only two states have any system that ties up to the geodetic surveys.

(*h*) Twenty-five states require the registration of land surveyors.

TABLE 1.—ANSWERS TO QUESTIONNAIRE

State	No. 1	No. 2	No. 3	No. 4	No. 5	No. 6	No. 7	No. 8
Alabama	Yes	Yes	No	No	No	Yes	No	Yes
Arizona	....	....	....	....	....	....	....	....
Arkansas	No	No	No[e]	No	Yes	Yes	No	No
California	Yes	Yes[a]	No[b]	No	Yes	Yes[a]	No?	Yes
Colorado	No	No	No	No	No	No	No	Yes
Connecticut	Yes	No	No	No	No	No	No	Yes
Delaware	No	No	No	No	No	No	No	No
District of Columbia	Yes	No	Yes	Yes	Yes	Yes[c]	No	No
Florida	Yes	Yes	Yes	Yes	Yes	Yes	No	Yes
Georgia	Yes	No	No	No	Yes[c]	No	No	No
Idaho	No	Yes[c]	No	No	No	Yes[c]	Yes[c]	Yes
Illinois	Yes	No	No	No	No	No	No	No
Indiana	Yes	No	Yes[c]	No	No	Yes	Yes[c,d]	Yes
Iowa	?	Yes[c]	No	No	No	No	No	Yes
Kansas	....	....	....	....	....	....	....	....
Kentucky	No	No	No	No	No	No	No	No
Louisiana	Yes	No	No	Yes[e]	Yes[e]	No?	Yes[c]	Yes
Maine	....	....	....	....	....	....	....	....
Maryland	No	No	?	?	?	?	No	Yes
Massachusetts	No	No	No	No	Yes	No[b]	No	No
Michigan	Yes	Yes	No	No	Yes	Yes	No	Yes
Minnesota	Yes	Yes	No	No	Yes	No	No	Yes
Mississippi	....	....	....	....	....	....	....	....
Missouri	No	No	No	No	Yes	No	No	No
Montana	No	No	No	No	Yes	No	No	No
Nebraska	Yes[f]	No	Yes	Yes	Yes	No[c]	No	No
Nevada	No	No	No	No	No	No	Yes	Yes
New Hampshire	No	No	No	No	No	No	No	No
New Jersey	No	Yes	No	No	No	No	No	Yes
New Mexico	No	Yes	No	No	No	No	No	Yes
New York	Yes	?	No	No	Yes	No	Yes	Yes[e]
North Carolina	....	....	....	....	....	....	....	....
North Dakota	No	No	No	No	No	No	No	No
Ohio	Yes	Yes	Yes[l]	No	No	No?	No	Yes
Oklahoma	No	No	Yes[g]	No	No	Yes[h]	Yes[h]	No
Oregon	No	No	No[b]	No	No	Yes	No	Yes
Pennsylvania	Yes	No	No[c]	No	Yes?[c]	No	....[c]	Yes
Rhode Island	No	No	No	No	No	No	No	No
South Carolina	No	No	No	No	No	No	No	No
South Dakota	No	No	No	No	No	No	No	Yes
Tennessee	No[i]	No	No	No	No[c]	No	No	No
Texas	No	No	No	No	No	No	No	No?
Utah	No	No	Yes	No?[j]	No?	No	No	Yes
Vermont	No[i]	....	....	....	....	....	....	N.Y.
Virginia	Yes	No	No	No	No	Yes[e]	Yes[e]	Yes
Washington	Yes[a]	Yes[a]	No	No	....[k]	Yes?	No	Yes
West Virginia	Yes	Yes	No	No	No	No	....	Yes
Wisconsin	Yes	No	No	?	No	Yes	No	Yes
Wyoming	No?	?	No	No	No	No	No	No

[a] Partly. [b] Except Land Court. [c] See reference to "Folder" of replies in "Summary." [d] Yes and no, folder submitted. [e] Limited. [f] Not uniform. [g] Yes, as to government lands. [h] Yes, in case of difficulty. [i] Custom? [j] County. [k] Commissioner of Public Lands. [l] Land Court.

The folder of replies cited in Table 1 is not included in this report but the replies received in addition to the answers to the questionnaire can be summarized as follows:

1. The principal subdivisions of government within the states, with two exceptions, are counties. Practically all states require recording of deeds in either registries of deeds or with the clerks of courts within such counties. Many states have county surveyors or engineers.

2. There is no provision for the recording of engineering data in most of the jurisdictions.
3. There is almost a total lack of facilities for the central recording of such data.
4. In most of the states the machinery set up for the proving of bounds and boundaries is rather cumbersome and could and should be corrected.
5. There is a decided lack of uniformity in the requirements for making land surveys.

Since it is evident that there is an obvious necessity for the correlation of survey data contained in deeds and also survey data pertinent to boundary location but not contained in deeds, and since the Committee's investigation has shown that this necessity has been seriously neglected in the various states, the Committee recommends:

A. That the Enabling Act designed for the establishment of state plane coordinate systems recommended by this Committee in the First Progress Report[1] be pressed for adoption with as much vigor as possible;

B. That registration of land surveyors be required in all of the states; and

C. That a model act be prepared by this Committee designed to provide official recognition and correlation of all survey data pertinent to boundary surveys.

Respectfully submitted,

A. H. Holt, *Secretary*

Committee of the American Society of Civil Engineers:

Philip Kissam, *Chairman*
A. W. Dean
H. W. Hemple
C. B. Humphrey
C. T. Johnston
S. S. Steinberg
W. C. Taylor

Committee of the American Bar Association:

Dorr Viele, *Chairman*
Charles H. Houghton
Charles M. Lyman
R. G. Patton
Theodore Spector
Wallace Haynes Walker

April 20, 1941

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

MASONRY DAMS
A SYMPOSIUM

Discussion

By Messrs. Charles H. Paul and Joseph Jacobs, Irving B. Crosby, I. L. Tyler, and Byram W. Steele

Charles H. Paul,[121] and Joseph Jacobs,[122] Members, Am. Soc. C. E. (by letter).[122a]—The writers are glad to note that all of the discussers are practically in agreement as to the correctness of the principal suggestions and assertions of the paper. The writers are in full accord with Mr. Riegel in his statement that "The most competent and continuous supervision possible should be a feature of dam site exploration." Vigilance and competent judgment are essential throughout the entire schedule of foundation preparation, and the exploratory period is, by no means, exempt from these requirements. To Mr. Riegel's suggestion that deep seating of the dam structure is aimed at utilizing shearing strength to develop resistance against lateral movement, the writers would repeat what is already stated in the paper—namely, that it is also aimed at providing, at the toe of the dam, direct compressive resistance to lateral movement.

Mr. Jones' discussion is mainly a plea in behalf of lower grouting pressures, indorsing and emphasizing the writers' advocacy of the same consideration, and he supports his emphasis splendidly with sound argument and illustration. The plea for lower grouting pressures is timely and highly desirable. There is some contention that the higher pressures, with their resultant slight rock displacements, are not objectionable, in that they insure a more definite injection and a more solid and continuous placement of the grout. Something may be said for this contention but the writers, and apparently Mr. Jones also,

Note.—This Symposium was published in May, 1940, *Proceedings*. Discussion on this Symposium has appeared in *Proceedings*, as follows: September, 1940, by Messrs. William P. Creager, J. R. Shank, George R. Rich, Robert A. Sutherland, Ross M. Riegel, Paul Baumann, W. A. Perkins, L. J. Mensch, and Lewis H. Tuthill; October, 1940, by Messrs. F. A. Nickell, Leslie W. Stocker, Barton M. Jones, P. E. Gisiger, Joseph A. Kitts, S. O. Harper, and R. F. Blanks; November, 1940, by Messrs. Berlen C. Moneymaker, A. Warren Simonds, and W. J. E. Binnie; December, 1940, by Homer M. Hadley, Assoc. M. Am. Soc. C. E.; January, 1941, by Messrs. I. Nelidov, and James B. Hays; and February, 1941, by James S. Lewis, Jr., Assoc. M. Am. Soc. C. E.

[121] Cons. Engr., Dayton, Ohio.

[122] Cons. Engr., Seattle, Wash.

[122a] Received by the Secretary January 6, 1941.

favor the more conservative policy of limiting the pressure sufficiently to insure against possible rock displacement. It is believed that the other procedure involves some elements of uncertainty and hazard in that it may unduly widen and extend existent fracture planes and possibly open up new fracture planes. Mr. Jones' description of the uplift gage used at Norris Dam and his discussion of the need of such gages in general are of great interest.

Mr. Binnie's experience at Silent Valley Reservoir in finding preliminary borings entirely deceptive as to the actual position of bedrock is, by no means, an unusual experience. The history of dam site explorations is replete with such occurrences. It emphasizes the necessity of realizing in advance that preliminary borings may be deceptive and, for important structures, the necessity of making such borings quite thorough and complete. Fully as dangerous as the boulder deception is that of a relatively thin stratum of rock which, for lack of deep drilling, is assumed to be the true ledge bedrock. In morainal foundations it is important, when bedrock is apparently encountered, to continue the drilling to a depth somewhat greater than the possible thickness of boulders in that particular morainal deposit, relying upon the best judgment of competent geologists as to what that further depth should be.

Mr. Lewis' reference to the preparation of the foundation for Watt's Bar navigation lock is a good illustration of modern practice in preparation of shale foundations.

His advocacy of the diamond core drill is somewhat broader than the writers could express it. Where the character of the rock is such that large cores (3 in. to 5 in.) are required for most complete recovery, the shot drill has its place and often will yield a higher percentage of core at lower cost. Skilful use of the most suitable equipment is the main consideration. It should be borne in mind constantly that the only purpose of exploratory drilling, and the only justification for the cost involved, is to determine, as fully as is practicable, the actual nature of subsurface conditions. The first step in that determination is to secure the highest possible percentage of core recovery. Then follows the required correct interpretation of drilling experience and results, the importance of which can scarcely be overemphasized.

The writers agree with Mr. Lewis as to a permissible closer drilling for excavation of vertical faces where no damage to the rock below can be involved. They also are in accord with his observations on line drilling. With respect to all such matters, good judgment must be used both in the preparation of specifications and in the actual construction operations. Where some overbreakage is not in itself objectionable structurally, and the cost of the extra concrete involved is less than the extra cost of line drilling, then, of course, line drilling should not be used.

It is noted that Mr. Hays concurs in the writers' statement as to the importance of correct core drilling interpretations. His comments on the selection of suitable equipment, and on some of the details of continuous observation during drilling operations, are constructive and entirely correct. Regarding the relation of contiguous lines of grout holes and drainage holes, the relative inclinations of the holes as outlined by Mr. Hays are now the usual practice.

As to the possibility of too close proximity, it may be observed that, while they should be far enough apart to avoid interference, their basic purpose requires that they both be located relatively near the upstream face of dam—the grout curtain to prevent, as far as practicable, initial seepage under the dam, and the drainage holes to catch that seepage as early and as far upstream as possible so as to reduce upward pressure to a minimum.

IRVING B. CROSBY,[123] AFFILIATE AM. SOC. C. E. (by letter).[123a]—Mr. Nickell and Mr. Moneymaker have effectively supplemented this writer's outline (which could not cover all details on account of limitations of space), and they have also discussed the methods of the Bureau of Reclamation and the Tennessee Valley Authority. The examples described by the writer were given as illustrations of the types of problems most commonly encountered with the different rocks—not as explicit instructions for the handling of specific conditions. Every dam site is a new problem requiring individual study; no two are exactly alike. The use of analogy may be helpful where it is difficult to obtain all the facts, but it may be dangerous due to the infinite variety of geological conditions. The omission of dams on pervious foundations of sand or other soils was intentional, since the adequate discussion of this case would have greatly extended the paper.

The writer is heartily in accord with the practice of the TVA of assigning work in geophysics, soil mechanics, and allied sciences to specialists in those fields. The geologist should recognize those problems, however, call attention to the need of allied specialists, and then outline the subsurface conditions and interpret the results. This is necessary because the work of these specialists can be no more accurate than the interpretations of geologic conditions to which it is applied.

Although a geological investigation has not always insured against difficulties with dam sites, increasing experience and added facilities of investigation have reached the point where the danger of disaster should be negligible provided the geologists are sufficiently experienced and receive adequate cooperation from the engineers. The value of geological investigations of dam sites is accepted generally; but the idea (still held by some) that geological defects are unpredictable is erroneous and pernicious. Geological conditions can be determined in detail if sufficient investigation is made. It may be difficult or uneconomic to do so, but even in the most difficult cases where it may not be feasible to determine all details, the geologist can outline the limiting possibilities and explain the conditions that must be met.

I. L. TYLER,[124] M. AM. SOC. C. E. (by letter).[124a]—Those who have commented on the paper deserve credit for their interest and for their aid in attempting to bring to the attention of engineers generally the importance of proper control of concrete and concreting procedures, particularly on projects in which the

[123] Cons. Eng. Geologist, Boston, Mass.

[123a] Received by the Secretary March 6, 1941.

[124] Research Engr., Portland Cement Assn., Chicago, Ill.

[124a] Received by the Secretary April 24, 1941.

structures are expected to be reasonably permanent. It is readily agreed that there is still much to be learned about concrete in all its phases, but it is certain that better use of what is known would go far toward bringing average concrete construction to a level of quality well above that which now exists. If the paper and the comments have aided in this respect, the writer is indeed gratified.

The problem of maintaining uniformity of concrete is one of the most difficult encountered once construction is under way. Mr. Tuthill's comment should furnish "food for thought" along the very troublesome line of aggregate-grading control, which is so easily handled in the office, but which is entirely another problem on the job.

Mr. Kitts apparently disapproves the generally accepted trend of thought on mass concrete, classifying it as a product of a misguided "school" of thought, although it is doubtful that the viewpoints are really so far apart as might at first appear. Mr. Kitts' attraction to absolute volume measurements appears to illustrate a general impression that the sum of the absolute volumes of cement, water, and aggregates always equals the volume of mixed concrete. In few, if any, cases is this exactly true; in some cases it is not even a good approximation, even for plastic mixes. Effects of bleeding, evaporation, entrained air, and other variables combine to nullify, not the principles to which Mr. Kitts refers, but an appreciable part of their usefulness in calculating concrete mixes. Absolute volumes, however, are almost always used, at least in preliminary studies of concrete mixes. Even in setting the weight-batcher scales, the concrete engineer is thinking about an absolute volume when he adjusts the weighing apparatus to deliver, say, 1,675 lb of material.

Consistency of concrete must of necessity be governed by placing conditions and in this respect, if in no other, the "old army specification," as quoted by Mr. Kitts, fails entirely. (Inspection of any current construction project under the direction of the Army Engineers suggests that these engineers are not familiar with the "specification" attributed to them by Mr. Kitts.) The proposal of Mr. Binnie to use very thin layers of extremely dry concrete compacted by surface vibration is interesting in this connection. One of the oldest (and best preserved) dams in the United States was constructed in a similar manner, steel tamps operated by hand taking the place of the pneumatic hammers. Whether this procedure could be used with the present demand in the United States for speed of construction is a little doubtful, but the performance of dams built in this manner compared with those of the chuting era, which Mr. Kitts seems to prefer, is an indication that the present tendency in mass concrete is at least in the right direction.

Comments by Mr. Blanks need no amplification except possibly to observe that if construction joints were not required one serious problem in dam construction would be just about eliminated.

Control of cracking has been commented on by Messrs. Hadley, Hays, and others, in connection with this paper of the Symposium and by several in discussions of the papers on dam design. It has been pointed out that certain types of cracks in some locations of a structure may have little or no effect on stability, and it has been suggested that possibly too much attention is being

given to the control of cracking. A review of the designer's position regarding cracking reveals considerable concern, however, and the very fact that the effect of most cracks on the stability of a structure cannot be determined appears to be sufficient reason for crack elimination if design and construction procedures will permit this at a reasonable cost. Present indications are that much can be done along this line without greatly increasing construction costs.

BYRAM W. STEELE,[125] ASSOC. M. AM. SOC. C. E. (by letter).[125a]—The various discussions relative to construction joint problems have given evidence that there is a growing interest in the elimination of the objectionable results that too often accompany joints of any kind in concrete. The solution of most of these problems must be demonstrated over and over again in the realm of actual structure performance before they will be accepted as the basis of design assumptions. The number of variables is too great to permit of an easy mathematical treatment. What is urgently needed is more field study and analysis of actual structure performance instead of the acceptance of design criteria based on assumed theoretical conditions that are not founded on facts.

It is desired to call attention to the point which Mr. Hays made in his discussion relative to the effect of removal of forms in twenty-four hours on surface cracking. Mr. Hays has made a valuable observation and one that should be given careful consideration because of the possibilities for crack control, providing construction difficulties can be eliminated. It is often difficult to remove forms in twenty-four hours, or early enough to control surface and near-surface temperatures properly; and it is also not economical to leave forms in place long enough to be of any particular benefit. However, the detrimental effect of high-surface and near-surface temperatures must not be ignored or minimized if surface-crack inception is to be prevented. If forms were so designed that they could be loosened at twenty-four hours to begin the spraying of concrete with water, it undoubtedly would be worth the cost in crack elimination; but it is so easy to mar the surface or break chunks out of chamfers and edges in such green concrete that, to date, early removal of forms for mass concrete has not been permitted.

In closing this discussion on construction joints the writer urges that the study of joints and cracks be given a major rôle in the future programs of field and laboratory research to the end that joints may be designed in so far as practicable to displace volume-change cracks that cannot be eliminated, and that cracks—however insignificant—be discouraged from starting by every means that is within the realm of economic practicability.

It is not beyond the realm of probability that within a few years engineers will be specifying that cement and aggregate shall be so selected that the resulting concrete will withstand the temperature changes produced by an extremely rapid rate of freezing and thawing during several hundred cycles without showing any evidence of disruption. Such a requirement would demand that the volume-change characteristics, whatever they may be called, of the various ingredients in the concrete shall be practically identical.

[125] Head Engr., Office, Chf. of Engrs., War Dept., Washington, D. C.

[125a] Received by the Secretary April 18, 1941.

The writer cannot agree with Mr. Hadley's philosophy of the harmlessness of a great deal of the present-day cracking. Surface cracks are the entering wedges of wholesale disintegration, and many of them—in fact, too many—cannot be considered otherwise than the indication of lack of care and attention to details. If crackless, or at least near-crackless, dams did not now exist this might be classed as rather a bold statement, but since there are a few specimens approaching that state of perfection it appears that the technique of producing crack-free structures is within the reach of the art.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

RECOMMENDED PRACTICE AND STANDARD SPECIFICATIONS FOR CONCRETE AND REINFORCED CONCRETE

Discussion

BY MESSRS. WALTER H. WHEELER, DUFF A. ABRAMS, AND F. E. RICHART

WALTER H. WHEELER,[82] M. AM. SOC. C. E. (by letter).[82a]—More study might have been given by the Committee to the Flat Slab Division of the Report, Sections 831 to 850, inclusive. For ten years flat slab buildings have been designed and constructed in which the concrete column capitals have been entirely eliminated, and steel grillages have been substituted which are built into the slabs. More than sixty-five buildings have been so constructed, scattered over the United States. The column spacing has ranged from 12 ft to 35 ft and the live loads from 40 lb per sq ft to 400 lb per sq ft. No mention of this type of construction is made in the Report.

The limitations for flat slabs, Section 832, remain unchanged, as do also the moment factors in Table 6. It is specified that "special analysis" is to be made of flat slabs that are outside of the limitations defined in the Report. No suggestion is offered as to how such an analysis may be made. "Conditions of Restraint at Discontinuous Edges" (Section 837) and "Moments in Discontinuous Panels" (Section 838) are arbitrarily defined. The Report does not suggest that elastic or rigid-frame analysis has any application to "Flat Slabs."

Elastic analysis can be applied to flat slabs as well as to other types of structures, and it is being so applied in an increasing number of cases. It removes uncertainties and places a tool in the hands of structural engineers that enables them to disregard empirical formulas and design for the existing conditions.

NOTE.—This Report was published in June, 1940, *Proceedings*, Part 2. Discussion on this Report has appeared in *Proceedings*, as follows: September, 1940, by L. J. Mensch, M. Am. Soc. C. E.; November, 1940, by Messrs. John C. Sprague, and Walter R. Hnot; December, 1940, by Edward C. Gould, Assoc. M. Am. Soc. C. E.; February, 1941, by O. G. Julian, M. Am. Soc. C. E.; March, 1941, by Jacob Feld, M. Am. Soc. C. E.; April, 1941, by Messrs. Harold E. Wessman, N. T. Stadtfeld, C. A. Ellis, R. H. Sherlock, S. C. Hollister, Thomas K. A. Hendrick, Morris Berman, F. R. McMillan, and Meyer Hirschthal; and May, 1941, by Messrs. Egidio O. Di Genova, A. J. Boase, William Richard Wallis, and Elwyn E. Seelye.

[82] Designing and Cons. Engr., Minneapolis, Minn.

[82a] Received by the Secretary May 7, 1941.

In 1941 a load test was made by the City of New York on a flat slab building that was designed by elastic analysis. The building, which was formerly a theater at the northeast corner of 7th Avenue and 50th Street, was remodeled into a store building. The interior floors and framing were removed, except the columns that supported the proscenium arch. The exterior walls and framing and the roof were left in place. The new first and second floors are of flat slab construction with tile fillers in the slabs, supported on steel columns made by welding two 8 in. by 8 in. steel angles at the corners to form box-shaped columns about $8\frac{1}{2}$ in. square. The first-floor slab is supported on the old walls at the outside. The second-floor slab is supported on the old walls at the north end and south side, and on new steel beams connected to the columns on the 50th Street side and the south end. There are two elevator openings through the slabs, one of which is framed in the slabs and the other connected by steel beams suspended from the third-floor framing. There are two stair openings through the first floor, framed in the slab, as well as a number of large vent openings through the first floor, all framed in the slab. The building is seven bays long, and the spans range from 13 ft 8 in. to 26 ft 8 in. It is four bays wide, and the spans in this direction vary from 19 ft 8 in. to 28 ft 0 in. The 28-ft span is discontinuous at the 50th Street wall and is wholly unrestrained at the wall. The basement and first story are each about 18 ft 0 in. high. The test was made on one panel in the 50th Street bay. The span to the wall was 26 ft 10 in. and between interior columns it was 24 ft 0 in. There were stair openings not framed by beams on either side of the panel tested.

In the Seventh Avenue building the grillages were welded to the steel columns. The slab was 12 in. thick, with 12-in. by 12-in. by 9-in. tile fillers spaced about 16 in. on centers both ways; and 3,000-lb concrete was specified. Table 14 shows the loads applied and deflections at the eight reading points under the loads and after removal of the load in each case. Points 1, 2, and 3, Table 14, were at the center of the panel; points 4 and 5 were on the diagonal,

TABLE 14.—RESULTS OF LOAD TESTS ON FLAT SLABS (DEFLECTIONS IN INCHES)

Date (1941)	Time	Load		Deflection Points:							
		Total (tons)	Unit (lb per sq ft)	1	2	3	4	5	6	7	8
3-12	12:20 p.m.	32	100	0.0625	0.1875	0.0625	0.0625	0.0625	0.0625	0.0625	0.0625
3-12	3:15 p.m.	74[a]	230	0.3125	0.3750	0.3125	0.1875	0.1875	0.1875	0.1875	0.1875
3-13	3:20 p.m.	74[a]	230	0.3125	0.4375	0.3125	0.1875	0.1875	0.1875	0.1875	0.1875
3-13	5:45 p.m.	....[b]	....	0.0625	0.1875	0.2500[d]	0.0625	0.0625	0.0625	0.0625	0.0000
4-8	10:45 a.m.	5[c]	....	0.0625	0.1875	0.0625	0.0625	0.0625	0.0625	0.0625	0.0000

[a] Full load. [b] Prior load removed. [c] Estimated load. [d] This reading thought to be in error.

7.5 ft from the column centers; point 6 was in the column strip midway between columns; and points 7 and 8 were at the center of the column strips normal to the wall. The average maximum deflection at points 1, 2, and 3 at the center of the panel was less than one half the deflection permitted by the New

York City Code. In this connection it should be noted that unless a slab is designed for stiffness as well as strength the deflection, when there is no end restraint and no load on the adjacent panels, is two or three times the deflection of a slab designed to support the same load but restrained or continuous at the ends when the adjacent panels are also uniformly loaded. This slab was not designed for stiffness—only for strength.

It was designed for a live load of 120 lb per sq ft, plus applied dead load of 31 lb per sq ft, plus a slab dead load of 97 lb per sq ft. The 6-in. tile wall around the stair at one edge of the panel tested, which weighs about 450 lb per lin ft, was not included in the load computations. Floor finish and plaster had not been placed when the test was made. It was estimated that about 5 or 6 tons of building material were on the slab when the final readings were taken.

The test was made under the direction of Samuel Cohen of the New York City Building Department, who took the readings except the final set. The readings were checked by Dana Reed, superintendent for the general contractor, M. Shapiro and Son Construction Company, Inc.

Several other load tests have been made on flat slab buildings of similar design, some of which have been published. Solid slab construction and slabs with tile fillers have been tested. Some of these buildings had steel columns and others had reinforced concrete columns.[83] In one test, strain-gage measurements were made of the deformations in the concrete, reinforcing steel, and steel floor grillages from which the unit stresses were calculated.[84]

As stated, the 50th street building was designed by elastic analysis. The test results indicate that the method can be depended upon. The slab tested could not be satisfactorily designed by the Report of the Committee.

Duff A. Abrams,[85] M. Am. Soc. C. E.[85a]—The sections in the Recommended Practice and Standard Specifications covering the grading of fine aggregate for concrete are confusing and contradictory. Until these contradictions and confusions are cleared up the sections under the heading "Fine Aggregate Grading" will remain largely inoperative.

Fineness Modulus of Aggregate.—This function was derived by the writer about twenty-five years ago, based on the studies of concrete conducted at Lewis Institute, in Chicago, Ill. (later, part of Illinois Institute of Technology).[86] The fineness modulus was shown to have three distinct functions when applied to aggregates and concrete:

(*a*) A measure of size and grading of separate fine and coarse aggregates;

(*b*) An index to the optimum percentages in which two or more sizes of aggregate may be combined in a concrete mix having a given cement content; and

[83] "Test of Thin Flat Slabs," by Walter H. Wheeler, *Engineering News-Record*, January 9, 1936, p. 49; also "Flat Slab of Tile and Concrete," *loc. cit.*, April 7, 1938, p. 511.

[84] "New Type Flat-Slab Floor in Baltimore Building," *loc. cit.*, October 18, 1934.

[85] Cons. Engr., New York, N. Y.

[85a] Received by the Secretary May 20, 1941.

[86] "Design of Concrete Mixtures," by Duff A. Abrams, *Bulletin No. 1*, Structural Materials Research Lab., 1918.

(*c*) A term in a formula for predetermining the quantity of mixing water required for a given mix.

The Joint Committee restricts its usage of this term to function (*a*) and then as applied to limits of grading fine aggregate only; this seems to the writer to be one of the least significant of its many uses.

Definition of Fineness Modulus.—The fineness modulus was originally defined by the writer as "The sum of the percentages in the sieve analysis of the aggregate divided by 100." The sieve analysis was expressed in terms of percentages by weight coarser than each of the same standard sieves used in the present report. The Joint Committee has attempted to improve on the foregoing in Section 102. This definition starts by giving the fineness modulus a "bad odor" by calling it an "empirical factor"; it then gives a method of determining the fineness modulus which at best is incomplete and confusing, and at worst is entirely erroneous.

Fineness Modulus Based on "Coarser-Than" Grading.—If a nest of standard sieves are arranged with No. 100 at the bottom, then in ascending order of size, with No. 4 at the top, and a sample of sand is shaken to refusal, a certain percentage of the sand will be "retained on" each sieve. This is the expression used in practically all industries where sieving is done. Yet this is not at all what the Joint Committee means by "retained on." It is impossible to compute the fineness modulus of a sand by taking the sum of the "retained-on" percentages, since every sand would give a sum of almost exactly 100%. It is only by a far-fetched inference that a designer can guess at what the Joint Committee means by "retained on."

In Section 208-S the grading is expressed in still other terms—namely, "Total Passing (Percentage by Weight)." Those who attempt to use these Specifications are left entirely to their own devices, and finally must guess that the values of "retained on" (as erroneously used in the definition of fineness modulus) must be obtained by taking 100% minus the "Total Passing (Percentage by Weight)." The report should be consistent with reference to this usage; the "cumulative fraction coarser than" or the "percentage coarser than" usage as illustrated in Tables 15 and 17 is recommended.

The "improved" definition in the Joint Committee Report makes the fineness modulus practically useless here, and entirely useless if the definition should be quoted elsewhere.

Many different expressions have been used for designating the separate fractions or cumulative fractions of materials in conducting a sieve analysis. The expression of the Joint Committee "Total Passing (Percentage by Weight)" is one that the writer had not heretofore encountered. In developing a new term the Joint Committee has unfortunately given the designer a contradiction that may prove extremely troublesome if a contest ever arises over the meanings involved by the fine aggregate grading specifications.

Section 208-S contains (for example) the requirement that, for sieve No. 50, the "Total Passing (Percentage by Weight)" is "5–30." It is obvious that neither of the values in the expression "5–30" is the "total" of anything; in fact the heading states explicitly that they are "percentages." The point

is that a quantity cannot be a total and a percentage at the same time; 5% lacks exactly 95% of being total; 30% falls 70% short of being total. The wording of 215-S, grading of coarse aggregate ("percentage by weight passing laboratory sieves"), is not open to this objection.

Methods of Computing Fineness Modulus.—In order to bring out the true nature of the fineness modulus and to correct the erroneous definition and method in the report, four different methods of computing this function are presented.

Method 1.—The simplest and most direct method of computing the fineness modulus of a concrete sand is shown in Table 15.

TABLE 15.—COMPUTATION OF FINENESS MODULUS; METHOD 1

Sieve No.	Cumulative fraction coarser than sieve
100	0.99
50	0.92
30	0.63
16	0.42
8	0.20
4	0
Fineness modulus	3.16

If the cumulative fractions are expressed in percentages, the sum must be divided by 100 in order to secure the fineness modulus. In making a sieve test the values for Method 1 may be secured almost automatically. After sieving is completed, weigh the material retained on the coarsest sieve, then place each smaller size group in turn on the balance, without removing any size from the pan, and weigh the cumulative fractions; reduce weights to fractions of the total sample. The values then may be entered in the table. The same method is applicable to any size or size range of graded material.

TABLE 16.—COMPUTATION OF FINENESS MODULUS; METHOD 2

Size Group				Separate fractions of sand, f	Fineness modulus of fraction (fm)
Sieve Size (in.)		Mid-diameter, D (in.); log scale	Fineness modulus, m		
Minimum	Maximum				
(1)	(2)	(3)	(4)	(5)	(6)
200	100	0.0041	0	0.01	0.00
100	50	0.0082	1.00	0.07	0.07
50	30	0.0164	2.00	0.29	0.58
30	16	0.0328	3.00	0.21	0.63
16	8	0.0656	4.00	0.22	0.88
8	4	0.131	5.00	0.20	1.00
4	$\frac{3}{8}$ in.	0.262	6.00	....	....
$\frac{3}{8}$ in.	$\frac{3}{4}$ in.	0.525	7.00	....	....
$\frac{3}{4}$ in.	$1\frac{1}{2}$ in.	1.05	8.00	....	....
$1\frac{1}{2}$ in.	3 in	2.1	9.00	....	....
3 in.	6 in.	4.2	10.00	....	....
Sum	....	....	....	1.00	3.16[a]

[a] Fineness modulus of sand.

Method 2.—The fundamental relationships involved in the fineness modulus are brought out more fully in Table 16. Here each size group, as measured by the standard sieves, is treated as a single size, represented by the mid-

diameter of that group (to log scale). The whole-number values of fineness modulus correspond to the mid-diameters of the size groups; however, like logarithms, there are any number of values between the integral values shown which correspond to intermediate sizes. The fineness modulus is the summation of the fraction times the fineness modulus of that group, obtained by multiplying f by m and totaling these products. The resulting fineness modulus of the sand, in Table 16, is the same as that given by the more direct method of Table 15.

The values for D and m in Table 16 show that the size of a group is:

$$D = (0.0041)\, 2^m \quad \text{.....................(28)}$$

in which D = the mid-diameter of the group (to log scale); and m = fineness modulus (an exponent). The constant 0.0041 is the mid-diameter (to log scale) of the smallest size group (200 to 100 sieve) expressed in inches; this is a unit of measure. In metric units this constant becomes 0.104 mm or 104 microns.

Method 3.—Table 16 shows that the fineness modulus of a 1-in. sphere is a little less than 8. In rare instances it may be desirable to compute the fineness modulus of particles of a given diameter or of a size range different from that given by standard sieves. In solving for m in Eq. 28, the following general relationship is secured:

$$m = 7.94 + 3.32 \log D \quad \text{...................(29)}$$

For a 1-in. sphere, $\log D = 0$; hence the second term drops out, and the fineness modulus becomes 7.94, which confirms the approximate value arrived at from an examination of Table 16 Eq. 29 is not convenient for computing the fineness modulus of granular materials; it gives correct results, but is too laborious, since m must be determined separately for each size group; these values must then be multiplied by the corresponding fraction and the products added. Simplified Method 1 should be used in dealing with aggregates.

Method 4 (Graphic).—A curve showing the relation between D and m in Table 16 enables one to read off the fineness modulus of any desired size.

Sieve Analysis Curves.—If standard sieve sizes are plotted as abscissas to a log scale (sieves equally spaced) and cumulative percentages coarser than the sieves as ordinates, and a smooth curve is drawn through the points, what is known as a sieve analysis curve is secured. These curves are of value in showing the grading characteristics of an aggregate. If non-standard sieves were used, the fineness modulus may be determined by locating the non-standard sieves in their proper place on the aforementioned chart, plotting the values for these sieves, drawing a smooth curve through the points, and reading off and tabulating the percentages for the standard sieves. For curves of this type, the fineness modulus is proportional to the area under the curve.

The Fineness Modulus Is Not Empirical.—There are dozens of formulas and expressions in the report that are empirical, but this term has been applied

only to the fineness modulus, which is (probably) the only function in the report that has a rational basis. The fineness modulus is a real property of a granular material; Eqs. 28 and 29 show that it is a function of the logarithm of the diameter of a particle or of the mid-diameter of the size group (86); it is not an empirical factor, as stated by the Joint Committee. It is no more empirical than is a table of logarithms, or the area of a circle.

The fineness modulus is approximate, due to the facts that: (*a*) Sampling, sieves, and sieve analyses are all approximate; (*b*) it uses only six to nine sieves; (*c*) no separation is made below 100 sieve; and (*d*) the particles in a granular material are not always of uniform shape. The fineness modulus as determined by Method 1 and as illustrated in Tables 15 and 17 is sufficiently precise for the purpose.

Grading Limits for Fine Aggregate.—Section 208-S requires that fine aggregate shall be graded within the limits shown in Col. 2 of Table 17. It is more

TABLE 17.—GRADING LIMITS FOR FINE AGGREGATE

Sieve size	Total passing (percentage by weight)	PERCENTAGE COARSER THAN SIEVE	
		Maximum	Minimum
(1)	(2)	(3)	(4)
100	0–8	100	92
50	5–30	95	70
30[a]	20[a]–60[a]	80	40
16	45–80	55	20
8[a]	70[a]–95[a]	30	5
4	95–100	5	0
3/8	100	0	0
Fineness modulus	....	3.65	2.27

[a] Not given in Joint Committee Report (see text).

logical to tabulate the values with the smallest sieve at the top. Table 17 then may be extended without change to include aggregate sizes up to 6 in. or 12 in. if necessary.

The Joint Committee missed an excellent opportunity to give an example of the method of computing the fineness modulus of a sand; this could readily have been done by expressing the grading in terms of "percentage coarser than sieve" as shown in Table 17. Col. 2 of the table gives the limits of sand percentages specified in Section 208-S; Cols. 3 and 4 give the same limits on a "percentage coarser-than" basis. In order to compute the fineness modulus it was necessary to supply minimum and maximum values for the No. 30 and No. 8 sieves. This was done by plotting the given values on a chart, then drawing smooth curves through the points, and finally taking off the approximate values for the two missing sieves.

One is struck by the wide range of fine aggregate gradings that are approved by the Joint Committee, as represented by a low value of the fineness modulus of 2.27 and a high value of 3.65. The recognition of the feasibility of using sands of a wide range in grading is in exact accord with the recommendations

of the writer[86] in 1918, when he introduced a chart for selecting the optimum percentages of sands of widely varying fineness modulus, when the fineness modulus of the sand and of the combined fine and coarse aggregate in the mix is known. The optimum fineness modulus of the mixed aggregate was a function of the quantity of cement used; a lower value was used for "lean" mixes and a higher value for "rich" mixes. Unfortunately, the committee gave no clew as to what it means by "lean" and "rich" mixes.

Restrictive Limits on Sand Grading.—Section 206 states with reference to restrictions to be specified on spread of sand percentages on certain sieves:

> "However, in no case should a range in grading be specified more restrictive than indicated below:
>
> Passing No. 16 sieve—range 20% or less
> Passing No. 50 sieve—range 15% or less
> Passing No. 100 sieve—range 5% or less."

The writer confesses his inability to understand these provisions. It seems to begin by stating that the range in percentages of sand passing the No. 16 sieve in 208-S must not be limited to 5%, 10%, or 15%, but must be kept open to 20%; in other words, the Engineer should not specify minimum 55% and maximum 65% passing the No. 16 sieve, but should provide limits not less than 20% apart (say, minimum 50% and maximum 70%); however, this sensible and proper provision is entirely nullified by the "or less." The "or less" clauses, which have been added since the publication of the Progress Report of January, 1937, appear to make nonsense of this entire section of the Recommended Practice.

Permissible Range in Fineness Modulus of Sand.—The aforementioned restrictions on sand gradings are considered to be specified. Restrictions on grading "as delivered" under the Specifications are covered by Section 209-S. It is not clear why the Specifications provide for the rejection of a sand that falls outside the 0.20 variation of fineness modulus from the original sample, or why they imply that penalties may be involved in "such changes in concrete proportions as may be necessary * * *." The fineness modulus that varies more than 0.20 from the original sample may be better than the original. It is a simple matter to change the sand percentage in the mix to compensate for any reasonable fineness modulus of the sand, if the shipment of sand is essentially uniform.

Interpretation of Tolerance Clause.—The tolerance clause on the fineness modulus of sand has been variously interpreted. The exact interpretation may become of considerable importance as shown by the following example: The Specifications for the foundation section of a large dam contained the following provisions—

> "*Concrete Composition* * * *. The individual mixes will be based upon securing concrete having suitable workability, density, impermeability, and required strengths, without the use of an excessive amount of cement, and using, in so far as practicable, the entire yield of suitable materials from the natural deposits from which the concrete aggregates

are obtained. If, in the opinion of the contracting officer, it is impracticable to utilize in the concrete the entire pitrun yield of suitable material, the contractor shall be entitled to no additional compensation due to the necessity of wasting any of the excess material.

"The sand for concrete shall have a fineness modulus of not less than 2.50 nor more than 3.00 unless approval is given by the contracting officer to use sand not meeting this requirement."

The foregoing specification seems to imply that sands outside the fineness modulus range of 2.50 to 3.00 might be permitted; however, the engineers for the owners ruled that:

(*a*) Only an insignificant tolerance in fineness modulus should be permitted; all sand should be recombined after separation into 3 sizes by classifiers to uniform grading of fineness modulus of about 2.64; and

(*b*) Sand should be held throughout to 25.8% or 26.9% of total aggregate.

Thus, it may be noted that 50% of "the entire yield of suitable materials from the natural deposits" was wasted.

Actual gradings of sand used in this dam are given in Table 18:

TABLE 18.—SAND GRADING

No.	Year	Percentage Coarser Than Sieve						
		100	50	30	16	8	4	Fineness modulus
1	1936	94	76	54	29	13	1	2.67
2	1937	93	75	53	32	13	1	2.67
3	1937	92	74	54	29	13	0	2.62
4	1937	93	75	55	28	12	0	2.63

Whether based on a single sample, an average of five samples on the same day, or an average of more than six months, the grading was practically identical. The wasted material consisted of high-quality sand; any desired grading or percentage could have been used. The 28-day strength of concrete was about twice that specified; experiments indicate that workability and permeability would have been improved by increasing the sand ratio.

Under the claim that the grading of the sand was not the optimum and that the sand percentages enforced by the owner were too restrictive, the contractors subsequently endeavored to collect a large sum on this item alone. Specifications similar to the foregoing, but with certain limits on individual sieves, have been used on a number of other large concrete dams constructed recently.

Shorthand Signs in Specifications.—The form of expression used by the Joint Committee to define limits of percentages of aggregate grading is highly objectionable. For example (see Section 208-S), what does the expression "5–30" mean? By eliminating impossible interpretations, and judging from the context, it is found that it means the upper and lower percentage limits.

The difficulty would be entirely removed by making the tables read:

Amount Passing (Percentages by Weight)	
Minimum	Maximum
5	30

The dash used in the expression "5–30" occurs hundreds of times in the Joint Committee Report. What has been said applies particularly to the em-dash. However, the en-dash is used interchangeably in the report, so that it should be avoided also. This shorthand symbol should be ruthlessly banished from the Joint Committee Report, except for its legitimate use of indicating subtraction or negative quantities. A specification that is recommended for national use, and that at any time may furnish the basis for litigation, should not be confused by shorthand signs; nothing should be left to guesswork.

F. E. Richart,[87] M. Am. Soc. C. E.[87a]—For various reasons, it has not been possible for the members of the Joint Committee to formulate an official closure to the discussion of the June, 1940, report. However, since it did seem desirable to place on record some of the information called for by the discussers, the writer has been requested to prepare an unofficial closure. The following remarks represent the writer's personal views, with a background of familiarity with much of the committee's studies and deliberations.

Mr. Mensch's discussion[88] covers many phases of the report and contributes many helpful statements and suggestions. However, it is so voluminous that a reply can be made only to those items from which a misunderstanding might result. Mr. Mensch criticizes the report for emphasizing the importance of strength and water-cement ratio as of primary importance in proportioning concrete (with the water-cement ratio used as a control for securing durability); but on the basis of a series of tests "made several years ago" he gives a table of water requirements for cement, sand, and 1-in. gravel, without limitations as to the source of the materials or the nature and gradation of the aggregates. He objects to the conventional straight-line theory of flexure because of certain inaccuracies as to linear strain distribution, variable modulus of elasticity, imperfect bond, etc., and implies that the so-called "plastic theory" of flexure would be preferable. It is well known by engineers that the straight-line theory is far from agreement with the actual conditions, but it is retained because it furnishes a simple design method and will produce safe designs. There is nothing new about the fact that different elements of a composite design may involve different factors of safety; such factors of safety naturally may be decreased as the uniformity of quality increases and danger of sudden failure is minimized. This would justify a lower factor of safety for the tensile steel stress than for the concrete fiber stress.

[87] Research Prof., Eng. Materials, Univ. of Illinois, Urbana, Ill.

[87a] Received by the Secretary May 19, 1941.

[88] *Proceedings*, Am. Soc. C. E., September, 1940, p. 1351.

The test results that Mr. Mensch quotes would be rather alarming except for the fact that the manner of failure is not described. It has been the writer's experience with most so-called shear tests that it is rather difficult to design beams with any considerable amount of web reinforcement, which will fail by diagonal tension when tested. Of the tests quoted in Mr. Mensch's discussion, many of the beams, including some which developed relatively high shearing stresses, failed initially by tension or bond. Beams must be very short and deep before high shearing stresses are needed in design, since tension and bond on the longitudinal steel usually furnish the governing considerations. In some of the writer's tests quoted by Mr. Mensch, beams containing as much as 3.7% of longitudinal steel of intermediate grade failed by tension in this steel, and not by diagonal tension. Professor Slater, in planning the tests reported in *Technologic Paper No. 314*, Bureau of Standards, used short, deep beams of I-section, with very heavy tension steel, in order to insure that diagonal tension failures would result. Of course, diagonal tension failure may result if web reinforcement is omitted; but if properly designed web reinforcement is used, it is difficult to produce this type of failure, with the relative values of working stresses given in the report.

Mr. Mensch quotes tests by Mr. Abrams to show that at a unit stress of 6,000 lb per sq in. in the steel, at the middle of a beam (where the shearing stress was zero), a considerable amount of slip between concrete and steel occurred. This was merely the phenomenon that Mr. Abrams called "anti-stretch slip," a local slipping of concrete on the bar adjacent to a tension crack. It has no significance with regard to general slipping due to bond failure.

With regard to columns, the writer takes exception to a number of Mr. Mensch's statements. There would seem to be no logical reason for including, in a formula for column strength, a part of the strength produced by the protective shell and a part due to the effect of the spiral steel; nor is there any good reason for assuming that the column will fail at a longitudinal strain of 0.0015 for all grades of steel. It is surely more logical to use either the entire shell strength, or the entire spiral contribution, whichever is larger. The strength produced by the spiral before failure of the shell is small and highly speculative, since none of the tests upon which Mr. Mensch's Table 8[89] is based were made on columns having protective shells.

The Joint Committee formula for the strength of spirally reinforced columns was based upon the results of the American Concrete Institute Column Investigation[90] and numerous other tests. Published reports of American and European tests of some 1,644 columns and many unpublished reports were reviewed before the A.C.I. investigation was started. The ultimate strength of a spirally reinforced concrete column was found to depend quite generally upon: (1) The concrete core, for which the strength is 85% of f_c' (the strength of a 6-in. by 12-in. cylinder); (2) the vertical steel, which will develop at least its yield point strength; and (3) either the strength of the shell concrete, at $0.85 f_c'$, or the strength added to the core by the presence of spiral reinforce-

[89] *Proceedings*, Am. Soc. C. E., September, 1940, p. 1364.

[90] See *Journal*, Am. Concrete Inst., April, 1930, p. 601; February, 1931, p. 675; March, 1931, p. 761; November, 1931, p. 157; January, 1932, p. 279; February, 1933, p. 283; June, 1933, p. 433. Also, *Bulletin No. 267*, Eng. Experiment Station, Univ. of Illinois, June, 1934.

ment (computed as equivalent vertical steel with about twice the effectiveness of vertical reinforcement). Item 3 has been the center of technical controversy for nearly forty years. The effectiveness of the spiral steel as compared to that of an equal volume of vertical steel has been found to be as much as 2.4; in the writer's tests it averaged about 2.0; and in the tests made at Lehigh University it was more nearly 1.5. The effectiveness is undoubtedly greatest for short columns; for long columns or columns with a large bending stress, the effectiveness of the spiral dwindles rapidly. This is to be expected, since, after the shell spalls off and the spiral comes into action, the column becomes a highly plastic member, held in unstable equilibrium. In this stage of spiral action bending due to initial eccentricity or to load is very likely to cause failure due to lateral deflection. The shortening of a column with a large amount of spiral may reach a value of 1.6% of its length.[91] In a 12-ft story height this would amount to 2.3 in. This excessive plastic deformation is the greatest drawback to the utilization of the strength due to spiral, except as insurance against complete collapse.

The spiral column formula of the Joint Committee Report recognizes the strength contributed by the concrete of both core and shell, and that contributed by the vertical steel. The effect of this is to permit load on the over-all or gross area of the concrete. This utilization of the strength of the shell concrete (not twice its value as Mr. Mensch states) as a load-carrying element is permitted only when sufficient spiral reinforcement is used to provide at least as much potential strength as would be lost if the shell were removed. Furthermore, to insure integrity of the shell concrete, the clear pitch of spirals is held to not less than $1\frac{3}{8}$ in., so that a surface of cleavage or weakness will not be produced between shell and core. With this type of design, the column strength could be based upon either core or gross area, but, from considerations of bending stress, there are decided advantages in using the gross area, and the column may be thought of as a rigid member having very small deformations right up to the ultimate load, where the shell begins to crack.

In considering bending stress, it is convenient to be able to consider the gross area of the column, just as in a beam. The formulas for combined bending and direct stress in columns are intended to bridge the gap between pure bending, wherein the allowable compressive fiber stress is $0.45 f_c'$, and axial compression, wherein the fiber stress is much smaller. For the combination of the two stresses, an intermediate fiber stress is allowed.

Mr. Mensch gives a formula for the permissible load on an eccentrically loaded column, with certain limitations on the eccentricity and percentage of steel. It is of interest that the Joint Committee formulas (Eqs. 16 and 17) that include the effect of percentage of reinforcement, eccentricity, and shape of column may be reduced to essentially the same form of expression as given by Mr. Mensch. The only difference is in the constant 3.14, which under the Joint Committee formula may vary from about 3 to 6 or more, depending on the details of the column.

[91] *Bulletin No. 267*, Eng. Experiment Station, Univ. of Illinois, June, 1934, p. 71.

Mr. Sprague offers the opinion[92] that Alternate A, in the method of proportioning, Chapter III, is a step in the right direction. Probably many more people will agree now than did when something of this kind was proposed in the 1924 Joint Committee Report. Since that time, contractors have learned much about the control of concrete, and specifications that permit them to use this knowledge should prove popular and generally satisfactory.

In reply to Mr. Hnot's inquiry,[93] the writer would say that he believes hooks on footing bars are generally desirable. Although the A.C.I. building regulations definitely require hooks in all footings, the Joint Committee has not made their use mandatory. However, bond considerations will lead to their use in a majority of cases.

With the comments of Mr. Gould[94] regarding the practical applications of design practice, the writer is in general accord. Undoubtedly when a structure is of sufficient importance to warrant a rigorous analysis, great refinement of design, and careful inspection of construction, there should be some compensation through higher allowable stresses. Unfortunately, in compiling a report such as the one under discussion, it is difficult to provide different degrees of refinement to meet the many grades of design and construction practice to be encountered. In general, the committee adopted the policy of writing a recommended practice for a relatively high grade of design and workmanship. There is no profit in writing a manual of poor practice. For cases in which designing must be done by hasty or haphazard methods, if the construction cannot be adequately inspected or the quality of materials cannot be determined, it is obvious that conservative working stresses should be used. Recognition of this and other considerations, such as the limitations on continuous structures, the effect of plastic yielding, and similar items mentioned by Mr. Gould, are essential requisites for intelligent and well-balanced design practice.

In his discussion, Mr. Julian calls for a high degree of careful design and supervision of reinforced concrete work.[95] No one should question his statement that "the cost of such supervision [that knows the best practice and will permit nothing but the best] on all parts of the work is the most profitable investment that can be made in the interest of obtaining durable structures." It is unfortunate that such supervision is lacking on a large part of current building construction. The comment regarding the Moment Distribution Method of Appendix 2 indicates the difference in viewpoint among engineers. Although other discussers consider the abbreviated method as "impracticable to apply," Mr. Julian's reaction is that he prefers the complete treatment of the problem, as given by Professor Cross.[5]

The writer does not share Mr. Julian's suspicion of vertical stirrups. There is a considerable volume of test data regarding vertical stirrups to vindicate their dependability and efficiency, when properly designed. As a

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92 *Proceedings*, Am. Soc. C. E., November, 1940, pp. 1710–1712.

93 *Loc. cit.*, p. 1712.

94 *Loc. cit.*, December, 1940, p. 1846.

95 *Loc. cit.*, February, 1941, pp. 247–254.

5 For a more complete treatment of this method, see "Continuous Frames of Reinforced Concrete," by Hardy Cross and N. D. Morgan (John Wiley and Sons, N. Y., 1932).

matter of fact, diagonal stirrups, although generally in a better position to resist stresses and deformations, may have their faults. Among these is a tendency to slip along the longitudinal steel unless rigidly attached thereto. Bent-up bars, while generally satisfactory, are difficult to handle and place, and require unusual care in detailing to furnish a complete system of shear reinforcement.

Mr. Julian dwells at length upon the problem of columns subject to biaxial flexure. The problem is simple when the applied moments are normal to axes of symmetry. The unsymmetrical case is only one of many problems necessarily outside the scope of this report.

Mr. Julian expresses regret that a more extensive treatment was not given to the effect of plastic flow, shrinkage, and the so-called plastic theory of flexure. The answer is that the effects of these phenomena were given a great deal of study, but that it was felt that safe designs were being secured under the familiar "straight-line" theory, and the actual departures from such theory could be as well related to that theory as to any other. The effect of plastic flow in flexural members has relatively little effect upon their safety[96] and the relation between the fiber stress by straight-line theory and the strength of control cylinders has been quite clearly defined.[97]

Mr. Feld[98] has offered many suggestions and statements of fact which form a useful supplement to the information in the report, and the writer has no quarrel with most of them. In the case of others, although he might agree personally (as in the case of the recommendation that spirally reinforced columns less than 16 in. in least lateral dimension be not permitted), the writer would be inclined to leave that question to the designer to decide for his individual situation.

Messrs. Stadtfeld and Hendrick[99] emphasize the need for information on the history of the cement during manufacture, in addition to the requirements of the A.S.T.M. specifications. It is natural that engineers of the New York (N.Y.) Board of Water Supply should take a keen interest in this subject, since they have pioneered in this phase of cement research.

Unfortunately, the report does not reflect the latest developments in cement specifications. Activity in this field during several years past has resulted in a new tentative specification recognizing five types of portland cement. However, this specification was adopted after the Joint Committee Report was published. It may be useful to mention here that this new specification resulted from discussions on the need for special purpose cements and for a normal portland cement with increased resistance to weathering. In the specification, the requirements for Type II cement have been adopted in the attempt to produce a more uniform and more durable normal cement.

The remarks of Professor Wessman[100] are of interest. Apparently he is the one discusser who is satisfied with the report.

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[96] *Proceedings*, Am. Concrete Inst., Vol. 30, 1934, p. 181.

[97] *Loc. cit.*, Vol. 26, 1930, p. 831.

[98] *Proceedings*, Am. Soc. C. E., March, 1941, pp. 457–464.

[99] *Loc. cit.*, April, 1941, pp. 728, 730.

[100] *Loc. cit.*, p. 727.

The writer is somewhat mystified by the criticisms of Professor Ellis,[101] who states that a column with 13-in. diameter and 10-in. core, and with 4% of vertical steel, is assigned an allowable load under this report that is approximately twice that permitted under the 1924 Joint Committee Report. Although this description is not specific, it might be assumed that a 3,000-lb concrete and intermediate grade reinforcing steel would be used, and that the steel percentage is based on core area. With these conditions, the 1924 Report allows a working load of 115,500 lb; and the 1941 Report, 140,000 lb, or 21% more. This is far from the 100% increase alleged. Furthermore, this depends upon the use of intermediate grade steel under the 1941 Report, whereas the 1924 Report allowed the use of structural grade, with the same allowable load. Had structural grade steel been considered in both cases, the allowable load under the 1941 requirements would become 129,800 lb, or only 12% more than under the 1924 Report. Furthermore, the 1941 Report would require the column to have 2.3% of spiral reinforcement, as compared with 1.0% under the 1924 Report. Hence, it does not appear, for the example chosen by Professor Ellis, that a fair comparison will show any such radical change as he has charged. Had he wished to demonstrate the greatest difference between the two column provisions, he should have chosen an even smaller column, with a low steel percentage and concrete of high strength. However, in the writer's opinion, such a column would represent a very poor design, and would have an abnormally high factor of safety against failure according to the 1924 provisions. As indicated by Mr. Feld, the use of spiral columns in diameters below 16 in. probably does not represent an economical or well-balanced design.

Professor Sherlock[101] inquires about the rationality of the formulas for columns with combined bending and axial stress. As he surmises, the relation is an empirical one, devised to provide a smooth transition from the allowable concrete stress in an axially loaded column, on one hand, and the allowable stress in simple flexure, on the other.[102] However, the equation has been checked against the results of various tests of eccentrically loaded columns, and provides a factor of safety comparable with that existing for the axially loaded column.

The discussion by Mr. Berman[103] calls attention to conditions in reinforced concrete walls produced by continuous slab and frame action. The report is undoubtedly lacking in its treatment of wall design; this is probably due to the fact that, although the committee originally hoped to cover a wide field of concrete structures, it finally found it necessary to limit the scope of its work in order to bring the report to completion.

The discussion by Mr. Seelye[104] provides an interesting contribution on the subject of walls bearing on masonry walls, and his recommendations as the result of experience on housing projects are appreciated. Likewise, the comments of Mr. Wallis[105] are of value in supplementing certain provisions

[101] *Proceedings*, Am. Soc. C. E., April, 1941, p. 729.

[102] *Proceedings*, Am. Concrete Inst., 1938, pp. 401–420.

[103] *Proceedings*, Am. Soc. C. E., April, 1941, p. 731.

[104] *Loc. cit.*, p. 884.

[105] *Loc. cit.*, p. 883.

of the report. Mr. Wallis, like Mr. Feld, calls attention to the omission from the report of recommendations on temperature reinforcement. It is the writer's belief that this was an unintentional omission, although the reason may have been that mentioned in the preceding paragraph.

Mr. Di Genova[106] raises a question concerning the effectiveness of the spiral reinforcement in a column when subjected to a sufficiently large bending moment to produce tensile stresses in the steel, and expresses the opinion that the spiral reinforcement cannot be as effective as in a column wherein all stress is compressive. With this opinion the writer agrees. It will be noted that for a column with such a large eccentricity of loading the formulas for the extreme fiber stress in compression give a value approaching the allowable fiber stress in flexure, for either tied or spirally reinforced columns. In other words, with such large flexural stresses, the presence of the spiral is largely neglected in the application of Eq. 17, Section 861. As the axial stress in the column approaches zero the formula approaches the expression for flexural fiber stress, $f_c = 0.45 f_c'$. Naturally in such a column, the tensile stress in the column steel would need to be investigated and properly provided for in the design.[107]

Mr. Wheeler[108] describes a type of flat-slab floor that he has used to a considerable extent, in which the concrete column capital and drop panel are eliminated; and he advocates elastic frame analysis for special conditions of slab arrangement not covered in the report. The committee has been aware of new developments of this kind, but since the present design procedure, which is quite favorable to flat slab structures, is based largely upon experience and extensive tests of slabs of the more familiar type, it seemed advisable to confine its recommendations to such structures.

Mr. Abrams objects to having the fineness modulus termed an empirical factor, although he has shown no rational significance to this quantity except that it can be represented by the area under a sieve-analysis curve. Perhaps this area is an empirical function, also. He objects strenuously to the substitution of the term "retained on" for "coarser than" in the definition of the fineness modulus. If a sample of sand is passed through each of a set of sieves, the two expressions are synonymous; it is only where the sieves are nested so as to perform the sieving in one operation that any question as to the definition could arise. However, since many thousands of copies of Mr. Abrams' bulletins have been distributed, it is very unlikely that any one would mistake this definition. One might agree that the table heading in Section 208-S, "Total Passing, Percentage by Weight" would have been clearer if stated "Percentage of Total Passing, by Weight," but here again, there is little likelihood of any one misunderstanding the table. Criticism of the term "5–30" used to indicate a range of 5% to 30% might also be applied to many of Mr. Abrams' bulletins, in which he refers to a range of sizes of aggregates as 0–28, 0–14, etc.[109] However, such a criticism would be invalid,

[106] *Proceedings*, Am. Soc. C. E., May, 1941, p. 879.

[107] See, for example, *Journal*, Am. Concrete Inst., April, 1940, pp. 513–517.

[108] *Proceedings*, Am. Soc. C. E., June, 1941, p. 1087.

[109] See *Bulletin No. 1*, p. 13, or *Bulletin No. 11*, p. 12, Structural Materials Research Lab., Lewis Inst. Chicago, Ill.

since the usage of many years has established clearly the meaning of this notation.

In conclusion, the writer would like to comment on Section 866 of the chapter on footings. In Section 866(*b*) it is stated that "the critical sections for bond should be taken at the same plane as those for bending, and the shear used for computing bond should be based on the same loading and section as for bending." This was evidently intended to apply to the usual case in which the critical section for bending is taken at the face of the column. However, Section 866(*a*) provides for two special cases, footings under masonry walls, and footings under metallic column bases, for which special positions of the critical section are specified. For these cases, the writer recommends that bond be investigated at both the critical section and at the face of column, wall, or metallic base. That is, the section of maximum shear should be considered in computing bond stress.

The writer regrets that the time allowed for preparation of this closure has not permitted him opportunity to discuss it with any other members of the Joint Committee. This closure must therefore be submitted as a statement of his understanding of committee actions and of his personal views.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

EXPANSION OF CONCRETE THROUGH REACTION BETWEEN CEMENT AND AGGREGATE

Discussion

BY MESSRS. J. MACNEIL TURNBULL, AND ROBERT A. KINZIE, JR.

J. MACNEIL TURNBULL,[22] ASSOC. M. AM. SOC. C. E. (by letter).[22a]—Investigations and tests of great interest are presented in this paper and, by demonstrating one effect of the presence of the alkalis in portland cement, the importance of these minor and hitherto neglected constituents has been emphasized.

F. M. Lea and C. H. Desch[23] have given the percentage of the total alkali content of various types of portland cement that are dissolved on shaking the cement in water for forty-eight hours. The amounts of the alkalis that can go into solution under these conditions exhibit a wide range of values—20% to 85%.

R. H. Bogue[24] has given some of the possible compounds that the alkalis may form in portland cement.

The writer believes that the percentage of soluble alkalis depends upon the proportion of the alkalis which are combined with the cement in stable compounds. Any excess greater than this will be present in a free, or easily liberated, state, and available for extraction by water or reaction with any chemically active substance in the aggregate.

During 1936 the writer investigated the increase in the caustic alkalinity of water in contact with hardened concrete. He had hoped to be able to remove a considerable proportion of the alkalis from the cement by washing it in water before addition to the aggregate. This was done by adding the cement to three and one half times its weight of water and keeping the mixture

NOTE.—This paper by Thomas E. Stanton, M. Am. Soc. C. E., was published in December, 1940, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: February, 1941, by R. W. Carlson, Assoc. M. Am. Soc. C. E.; March, 1941, by Bailey Tremper, Esq.; April, 1941, by Messrs. Hubert Woods, and N. T. Stadtfeld; and May, 1941, by Messrs. W. C. Hanna, J. C. Witt, and R. F. Blanks.

[22] Asst. Engr., Hume Pipe Co. (Australia) Ltd., Melbourne, Victoria, Australia.

[22a] Received by the Secretary March 28, 1941.

[23] "The Chemistry of Cement and Concrete," by F. M. Lea and C. H. Desch, Edward Arnold and Co., London, 1935, p. 330.

[24] "Constitution of Portland Cement Clinker," by R. H. Bogue, *Proceedings of the Symposium on the Chemistry of Cements*, Stockholm, 1938, pp. 69, 70, and 88.

well stirred for a period of 30 min or longer. The cement was allowed to settle and the free water decanted, the surplus, greater than the water-cement ratio desired, being removed by centrifuging. The wash water was filtered and submitted to chemical analysis. Table 8 gives the composition of this wash water after washing 1,400 g of cement with 4,900 ml of tap water for 30 min; 4,275 ml of water were recovered, leaving a water-cement ratio of 0.45 by weight in the cement paste.

TABLE 8.—CHEMICAL COMPOSITION OF WASH WATER FROM A MEDIUM-ALKALI CEMENT

In Parts per Million

Cl	CO_3	OH	SO_4	SiO_2	Ca	Mg	Na	K	NO_3
21	15	984	1,325	2.0	1,444	0.4	19	492	trace

The cement used was cement *C* of Table 9, from which it will be seen that this cement contains considerably more potash than soda. The principal compounds dissolved out of the cement appear to be calcium sulfate, calcium hydroxide, and potassium hydroxide.

TABLE 9.—PERCENTAGE OF ALKALI IN PORTLAND CEMENT

Cement	Na_2O	K_2O	Total alkali[a]
A (low-heat)	0.11	0.23	0.26
B (low-heat)	0.64	1.22	1.44
C (standard)	0.14	0.78	0.65
D (standard)	0.70	0.40	0.96
E (standard)	0.98	0.55	1.34

[a] Calcined as Na_2O.

Tests of hardened cement mortar, 1 : 2.2 by weight, in the form of a pipe lining ⅜ in. thick, 3¼ in. interior diameter, 12 in. long, made of cement *C*, with and without washing, confirm the analysis given in Table 8. After hardening, the pipes were filled with tap water and left standing for a period of seven days. The water was then removed and replaced with fresh water, and left for a further period of seven days. The composition of this water is given in Table 10.

The washing of cements *A* and *D*, Table 10, had no effect in reducing the caustic alkalinity, but cement *C* was considerably improved. Cements *B* and *E* were not tested but, due to their very high alkali content, are included in Table 9 as a matter of interest.

Although the soda cannot be washed out of the cement itself, it can be leached out of the hardened cement mortar or removed by washing the cement for longer than the normal period of final set. L. Forsén[25] mentions that the

[25] "The Chemistry of Retarders and Accelerators," by L. Forsén, *Proceedings of the Symposium on the Chemistry of Cements*, Stockholm, 1938, p. 301.

alkali appears as free hydroxide as soon as the gypsum or any other calcium salt is consumed by the aluminates.

The writer suggests that these differences between the behavior of the sodium and potassium hydroxides in unhardened and hardened cement may

TABLE 10.—CHEMICAL COMPOSITION OF TAP WATER AFTER CONTACT WITH CEMENT MORTAR FOR SEVEN DAYS (AVERAGE OF TWO SUCCESSIVE PERIODS)

In Parts per Million

Cement	Prior treatment of cement	Age, in weeks[a]	Cl	CO_3	OH	SO_4	SiO_2	Ca	Mg	Na	K	NaOH[b]	Remarks
A	None	11	9	24	33	51	58	55	Trace	11	33	84	Palatable water[c]
A	Washed	11	9	25	44	16	82	53	Trace	11	33	93	Palatable water[c]
C	None	4	8	39	86	35	105	37	Trace	26	166	223	Objectionable taste[d]
C	Washed	4	8	21	47	62	78	42	Trace	26	70	109	Palatable water[e]
D[f]	None	4	..	..	..	..	...	..	....	..	..	329	Unpalatable water
D[f]	Washed[g]	4	..	..	..	..	...	..	....	..	..	340	Unpalatable water[h]
Tap water			12	10[i]	..	..	...	3	2	7	2	....	....

[a] Age of lining at test. [b] Caustic alkalinity computed as sodium hydroxide. [c] Caustic alkalinity due largely to calcium hydroxide. [d] Caustic alkalinity due largely to potassium hydroxide. [e] Potassium hydroxide reduced 58%. [f] Chemical composition not determined. [g] Washed for 1.5 hr. [h] No reduction of caustic alkalinity. [i] As HCO_3.

explain the lack of coordination between the author's results and the Merriman test.

The results of the author's tests with finely ground siliceous magnesian mineral No. 28039 suggest that the use of this material as an admixture (apparently by inducing a rapid reaction with the alkalis in the cement) may prevent any subsequent reaction with the chemically active material in the aggregate.

Acknowledgment is made to D. Avery and V. G. Anderson for the chemical analyses.

ROBERT A. KINZIE, JR.,[26] ESQ. (by letter).[26a]—The results of Mr. Stanton's work, and his interpretation of the results, were both interesting. Since the test method was new, the writer was mainly interested in the problems of reproducing the results, and to this end he secured aggregates from the same source.

The work was arranged so that the effect of the sand, method of curing, and the alkali in the cement could be determined. Mr. Stanton's tests showed that a portland-puzzolan cement which he tested had a much smaller expansion than would be predicted from its alkali content. This cement was included in the writer's series.

The bars in this series of tests were made according to the method given by Mr. Stanton; that is, they were cured in four ways:

[26] Supt., Santa Cruz Portland Cement Co., Davenport, Calif.

[26a] Received by the Secretary April 14, 1941.

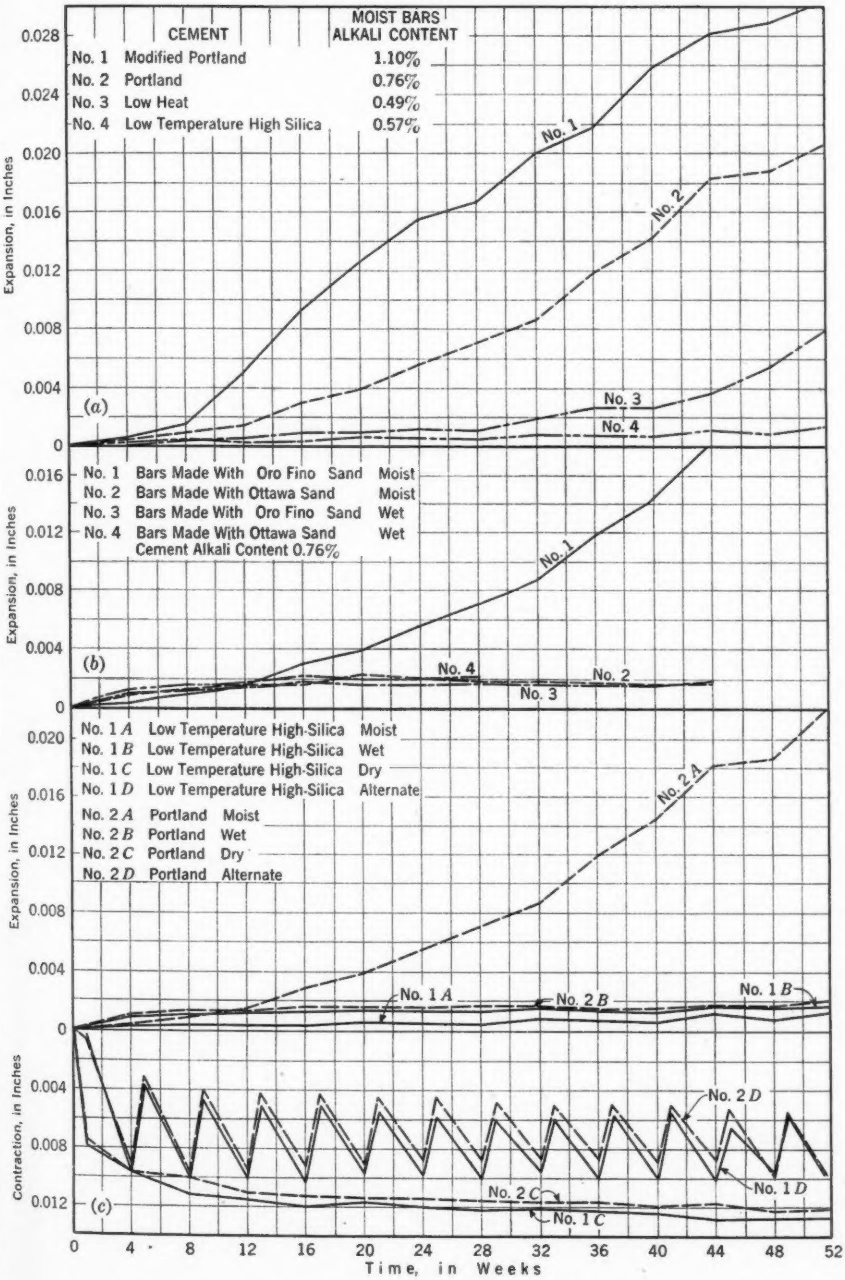

FIG. 15.—REPRODUCTION OF THE STANTON TESTS (ORO FINO SAND, GROUP I, NO. 4)

(1) Wet............Immersed in water.
(2) Moist...........Cured in sealed cans, over but not touching a layer of water on the bottom of the can.
(3) Alternate........One week immersed in water and then dried in air at 70° F for three weeks. The relative humidity was not closely controlled, but was about 60%.
(4) Dry............Kept in air at 70° F and a relative humidity of about 60%.

Portland cements, made to Government specifications, were used, with the exception of the low-temperature high-silica cement, which was a portland-puzzolan cement. The results of the writer's tests may be summarized as follows:

A test on bars made with Oro Fino sand (Group I, No. 4) was designed to show the effect of alkali in the cement in relation to this sand. Therefore, three cements of different alkali content were selected.

The portland-puzzolan cement was included to check Mr. Stanton's results. The curves in Fig. 15(*a*) show that with "moist curing" the expansion of the mortar made with Oro Fino sand increased with increased alkali content of the cement. The portland-puzzolan cement had a much lower expansion than would be expected from its alkali content.

The curves in Fig. 15(*b*) were drawn to show that the expansion is caused by a combination of "moist curing" and Oro Fino sand. The cement is the same as cement No. 2 in Fig. 15(*a*). With "wet curing" there was an initial expansion of some magnitude in all cases and then a much slower change. For both types of curing the Ottawa sand bars behaved normally.

The curves in Fig. 15(*c*) were included as a further check on the results of Mr. Stanton. The cements were low-temperature high-silica, the same as cement No. 4, Fig. 15(*a*), and the portland cement was the same as cement No. 2, Fig. 15(*a*). The bars were cured in four ways to determine whether the method of curing was the main factor in the expansion and to check further the behavior of the low-temperature high-silica cement. All of the bars were made with Oro Fino sand.

This series of tests showed that the method of testing developed by Mr. Stanton's laboratory will give results that can be checked by other laboratories.

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DISCUSSIONS

EARTHQUAKE STRESSES IN THE SAN FRANCISCO-OAKLAND BAY BRIDGE

Discussion

By Norman C. Raab, M. Am. Soc. C. E., and Howard C. Wood, Assoc. M. Am. Soc. C. E.

Norman C. Raab,[9] M. Am. Soc. C. E., and Howard C. Wood,[10] Assoc. M. Am. Soc. C. E. (by letter).[10a]—The study of earthquake stresses and effects has been advanced considerably by the valuable discussions of this paper.

Professor Biot has presented the outline of a method that appears to have the important advantage of simplicity of application after the characteristic curve or spectrum has been established. In his conclusions he very properly calls attention to the fact that damping has been neglected. The writers agree that damping can have a considerable effect on stresses and regret that no field measurements are available from which the damping characteristics of the San Francisco-Oakland Bay Bridge might be determined.

It is to be hoped that the necessary data on a number of structures will be assembled and careful research and analytical studies made so that these effects can be evaluated correctly. It seems particularly important to formulate, if possible, some general procedure on damping effects which could be followed during the design stages of a structure, with assurance that it would be reasonably dependable.

Mr. Ulrich has drawn upon his wide experience and studies, making particular use of his extensive knowledge of the El Centro shock of 1940 and of the seismic conditions in the region of the San Francisco-Oakland Bay Bridge. The writers are in accord with his logical and well-grounded conclusions.

Mr. Moisseiff has discussed very clearly some of the difficulties confronting the engineer engaged in the design of earthquake-resistant structures. While better and more complete information is being accumulated continuously on seismic motion and its effects on structures, the engineer is still faced with

Note.—This paper by Norman C. Raab, M. Am. Soc. C. E., and Howard C. Wood, Assoc. M. Am. Soc. C. E., was published in October, 1940, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: December, 1940, by Maurice A. Biot, Esq.; March, 1941, by Franklin P. Ulrich, M. Am. Soc. C. E.; and May, 1941, by Leon S. Moisseiff, M. Am. Soc. C. E.

[9] Senior Bridge Engr., State Dept. of Public Works, Sacremento, Calif.

[10] Senior Bridge Engr., San Francisco-Oakland Bay Bridge, Oakland, Calif.

[10a] Received by the Secretary April 7, 1941.

the necessity of exercising a large amount of discretion and judgment in the application of the available data to his particular problem. The basic assumptions used in calculating the magnitude and nature of stresses produced by earthquakes are of vital importance in estimating the adequacy of the design of a structure in the so-called "earthquake regions" and will usually have an influence on its cost as well. Fortunately it is found that structures can in many cases be designed to resist earthquake forces without adding materially to the cost of construction.

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DISCUSSIONS

TRANSATLANTIC SEAPLANE BASE, BALTIMORE, MARYLAND

Discussion

BY W. WATTERS PAGON, M. AM. SOC. C. E.

W. WATTERS PAGON,[6] M. AM. SOC. C. E. (by letter).[6a]—Professor Terzaghi asks two questions, which are interrelated. Wicks, as such, could not be used, but the principle involved was applied along the foundation that supports the rolling doors and the fixed walls. A part of this foundation, at the southwest corner, was located at a point where the depth of the granular fill was deficient; hence, some means of consolidating the mud below would stiffen the mud and create additional bearing power. Again, this foundation along its entire length was designed for the heavy concentrated loads of the seaplane and its beaching gear, whereas the granular fill that supports the floor has no such bearing power; hence, a means of drying the mud along this foundation is effective in bringing about more uniformity along the line where the beaching gear passes from the rigid foundation to the resilient fill. The means adopted was the insertion of frequent 1-in. holes through the side of the foundation to permit flow of water from the mud, through the granular fill, into the waterway located inside of the foundation. The result has been that there is a total settlement at the southeast corner of about 6 in., and at sporadic points along the perimeter there are similar settlements of from 2 in. to 4 in.

The settlement of the floor in general cannot be determined with accuracy, because the finished surface of the bituminous concrete floor covering was not accurate to within perhaps 2 in., due to the fact that it was put down and rolled with a steam roller. The approximate settlement varies from 0 to about 4 in., and the present contour is a gently rolling surface. Along one line, for a reason not yet determined, there is a settlement of about 3 in. to 4 in., forming in cross section an ogee type of curve. At one short length of the beaching railroad track there is a sag of about 2 in., but it is interesting to note that there is no evidence of settlement at the two points, one on each track,

NOTE.—This paper by W. Watters Pagon, M. Am. Soc. C. E., was published in October, 1940, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: February, 1941, by Karl Terzaghi, M. Am. Soc. C. E.; and March, 1941, by LeRoy L. Odell, Esq.

[6] Cons. Engr., Baltimore, Md.

[6a] Received by the Secretary May 2, 1941.

where the seaplanes and their gear stood for days at a time during overhaul. At one of these spots a seaplane stood continuously for about two months while repairs were being made.

Captain Odell very properly emphasizes the specification for uplift on the roof due to wind. Not only do tests show considerable suction, but the writer has repeatedly observed the eddy that covers about one half of the windward part of the roof, and the space within an eddy is under lower pressure than the streamline flow. In the design of hangars consideration must be given to the chance that a sudden thundersquall may develop at a time when there is a considerable area of open doors. Elsewhere,[7] the writer has discussed the pressure developed within a building when there are only small aperatures on the windward side, whereas the open doors of a hangar will permit an inside pressure of about 20 lb per sq ft when the velocity head of the wind velocity amounts to 30 lb per sq ft. With an inside pressure of 20 and an outside suction of 20 lb per sq ft, there may be a total outward force of 40 lb per sq ft over large areas.

The writer is indebted to the discussers for the thought which they have given to the paper, and is appreciative of their views.

[7] "Wind Tunnel Studies Reveal Pressure Distribution on Buildings," by W. Watters Pagon, *Engineering News-Record*, December 27, 1934.

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DISCUSSIONS

RELIABILITY OF STATION-YEAR RAINFALL-FREQUENCY DETERMINATIONS

Discussion

BY HERBERT C. S. THOM, ESQ.

HERBERT C. S. THOM,[36] ESQ. (by letter).[36a]—Statistical analysis has taken its rightful place among the tools of mathematical analysis that engineers and others constantly need for the solutions of their problems. As such it deserves and requires the same care in application as do other branches of mathematical analysis; that is, the elements of the problem must bear the closest possible relation to the elements of the analysis. In Euclidean geometry, for example, the elements are the point, line, and plane; in statistical analysis they are the probability postulates defining a random variable.[37]

The essential power of a mathematical procedure is that of making possible the elimination of the labor of intuition by allowing mathematics to do this work. However, one cannot be certain that mathematics is doing the work unless the condition of the correspondence of the elements is satisfied. Too often mathematical tools are used with undue enthusiasm in an attempt to obtain a solution in some form or other before first analyzing the problem sufficiently to determine whether or not a solution is possible with the data at hand. Statistical analysis has been perhaps the most abused tool of all in this respect because the concept of a random variable is somewhat difficult to comprehend. Nevertheless, it is a definite branch of mathematics and must be applied only to such problems for which it can give solutions—problems involving random variables.

The author classifies rainstorms into two types and then states that their fundamental distinction is based on the types of physical processes—namely, convective and frontal. The convective process is indeed an important cause of rainfall, but to say that frontal action is a physical cause of rainfall of the intensity for which frequencies are computed is to accentuate a rather prevalent

NOTE.—This paper by Katharine Clarke-Hafstad was published in November, 1940, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: January, 1941, by Paul V. Hodges, M. Am. Soc. C. E.; February, 1941, by Messrs. C. S. Jarvis, and Howard W. Brod; March, 1941, by Messrs. Merrill Bernard, and Charles F. Ruff; April, 1941, by Eugene L. Grant, M. Am. Soc. C. E.; and May, 1941, by Messrs. Waldo E. Smith, and Robert L. Lowry, Jr.

[36] Associate Hydrologic Engr., U. S. Weather Bureau, Washington, D. C.

[36a] Received by the Secretary April 18, 1941.

[37] "On the Statistical Analysis of Rainfall Data," *Transactions*, Am. Geophysical Union, 1940, p. 490.

misconception which is that all rainfall other than convective is caused by the simple process of lifting up a frontal surface. David Brunt and C. K. M. Douglas[38,39] have shown conclusively that, although light rainfall may be caused by frontal lifting, high intensities must necessarily be due, in most cases, to horizontal convergence, which process may have little relation to fronts.

The station-year method has been used for many years in various forms of engineering work and often with seemingly good results. The reasons for these good results can be shown in the following analysis: Let T = rainfall frequency (the period in years during which a depth y is equaled or exceeded); R = the number of years of record; $\bar{N}$ = average number of storms per year; p = the probability of an event equaling or exceeding y; q = the probability of an event being less than y; and N_p = the number of occurrences equaling or exceeding y. Now

$$T = \frac{R}{N_p} \qquad (4a)$$

and

$$N_p = \frac{R}{T} \qquad (4b)$$

so that

$$p = \frac{N_p}{R\bar{N}} \qquad (5)$$

It can be readily seen that if station records are to be combined by means of the station-year method, the frequency curves must be the same at every station. Under this condition $\bar{N}$ will be the same at all stations. When more stations are added, N_p and R will vary directly; hence p will remain practically constant regardless of how many stations, independent or not, are added. This does not mean, however, that new information proportional in amount to the increased number of stations has been added.

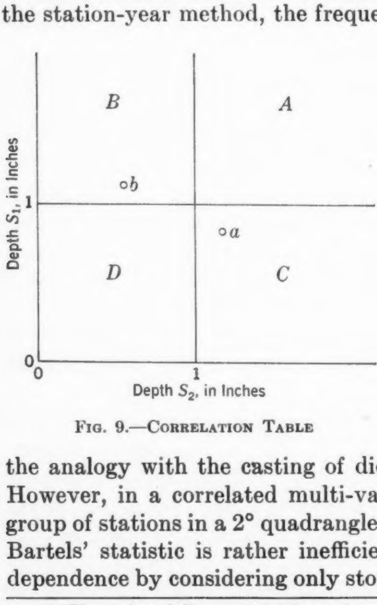

FIG. 9.—CORRELATION TABLE

The author purports to determine the dependence or correlation between stations by Bartels' technique[17] for obtaining the effective number of random components. Bartels claims his technique to be useful only in problems where the analogy with the casting of dice or drawing of random numbers is good. However, in a correlated multi-variate population such as is produced by a group of stations in a 2° quadrangle, the analogy is very weak and consequently Bartels' statistic is rather inefficient. The author has ignored much of the dependence by considering only storm events at the stations that exceeded 1 in.

[38] "Physical and Dynamical Meteorology," by David Brunt, Cambridge University Press, 1939, p. 354 *et seq.*

[39] *Memoirs*, Royal Meteorological Soc., Vol. III, No. 22, p. 34 *et seq.*

[17] "Zur Morphologie Geophysikalischer Zeitfunktionen," by J. Bartels, Sonderausg. aus den *Sitzber.* der Preussischen Akad. der Wiss. Phys.-Math., Klasse 30, 1935.

or some other base value. This can be seen clearly in Fig. 9, which shows a correlation table for depth at station S_1 against depth at station S_2. Thus, an event producing 0.8 in. at S_1 and 1.2 in. at S_2 would plot as point *a* in Fig. 9, whereas another event producing 1.2 in. at S_1 and 0.6 in. at S_2 would plot as point *b*. Since there are twenty-one stations in Table 1, there will be ${}_{21}C_2 = 210$ correlation tables. If the author's data were plotted in Fig. 9, only square *A* would contain observations. This procedure ignores any dependence introduced by the simultaneous occurrence of events in squares *B* and *C* as well as in *D*. The computation of the author's value of N_d for various other frequencies does not overcome this difficulty. A further and more serious criticism arises from the fact that the observations in square *A* do not constitute a random sample and, therefore, should not be operated on with statistical methods. The Bartels' statistic applied to completed tables of the form shown in Fig. 9 would be identical with the square correlation ratio. It is well known that the correlation ratio is an especially poor statistic when computed from tables with only two class intervals—for example, storms producing rains less than 1 in. and storms producing rains of more than 1 in.

R. A. Fisher[40] has decried the computation of a standard error for the correlation ratio because "even with indefinitely large samples the distribution of η for undifferentiated arrays does not tend to normality unless the number of arrays is also increased without limit." The standard error has little significance if the distribution is not normal.

Emphasis should be placed on the fact that only a single rainfall value at a particular station may be used from each storm; otherwise the successive events would not be mutually exclusive and no form of statistical analysis would then apply. If the author did not choose storm events so that they were mutually exclusive, then the results of her analysis would be very uncertain. The lack of mutual exclusiveness may be illustrated by the casting of a die which has a three and a six on the same face so that when that face turned up one could not tell whether to consider the event as a three or a six.

Although Table 1 cannot be analyzed by the analysis of the variance, it may be of interest to set up the variance table because the values can be used

TABLE 8.—VARIANCE TABLE (MEAN FREQUENCY, 7.27)

Variation (1)	Sum of squares (2)	Degrees of freedom (3)	Variance (4)
Between stations	254	20	12.7
Between years	2,334	31	75.2
Residual	4,029	620	6.50
Total	6,617	671	9.85

in other computations. The variance of a set of values is the expected value of the squared deviations from their mean. In the case of samples of greater than 30, it is equivalent to the squared standard deviation.

Table 8, Col. 2, gives the sum of squares of the differences of the frequencies from the appropriate mean frequency, stations, or years. Col. 3 is simply a

[40] "Statistical Methods for Research Workers," by R. A. Fisher, 4th Ed., 1932, p. 235 *et seq.*

number by which the "sum of squares" is divided in order to obtain the expected value of the variance. In this table "degrees of freedom" (Col. 3) is one less than the number of observations in the columns or rows for stations and years, one less than the total number of blocks for the total, and the remainder for the residual. Since the residual variance 6.50 represents the random variation in the table, it would be expected that, if samples were independent, the residual would be equal to, or greater than, the Poisson sampling variance, which is equal to the mean and less than the Bernoullian variance. The latter represents independent sampling conditions. Inasmuch as the residual is 10% less than the mean, it may be concluded that there is dependence within the table. It can be readily seen that this is true in Table 1.

This independence upsets the application of the analysis of variance because the essence of this analysis is to compare the residual, to the explained or year and station, variances; but as the residual variance is subnormal and the between-year variance is therefore supernormal, no comparison can be made. A further criticism of the application of the analysis of variance is that the distribution within the table is not normal and a transformation would have to be made to make it normal. However, by comparison with an analysis of a similar table,[41] it may be concluded, from comparing the station variance to the residual by means of the z-distribution, that there is a significant variation from station to station and that, therefore, the stations with greater frequency have been similar in the different years, at least indicating that the station classification is important. As to the highly supernormal variation between years, nothing can be said. It will be noted that the author's σ_1^2 is the total variance of Table 8, whereas σ_{21}^2 is approximately the variance between years divided by 21. If the "sum of squares" between years had been divided by 32 instead of by 31 it would have been exactly equal to it. A better estimate of the variance is obtained, however, by dividing by the "degrees of freedom" rather than by the number of observations.

It is seen further from Table 8 that the author's value of N_d is simply the between-years "sum of squares" divided by the total and multiplied by 21, or 0.353 times the number of stations. In fact, the author could have obtained N_d more easily in this way. However, the interesting fact is that 0.353, the explained "sum of squares" divided by the total, is like a squared correlation ratio of years on stations. The importance of this fact is that N_d is as inefficient as η^2 and further, having been estimated from a supernormal between-years "sum of squares" and a subnormal total "sum of squares," it is considerably too small. The squared correlation ratio can be interpreted only as the percentage of the total variation introduced by the year classification. It cannot even be interpreted as the percentage of common causes unless the causes are linearly related and *a fortiori*, not as the number of independent events. Since the distribution of η would be still skewed for thirty-two classifications,[40] its standard error has little meaning.

A word of caution might be inserted here about the standard error. The statistic "standard error" has sometimes been interpreted to be some standard

[41] "Statistical Methods for Research Workers," by R. A. Fisher, 4th Ed., 1932, p. 214 *et seq.*

or even mystical measure of error like the old probable error that has practically gone out of use. Neither the standard error nor the probable error, unfortunately, has any such connotations. The standard error is simply the standard deviation of a statistic, and the probable error is 0.6745 times the standard deviation. The standard deviation is a measure of the dispersion of the measures about their mean and only in the case of a normal distribution does it have any significance as a measure of error. Thus $a \pm \sigma$ means that the chances are about two to one that a measure or statistic will lie between $a - \sigma$ and $a + \sigma$, and $a \pm 2\,\sigma$ means that the chances are nineteen to one that it will lie between $a - 2\,\sigma$ and $a + 2\,\sigma$. Likewise, the probable error r is interpreted as even chances that a measure of the statistic will lie between $a + r$ and $a - r$ and nine to two that the measure lies in $a \pm 2\,r$. In modern statistical analysis the odds are unusually taken as nineteen to one (that is, $a \pm 2\,\sigma$); thus only one observation in twenty will fall outside of $a \pm 2\,\sigma$ due to random causes. Even if the various standard errors that the author has computed had significance, they would still only give odds of two to one against observations exceeding them, or one out of three observations would lie outside the limits set by the author.

In computing the standard deviation of the frequencies, the author has followed the questionable reasoning of Professor Grant[12] in assuming without *a priori* justification that the station frequencies follow a Poisson (exponential) distribution. The fact that large rainfalls are rare is not *a priori* justification for assuming that these events are distributed in a Poisson (exponential) distribution or the so-called "Law of Small Numbers." It can be shown[42] readily that if $p\ (= 1 - q)$ is the probability of a storm equaling or exceeding y then the frequency N_p will be distributed in a binomial distribution $(p + q)^n$, in which n is the total number of occurrences.

By Eq. 5, $p = \dfrac{N_p}{R\,\bar{N}}$; and

$$q = 1 - p = 1 - \frac{N_p}{R\,\bar{N}} \qquad (6)$$

Since it is well known that for the binomial distribution $\sigma^2 = n\,p\,q$, and by definition $n = R\,\bar{N}$, substitution gives:

$$\sigma^2 = R\,\bar{N}\,\frac{N_p}{R\,\bar{N}}\left(1 - \frac{N_p}{R\,\bar{N}}\right) = N_p\left(1 - \frac{N_p}{R\,\bar{N}}\right) \qquad (7)$$

Substituting:

$$\frac{1}{T} = \frac{N_p}{R} \qquad \text{and} \qquad N_p = \frac{R}{T}$$

$$\sigma^2 = \frac{R}{T}\left(1 - \frac{1}{T\,\bar{N}}\right) \qquad (8)$$

If $\bar{N} > 50$ and $T > 1$, then $\sigma^2 \cong \dfrac{R}{T}$. Thus, in Table 3, $T < 1.00$, and $\bar{N}$, the

[12] Discussion by Eugene L. Grant of "Rainfall Intensities and Frequencies," by A. J. Schafmayer, M. Am. Soc. C. E., and the late B. E. Grant, *Transactions*, Am. Soc. C. E., Vol. 103 (1938), p. 384.

[42] "Frequency Curves and Correlation," by W. P. Elderton, 3d Ed., 1938, p. 185.

(average annual) number of mutually exclusive and independent events, is not much more than 50, so that the standard errors for the two upper rows are not correct. The standard errors given in Fig. 2, however, are correct.

It is well known that the skewness of the binomial distribution[43] does not tend to zero as n increases if $p = 0\left(\frac{1}{n}\right)$, or if p is the order of magnitude of $\frac{1}{R\,N}$ or less. Under these conditions standard deviations cannot be interpreted as errors, as they can be with the normal distribution. Since $p = \frac{N_p}{R\,\bar{N}}$ (Eq. 5), this happens only when $N_p \leqq 1$, or when the frequency is defined by one storm.

Had the author computed the standard errors by the foregoing procedure there would have been less difficulty in interpreting the standard error for $N_p = 1$. Since this involves a binomial distribution with $p = \frac{1}{R\,\bar{N}}$, the standard error simply is the dispersion about the expected value of N_p, or $n\,p$.

[43] "Statistical Mathematics," by A. C. Aitken, Edinburgh, Oliver and Boyd, 1939, p. 58.

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DISCUSSIONS

CAVITATION IN OUTLET CONDUITS OF HIGH DAMS

Discussion

By Messrs. G. H. Hickox, and J. M. Mousson

G. H. Hickox,[28] M. Am. Soc. C. E. (by letter).[28a]—The authors of this paper have done the engineering profession a service in setting before it an analysis of some of the problems connected with cavitation in sluices of high dams. The writer wishes to add a few comments relative to the cause of cavitation in such sluices and some criticism and questions regarding the authors' methods of testing and calculation.

Cavitation will occur whenever and wherever the pressure at a point in a liquid is reduced to the vapor pressure of the liquid. At the vapor pressure of the liquid, evaporation occurs and cavities form which are filled with vapor and dissolved gases. These cavities pass downstream with the liquid, eventually entering a region of higher pressure where they collapse, generating extremely high pressures. If the cavities collapse in the vicinity of a solid wall, there is a considerable destructive effect known as pitting. It should be noted that cavitation and pitting are not synonymous terms. It should also be noted that, whereas the phenomenon of cavitation is always due to a lowering of liquid pressure to the vapor pressure, the corresponding pitting always occurs somewhere downstream in a region of considerably higher pressure.

The reduction of pressure may be due to two causes: (1) A general lowering of the pressure caused by an increase in the average velocity of flow or a rise in the conduit; or (2) a "local" pressure reduction caused by curvature of flow. A siphon spillway offers examples of all of these phenomena. The pressure at the throat is low because the throat is the highest part of the siphon. Frequently its area is less than that of either the intake or discharge legs, which causes an increase in velocity and a corresponding reduction in pressure. There

Note.—This paper by Harold A. Thomas, M. Am. Soc. C. E., and Emil P. Schuleen, Assoc. M. Am. Soc. C. E., was published in November, 1940, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: March, 1941, by Jerome Fee, Assoc. M. Am. Soc. C. E.; April, 1941, by Messrs. V. E. Leman, P. S. O'Shaughnessy and E. S. Randolph, and Carroll F. Merriam; and May, 1941, by Hunter Rouse, Assoc. M. Am. Soc. C. E.

[28] Senior Hydr. Engr., TVA, Hydraulic Laboratory, Norris, Tenn.

[28a] Received by the Secretary April 18, 1941.

is also curvature of flow that results in a further reduction of pressure on the inside of the curve. The cumulative result of these effects may be to lower the pressure to such a point that cavitation may exist.

A number of sluice conduits through high dams have been built in such a manner that cavitation occurred at their inlets. In a large conduit, the friction loss is not excessive and the hydraulic grade line may not be far above the roof, particularly if the conduit is inclined downward. The shape of the entrance is very important. If the entrance is gradually curved, or bell-mouthed, the local pressure reductions are low, and there may be no cavitation. If the curvature of the entrance is sharp, however, the local pressure reduction is much greater and cavitation may be suspected. In the case of the original Madden entrance shown in Fig. 1, there was an angle at the intersection of the conduit with the upstream face of the dam. Water flowing along the surface was required to make a sudden change of direction, necessitating an infinite acceleration at that point. The probable actual occurrence was that water flowing around the corner described as sharp a curve as could be effected by the maximum force available—that is, the difference between the pressure in the sluice and the vapor pressure of the water on the inside of the curve. In other words, the angular corner caused a region of extremely low pressure immediately below the entrance. Vapor cavities formed in this region and traveled downstream to a region of higher pressure where they collapsed, causing the pitting shown in Fig. 2.

A sharp-cornered entrance is not the only cause of local pressure reduction that may cause cavitation. Fig. 17 shows pitting that occurred below weep holes in the liner of the Norris sluice conduit during the first year of operation. The probable flow condition existing at these holes during operation is shown diagrammatically in Fig. 18. The downstream edge of the hole deflected the high velocity current away from the wall toward the center of the conduit. The flow returned to the wall almost immediately but the velocity and radius of curvature of flow were such that the pressure next to the wall was reduced to the cavitation point. Cavities forming in this region collapsed in the higher pressure area immediately downstream and attacked the liner with the result shown. Repairs were made by welding up the pitted areas, filling the holes with steel rods, welding them in place, and grinding the joints smooth. No further trouble is anticipated.

The writer is not quite in agreement with the authors' definition of a "cavitation pocket" as a void space filled with flying slugs of water. The photographs referred to[6] show clearly that there is not a cavity traversed by random slugs of water but rather a region in which cavities form, travel with the stream, and finally collapse. The rapid succession of these cavities has the appearance shown by the light areas in Fig. 10; but it is erroneous to refer to these regions as "pockets." More correctly, they should be called "cavitation regions" or "cavitation areas." The term "pocket" should be reserved for the individual cavities that traverse this region.

[6] "Applied Fluid Mechanics," by M. P. O'Brien and G. H. Hickox, Members, Am. Soc. C. E., p. 34.

The writer is not convinced that it is necessary to produce cavitation in models in order to design structures that will be free from cavitation. If the method proposed by the authors is to be followed, however, its limitations should be clearly stated. In general, the treatment is fairly clear, but there

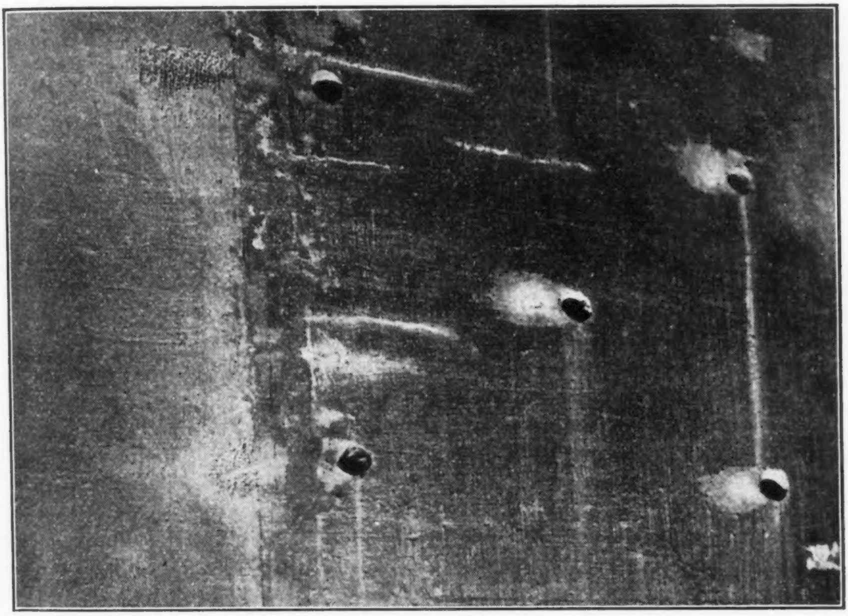

FIG. 17.—CAVITATION PITTING BELOW WEEP HOLES IN NORRIS SLUICE CONDUIT LINER

are a few places where the meaning is ambiguous and seems to require further explanation. At the end of the first paragraph in the section "Description of Apparatus for Cavitation Tests" it is stated:

> "In order that the basic conditions which produce cavitation in certain regions of the prototype may be correctly represented in the model, it is necessary that the pressures in these regions of the model be reduced to the vapor pressure of the liquid used in the model."

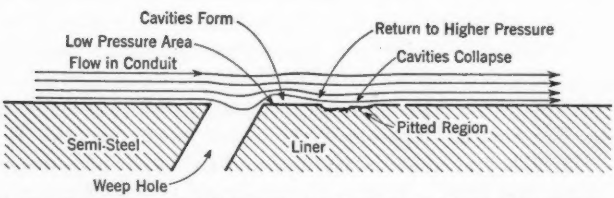

FIG. 18.—REGIONS OF LOW PRESSURE AND PITTING IN NORRIS SLUICE CONDUIT

This statement is correct but the reader who attempts to follow the authors' methods of representing prototype cavitation conditions in the model is likely to become confused. The cases in which such a model may be operated in

accordance with Froude's law while at the same time reducing the absolute pressures to the linear scale as indicated by Eq. 8c are the rare exceptions rather than the rule. Such operation can be accomplished only if the ratio of the vapor pressure of the model liquid to the vapor pressure of the prototype liquid is equal to the model scale. The authors have pointed this out in a discussion of "operational distortion of scale." It is also included in the assumptions immediately preceding Eq. 8c.

It is very seldom possible to adjust the model vapor pressure to a value that will be in the correct ratio to the prototype vapor pressure. The limitations of model testing make the largest practical model scale about 1 : 15. The water temperatures in the lower parts of deep reservoirs are relatively low. At Norris Dam, the average temperature of the water discharged through the sluices is approximately 50° F. The vapor pressure of water at this temperature is 0.36 in. of mercury. The vapor pressure at 32° F is 0.18 in. of mercury. It is evidently impossible to reproduce the vapor pressure to scale in a model of a Norris sluice using water as the model liquid. With water at 32° F for the model liquid at a scale of 1 : 15, the corresponding prototype vapor pressure is 2.70 in. of mercury, which occurs at 112° F. This temperature seldom occurs in prototype structures. The difficulty is further aggravated by the enclosed-tank apparatus. It is extremely difficult to work with low temperatures using this apparatus, as energy at the rate of several kilowatts is constantly being added to the system by the pump.

It might be assumed from the authors' remarks following Eq. 8c that operation of a model in accordance with Froude's law requires that the ratio of absolute pressures in model and prototype be equal to the model scale. This is not correct. It is sufficient that the ratio of pressure differences between corresponding points be equal to the model scale. This can be illustrated by writing Eq. 8c in the form

$$V\, v_T = c_T' \sqrt{2\, g \left[L\left(\frac{p_H - p_T}{w}\right) - L\, z_T \right]} \qquad (29)$$

In Eq. 29 p_H and p_T are not necessarily equal to p_H'/L and p_T'/L but may have any convenient values as long as $\frac{p_H - p_T}{w} = \frac{p_H' - p_T'}{w\, L}$. In fact this is the manner in which hydraulic models are usually operated. The results are completely satisfactory except in those cases where the model indicates pressures which would be below the vapor pressure in the prototype. Even in these cases, similarity can be obtained if the model pressure is adjusted so that the boundary of the vapor pressure region in the prototype corresponds with the boundary of the vapor pressure region in the model. It should be noted that point T is on the boundary dividing the vapor pressure region from the region of normal flow. This is necessary since the pressure gradient is discontinuous at this boundary and Eqs. 8 are not applicable across it. Hence, in order to produce similar performance in model and prototype, it is necessary to know the boundary of the vapor pressure region. Unfortunately the need for model tests of any particular structure means that the vapor pressure

region in the prototype is not known. Therefore, such adjustment, while theoretically possible, cannot be made by the experimenter.

The foregoing discussion has been confined to models geometrically similar to their prototypes and properly oriented. Rotating a vertical section into a horizontal plane would seem to introduce still further difficulty. The exact location of the cavitation region under the roof of a conduit must be influenced by the increasing pressures at greater depths. In a section rotated to a horizontal position, the pressures in the section cannot increase with depth, and consequently correct reproduction of the cavitation region cannot be expected. The relationships expressed in Eqs. 9 to 12 represent an attempt to overcome this difficulty. As pointed out by the authors, however, the best result that can be obtained is correspondence at only one point. For this reason, tests made with a rotated section of model are open to question, particularly with respect to the location of the pitted area.

It is noted that the use of Eq. 13 usually fails to result in model operation similar to that obtained by the use of Eqs. 9 to 12. This may be explained by the fact that in Eqs. 9 to 12 the velocity ratio is fixed in accordance with Froude's law, as may be seen by inspection of Eqs. 9. Eq. 12 expresses the adjustment of p_H' necessary to maintain this velocity ratio. Eq. 13, on the other hand, expresses the ratio of velocities that will exist for a given set of pressures and elevations in model and prototype. It would be a remarkable coincidence if this ratio did satisfy Froude's law.

The application of the equations to an actual model test raises further questions. In the example given, it is stated that a discharge of 2.343 cu ft per sec was assumed. It is doubtful if the discharge can be assumed arbitrarily. In order that the model flow pattern shall approximate that of the prototype, there should be at least a semblance of agreement with Froude's law. The discharge used gives a model velocity of 16.53 ft per sec which, compared with the prototype velocity of 74.0 ft per sec, gives a velocity scale of 1 : 4.47. This is exactly the square root of the length scale, 1 : 20, and indicates perfect agreement with Froude's law. One is led to believe that the authors meant to include this explanation since such good agreement could hardly be fortuitous.

The analysis following Eq. 14 is based on the assumption that the pressure at point T in the prototype for the reservoir elevation shown is the vapor pressure of the water. This was probably true in the case of the original Madden Dam sluices where there was an angle at the entrance. The application cannot be general, however, for the object of successful design is to build a conduit in which cavitation will not occur. The entire discussion is based on the assumption that the pressure at T is known, either in model or prototype, or both. When cavitation occurs, the pressure at this point is obviously the vapor pressure of the liquid and is thus fixed. When cavitation does not occur, as in a properly designed entrance, the pressure is much higher, and is not determinable by any of the equations of this paper. If the model is adjusted to operate in conformity with Froude's law, and the absolute pressures in reservoir and conduit barrel are adjusted to conform to the linear scale ratio, the model may indicate cavitation that would not occur in the prototype

because of the lack of conformity of the vapor pressure ratio. Assuming a vapor pressure of 1 ft of water in both model and prototype, cavitation would occur in the model at an absolute pressure corresponding to 15 ft of water in the prototype, a condition that would be quite safe. It is needless to state that the cavitation region would not be correctly reproduced. The authors seem to present no method of adjusting the model to insure correspondence if the pressures in the region where cavitation is suspected are not known in advance. Of course, if the pressures are known beforehand, the reason for experimental determination becomes rather obscure. The writer would like to know how the methods of the paper were applied to a rotated vertical diametrical section of the entrance of the Hiwassee sluice, for example, which was found to be entirely free from cavitation.

It is believed that Eqs. 8*a* and 8*b* are not in strict accordance with Torricelli's theorem. This theorem merely states that the velocity of efflux from an orifice under a given head is equal to the velocity that would be attained by a body falling freely through a distance equal to the head, and may be written as $V = \sqrt{2\,g\,h}$. Eqs. 8*a* and 8*b* are derived from Bernoulli's equation, of which Torricelli's theorem is a special case. The additional terms representing pressure were introduced by Bernoulli much later.

It is not intended that these criticisms should detract from the value of the paper as a whole. The authors have done the profession an invaluable service by pointing the way toward a solution of a very troublesome problem in the design of high dams. It is probable, however, that the experience of other designers and other hydraulic laboratories with similar problems may eventually lead to simpler techniques that will accomplish the same purpose.

J. M. Mousson,[29] M. Am. Soc. C. E. (by letter).[29a]—This paper is very timely in many respects. It is of particular interest to hydraulic engineers because the form-giving criteria for water passages are often overlooked and because, in other fields of endeavor, it needs an extreme case of unfavorable conditions entailing expensive repairs to bring the matter to the renewed attention of the profession as a whole. With no physical damage to structures, and with inadequacies limited to disappointing coefficients (that is, low efficiency), usually no one ever hears anything of it.

It is surprising how little thought is often given to entrance conditions on conduits or intakes for turbine installations. Although a negligible sum would provide ideal conditions free from vortices, the crudest forms of transitions persist at many new power plants; and they affect the efficiencies of the wheels adversely, due to trains of vortices or uneven flow distribution. The accrued deficiencies in power output, even if only 1% or 2% are involved, entail large monetary losses unnecessarily wasted to the detriment of the national economy.

As to the specific theories on cavitation proposed by the authors, some caution would seem advisable, particularly in the light of other researches. For instance, it is stated that (see heading, "Nature and Causes of Cavitation"):

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[29] Hydr. Engr., Safe Harbor Water Power Corp., Baltimore, Md.

[29a] Received by the Secretary May 15, 1941.

"The familiar effects of water hammering sufficiently intense to indent and disintegrate the surfaces of hard metals furnish abundant proof that pressures approaching or exceeding the higher values of Table 3 actually occur in engineering machines and structures."

(Table 3 gives values to 170,676 lb per sq in.)

It is well to remember that cavitation is a high-frequency phenomenon to the effect that any breakdown of material exposed to this punishment is primarily due to fatigue. The numbers of stress applications even in most severe cases may run into the tens of thousands,[30] and under these conditions it is reasonable to assume that the lower values in Table 3 (say 14, 223 ± lb per sq in.) may be entirely sufficient to cause severe damage rather than those near the upper limit. Furthermore, values in the upper part of the range are not likely to occur since damage would be so severe in a matter of minutes as to endanger any structure within the same time interval.

Additional items of evidence against the high pressures thought likely by the authors are the results obtained by means of a cavitation test stand at Holtwood, Pa., where steels and alloys of a large variety and in various conditions were exposed to cavitation.[31] The most resistant materials could be attacked within 16 hr with the adopted standard throat velocity of 265 ft per sec corresponding to the lower limit of the range in Table 3.

It could be argued, of course, that the throat velocity is not the primary criterion but that the velocity of cavity collapse and subsequent impact of water on water, or water on solid matter, is only to be considered. If this were true, then the degree of pitting found in any test stand would be little influenced by the throat velocity so long as cavitation exists. This, however, has not been indicated by any of the researches in the United States and abroad. The throat velocity was found to be one of the most important variables with respect to the severity of pitting.

Regarding the process of destruction by cavitation, it may be stated that the detailed microphotographic analyses of a large number of metallurgical specimens revealed that surface deformation with subsequent fatigue failures is but one form of disintegration. In many cases the breaking up or disintegration starts from within rather than from without, which again suggests caution with respect to the theory proposed by the authors as to the importance of cracks. Some of the reasons conducive to breakdown from within are the following: Uneven shape of crystals or grains, flake formations of metallic or nonmetallic substances in base matrix, metallic or nonmetallic inclusions, impurities, unfavorable stratifications, etc. All of these factors lead to possible failure due either to stress concentrations, or to surfaces providing little bond or poor interlocking. These same factors are certainly coming into play with concrete where fatigue cracks may develop due to unfavorable stress concentrations at sharp and pointed surfaces of the coarse or fine aggregate, or where failure may occur along the flat surfaces of sand particles or stones, providing little bond, or "break-throughs" of cracks may develop between inclusions. Al-

[30] "Ueber die Zerstoerung von Werkstoffen durch Tropfenschlag und Kavitation," by J. Ackeret and P. De Haller, *Schweizerische Bauzeitung*, September 5, 1936.

[31] "Practical Aspects of Cavitation and Pitting," Edison Electric Inst. *Bulletin*, September and October, 1937.

though possible surface deformations on concrete should not be ignored, nevertheless a microphotographic analysis would seem advisable to prove this point, particularly because the plasticity of concrete is low when compared with certain alloys.

Another factor that would appear to discredit the importance of surface cracks is the experience gained on turbine installations. For the repair of a badly pitted cast-iron turbine runner a straight stainless chrome steel of low ductility was used some years ago. In the course of welding, hair cracks developed, checking the entire welded areas. These cracks were so deep that they could not be smoothed off by grinding; yet despite these cracks no pitting whatever has developed, although stainless chrome nickel steel at the same location showed surface deformation. Similarly, on a turbine throat ring, with appreciably wider surface cracks due to contraction on an area welded with the same straight stainless chrome steel referred to, no cracks were found after a number of years.

In view of the foregoing, great caution seems advisable with respect to certain theories advocated by the authors regarding the necessity of stresses of extreme magnitude and the importance attributed to surface cracks and crevices.

Corrections for *Transactions:* In November, 1940, *Proceedings*, page 1651, line 2 below Fig. 11, change "56" to "65"; on page 1652, line 6, change the equation to read:

$$\left(\frac{16.53}{74.0}\right)^2 = \frac{p_H' - [62.4 \times (-0.26)] - 48.8}{8{,}949 - (62.4 \times 5.3) - 65}$$

in lines 1 and 2 below the equation change "464.8" to "459"; and "1631.2" to "1639."

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DISCUSSIONS

PLASTIC THEORY OF REINFORCED CONCRETE DESIGN

Discussion

By Messrs. Jaroslav J. Polivka, and Paul W. Abeles

Jaroslav J. Polivka,[60] M. Am. Soc. C. E. (by letter).[60a]—The characteristics of the plastic flow of concrete just before the failure of a structural member are different from those at working loads,[61] and more research is needed to prove whether, and how far, the stage preceding the failure can be considered as a reliable basis for reinforced concrete design. The adoption of the plastic theory is very attractive for its simplicity of analysis and design, compared with the generally accepted theory using the modular ratio n. A further simplifying step is suggested that may be extended for analysis and design of any irregular shape of the concrete section having unsymmetrical reinforcement and subject to any kind of loading.

Eq. 12 may be derived, assuming the value a equal to the distance between the centroidal axis of the prismatic cross section and the compression face:

$$a = \frac{t}{2} \qquad (56)$$

When the value a is substituted in the equation of equilibrium,

$$M = C_f \times f_c' \times a\,b \times c \qquad (57)$$

the ultimate resisting moment of the beam as controlled by concrete failure (Eq. 12) becomes equal to $\frac{f_c'}{3}$, maintaining the coefficient $C_f = 0.85$ and setting $d = 0.93\,t$. Then, $a = 0.538\,d$ and $c = 0.731\,d$, which values check very well

Note.—This paper by Charles S. Whitney, M. Am. Soc. C. E., was published in December, 1940, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: February, 1941, by Messrs. L. E. Grinter, and Basil Sourochnikoff; March, 1941, by Messrs. R. W. Stewart, George C. Ernst, Homer M. Hadley, and Robert W. Abbett; April, 1941, by Messrs. Paul Andersen, and R. A. Caughey; and May, 1941, by Messrs. Roberto Contini, and A. A. Eremin.

[60] Research Associate, Univ. of California, Berkeley, Calif.

[60a] Received by the Secretary April 17, 1941.

[61] "Design of Concrete Columns Reinforced by Steel of High Strength," by J. J. Polivka, *Stavitelské Listy* (*Structural Journal*), Vol. 29, 1933, Prague, pp. 175–178 and 188–191; see also "Le Béton Translucide," by J. J. Polivka—published by Les Études des Composés Siliceux, Bruxelles, 1934.

with those computed from tests reported by Professors Slater and Lyse[15] (0.537 d and 0.732 d).

The term $a\,b \times c$ in Eq. 57 is the statical moment of the compression area of the cross section with respect to the centroidal axis of the reinforcement:

$$M_c = A_c\,c = a\,b\,c \qquad (58)$$

This principle may be adopted for any shape of the cross section, considering the area of compression as acted upon by a uniformly distributed imaginary stress $C_f\,f_c'$. The compression area is determined by the neutral axis of the full homogeneous cross section (section of concrete). For pure bending the neutral axis is defined as a centroidal axis of the cross section, conjugate to the plane of bending. For eccentric loading the neutral axis is the antipolar of the point of application of the concentrated load with respect to the ellipse of inertia of the homogeneous cross section.

The coefficient C_f is equal to 0.85 when the compression face is parallel with the neutral axis and must be modified for other positions of the compression face according to the varying distance between the points of the boundary in compression and the neutral axis.

Example 1.—Consider an eccentrically loaded column 6 in. by 6 in., with symmetrical reinforcement of four $\frac{3}{8}$-in. round bars, and an eccentricity of the concentrated load $e = 2.50$ in. (see Fig. 24(b)).[62] The radius of gyration of

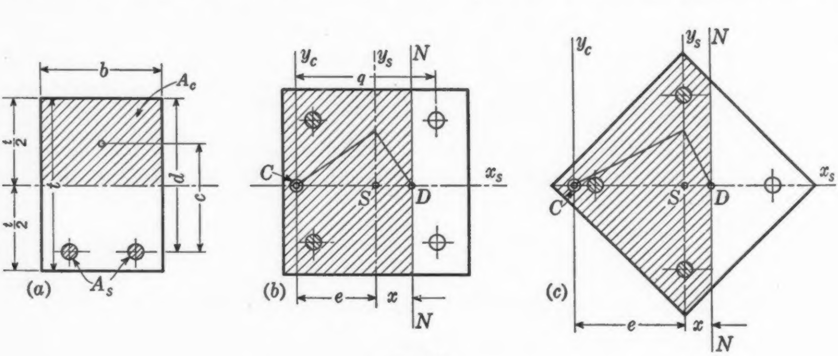

FIG. 24

the cross-sectional area is given by $r^2 = \dfrac{t^2}{12} = 3$ in.^{2} and $r = 0.289\,t$; and the distance between the neutral axis and the centroidal axis is $x = \dfrac{r^2}{e} = 1.2$ in.

For $f_c' = 4.02$ kips per sq in. ("kips" equal "kilo-pounds"; that is, 1,000 lb), $f_s = 43$ kips per sq in. (at the yield point), and $C_f = 0.85$, the statical moment of compressive stresses with respect to the centroidal axis of the tensile bars

[15] "Compressive Strength of Concrete in Flexure," by W. A. Slater and Inge Lyse, *Journal*, Am. Concrete Inst., June, 1930, p. 831.

[62] "Square Sections of Reinforced Concrete Under Thrust and Nonsymmetrical Bending," by Paul Andersen, Assoc. M. Am. Soc. C. E., *Bulletin No. 41*, Univ. of Minnesota, Minneapolis, Minn., Vol. XLII, 1939, Test Specimen No. 5.

(in kip-in.) is

$$\begin{aligned} 25.2 \times 0.85 \times 4.02 \times (2.1 + 0.8) &= 249.714 \\ 2 \times 0.11 \times 43 \times 4.0 &= \underline{37.840} \\ \text{Moment } M_c &= 287.554 \end{aligned}$$

The magnitude of the concentrated load (see Fig. 24(*b*)) is $P = \frac{M_c}{q} = \frac{287.554}{2.50 + 2.00} = 63.9$ kips. If the ultimate strength of the steel is used (62.5 kips per sq in.), the corresponding load P' is $P' = 68.6$ kips, which is very close to the ultimate load found by the test: $P' = 73.9$ kips, compared with $P = 53$ kips computed from Eq. 20 or $P = 57$ kips computed from Eq. 21.

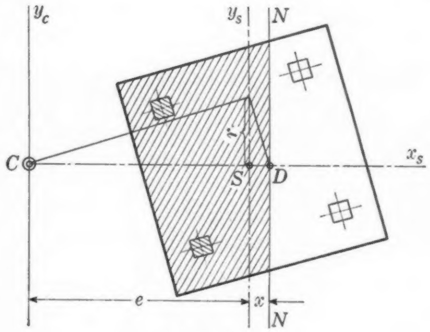

FIG. 25

Example 2.—Consider the same cross section as in Example 1, $e = 3.5$ in. along the diagonal of the square (see Fig. 24(*c*)).[63] In this case the constant C_f must be modified as previously stated. One method would consist in computing the stress volume for which the base is the area of compression, the stress remaining constant in planes perpendicular to the neutral axis and decreasing toward the corners. Then C_f becomes $\frac{2}{3} \times 0.85 = 0.567$.

Considering the fact that f_s is decreasing toward the neutral axis, the statical moment of compressive forces with respect to the centroidal axis of the tensile reinforcement (in kip-in.) is:

$$\begin{aligned} &\text{(concrete)} & 0.567 \times 3.98\,(18 \times 4.242 + 6.979 \times 2.387) &= 209.907 \\ &\text{(extreme bar)} & 0.11 \times 43 \times 5.656 &= 26.753 \\ &\text{(two bars in the compression zone)} & 0.22 \times 43 \times \frac{0.867}{3.695} \times 2.828 &= \underline{6.278} \\ & & \text{Moment } M_c &= 242.938 \end{aligned}$$

The intensity of the concentrated load is $P = \frac{242.938}{3.50 + 2.828} = 38.4$ kips.

If the ultimate strength of the steel is used, the corresponding load is $P' = 40.7$ kips. Both values are close to the ultimate load, 36.3 kips, of the test.

Example 3.—Comparison of the suggested method with the conventional analysis of eccentrically loaded concrete sections (Fig. 25) shows a square 12-in. by 12-in. section reinforced with four 1-in. square bars and subjected to a concentrated load $P = 10$ kips acting at a point C, which is 12 in. from the centroid S of the section; the plane of bending makes an angle of 15° with one

[63] "Square Sections of Reinforced Concrete Under Thrust and Nonsymmetrical Bending," by Paul Andersen, Assoc. M. Am. Soc. C. E., *Bulletin No. 41*, Univ. of Minnesota, Minneapolis, Minn., Vol. XLII, 1939, Test Specimen No. 1.

of the principal axes.[64] The following definition of the neutral axis has been formulated by the writer, relating to the conventional elastic theory:

> The neutral axis N–N is a line parallel to a diameter of the ellipse of inertia that is conjugate to the plane of eccentric bending. It is determined by the condition that the products of inertia of the areas in compression and tension with respect to the neutral axis and to a parallel y_C through the point of application are equal. The products of inertia are determined by two string polygons (Fig. 26), applying the statical moments of cross-sectional areas with respect to the line y_C as forces acting parallel to y_C.

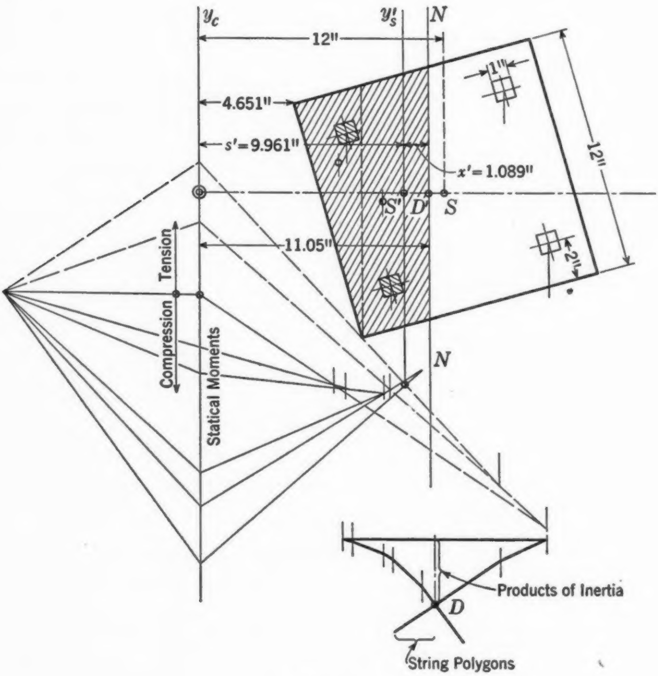

FIG. 26

Both string polygons intercept at a point D, which locates the neutral axis N–N. The distance of the neutral axis N–N from the point of application C is found graphically: $s' + x' = 11.05$ in. (compared with 10.99 in. found by Mr. Andersen).[64]

The products of inertia of the areas with respect to axes N–N and y_C check very well:

Zone	Product of inertia
Compression	1,552 in.^{4}
Tension	1,551 in.^{4}

[64] "A Graphical Method of Analyzing Eccentrically Loaded Concrete Sections," by Paul Andersen, *Civil Engineering*, January, 1940, p. 37.

Another check also gives good results—namely, the condition that the statical moments of the modified areas with respect to the modified centroidal axis y_s' (Fig. 26) must be equal, computing separately the statical moments in both zones which are separated by the centroidal axis y_s':

Zone	Sum of statical moments
Left	125.455 in.^{3}
Right	125.454 in.^{3}

The compressive stress in the concrete is $f_c = -\dfrac{10{,}000}{98.193}\left(1 + \dfrac{5.31}{1.09}\right) = -598$ lb per sq in. The tensile stress in the steel is $f_s = \dfrac{10{,}000}{98.193}\left(1 - \dfrac{6.94}{1.09}\right) \times 10 = 5{,}472$ lb per sq in. These stresses would mean a safety factor of 7.9, assuming $f_c' = 4.7$ kips per sq in., $f_s = 43$ kips per sq in., and $n = 10$.

For the plastic theory the compression zone is confined by the antipolar of the point of application with respect to the ellipse of inertia of the homogeneous cross section. The radius of gyration is given by $r^2 = \dfrac{t^2}{12} = 12$ in.,^{2} and the distance of the antipolar from the centroid of the homogeneous cross section is $x = \dfrac{12}{12} = 1$ in. The coefficient C_f is computed in a manner similar to that of Example 2: $C_f = 0.335 \times 0.85 = 0.285$. Then the ultimate load of the column is $P = 51.6$ kips and, since the working load considered in the conventional theory was 10.0 kips, the safety factor is only 5.2.

PAUL W. ABELES,[65] ESQ. (by letter).[65a]—As explained by Mr. Whitney, the formulas of this paper do not take into account the excess strength that has been reported in beams and slabs with small percentages of steel. The reason for this neglect is that, in Mr. Whitney's opinion, the use of small percentages of steel is not likely to be as satisfactory in practical construction as in the laboratory; and, furthermore, it has not been demonstrated that the excess strength will remain after repeated loading. Against this statement it may be emphasized that small percentages of steel have become more and more common owing to the use of medium and high-grade steel. Before the use of new steel grades of unusually high yield point was permitted, many tests had to be made in various countries. In this connection (for example, on structures reinforced with twin twisted steel bars), tests with repeated loads were conducted which have proved that the behavior with such a loading is similar to that of structures reinforced with mild steel. It may be stated that a repeated loading in cases of higher percentages of steel also causes less favorable results than tests on beams, whose loading is gradually increased until failure.

The excess of strength in cases of small percentages of steel is especially great when high-grade materials are used. The excess of strength can be

[65] Cons. Engr., London, England.

[65a] Received by the Secretary May 7, 1941.

shown best by the theoretical lever arm of test specimens at failure c_t calculated for the breaking moment:

$$c_t = \frac{M}{A_s f_s} \qquad (59)$$

This value c_t may be compared with the value c according to Eq. 6

$$c = \left(1 - \frac{p\,m}{2}\right) d = \left(1 - \frac{p f_s}{2 \times 0.85 \times f_c'}\right) d = \left(1 - \frac{p}{1.7} k\right) d \qquad (60)$$

in which $k = \frac{f_s}{f_c'}$ represents the ratio of the steel yield point stress and the concrete cylinder strength. Although the ratios $\frac{c}{d}$ in Mr. Whitney's theory represent an improvement over the values j determined by usual designing method, they differ materially from the values $\frac{c_t}{d}$, calculated on the basis of test results, which may be shown in the following. In Table 1 only tests are dealt with whose percentages of steel are greater than 2%. Table 2 contains tests, conducted in 1912, of small percentages of steel. In this case, the ratio $\frac{c_t}{c}$ (which demonstrates the difference between real behavior and theory according to Mr. Whitney) varies between 1.006 and 1.188 for percentages of 0.49 and 0.98. As long as no greater differences are in question there would be no reason for any improvement; but newer tests, especially in cases of high-grade materials, give much greater differences. This may be seen by reference to Table 13, which shows the relation between the ratio $\frac{c}{d}$ of beams or slabs and their reinforcement (between 0 and 2%). The values of $\frac{c_t}{d}$ calculated for several test results are tabulated. The ratio $\frac{c_t}{d} = 1$ represents a limit, the lever arm being identical with the distance of the reinforcement from the upper fiber, which case actually would be impossible should Eq. 59 be correct. This limit is materially surmounted, as shown in Table 13. The following test results have been taken into account: (1) Tests on slabs, conducted at Lehigh University,[19] Bethlehem, Pa.; (2) tests on rectangular and T-beams at Columbia University,[66] New York; (3) tests on rectangular beams by the writer[67] in Vienna, Austria; and (4) tests on tubular spun beams by Prof. C. Král[68] at Ljubljana, Yugoslavia.

Table 13(*a*) contains a selection of the Lehigh tests.[19] Two steel types were taken into account, one type with a yield point stress between 46,000

[19] "A Study of Reinforcement in Concrete Slabs," by Inge Lyse and George R. Wernisch, *Journal*, Am. Concrete Inst., September–October, 1936, p. 1.

[66] "The Modular Ratio," by K. Hajnal-Kónyi, *Concrete and Constructional Engineering*, Vol. 32, 1937, pp. 129, 189, 521.

[67] "Versuche mit Rechteckbalken, bewehrt mit besonders hochwertigem Stahl," by Paul Abeles, *Beton und Eisen*, Vol. 36, 1937, p. 295; also "Elastizität des Betons," by Paul Abeles, *Zement*, Vol. 26, 1937 p. 619.

[68] "Spun Tubular Beam," by P. Abeles, *Concrete and Constructional Engineering*, Vol. 35, 1940, p. 233.

TABLE 13.—COMPARISON OF THEORETICAL STEEL LEVER ARM WITH TEST RESULTS

Beam No.	100 p	Stresses (Lb per Sq In.): Yield point, f_s	Stresses (Lb per Sq In.): Cylinder strength, f_c	k	$\frac{c_t}{d}$	$\frac{c}{d}$	j	$\frac{c_t}{c}$	$\frac{c_t}{j}$
(a) Slabs (Lyse and Wernisch); $f_c = f_c'$; and $k = \frac{f_s}{f_c'}$									
9	0.35	46,000	2,890	15.92	1.45	0.976	.....	1.50	.....
10	0.303	55,300	2,795	19.66	1.47	0.965	.....	1.52	.....
6	0.755	47,500	2,784	17.06	1.31	0.924	.....	1.42	.....
7	0.754	50,000	2,640	18.95	1.32	0.916	.....	1.44	.....
32	0.725	50,000	5,750	8.76	1.30	0.963	.....	1.35	.....
22	1.32	48,500	3,140	15.45	1.02	0.880	.....	1.16	.....
24	1.328	50,300	3,140	16.20	0.985	0.874	.....	1.13	.....
12[a]	0.343	95,500	3,390	28.20	1.20	0.943	.....	1.275	.....
29[a]	0.77	93,000	5,230	17.70	1.04	0.920	.....	1.130	.....
(b) Rectangular Beams and T-Beams (Columbia Tests); $f_c = f_c'$ and $k = \frac{f_s}{f_c'}$									
C_4	0.451	56,330	3,376	16.62	1.135	0.956	.....	1.185	.....
C_{14}	0.456	56,330	2,837	19.85	1.13	0.947	.....	1.18	.....
C_7	0.798	55,730	3,304	16.87	1.03	0.921	.....	1.13	.....
C_{17}	0.820	55,730	3,340	16.78	1.02	0.919	.....	1.11	.....
D_3[b]	0.843	50,470	3,247	15.52	1.03	0.923	.....	1.115	.....
D_{13}[b]	0.812	50,470	2,960	17.04	1.04	0.918	.....	1.13	.....
C_1	0.620	35,600	2,510	14.18	1.125	0.948	.....	1.188	.....
C_{11}	0.623	35,600	3,320	10.7	1.141	0.961	.....	1.18	.....
C_5	1.245	35,800	3,243	11.04	1.0	0.919	.....	1.09	.....
C_{15}	1.230	35,800	3,028	11.82	0.97	0.914	.....	1.06	.....
(c) Rectangular Beams (Abeles); $k = \frac{4}{3}\frac{f_s}{f_c}$; and $n = 15$ for j and $\frac{c_t}{j}$									
Concrete 1:									
1	0.38	37,600	2,060[c]	24.32	1.025	0.946	0.905	1.085	1.13
2	0.85	37,100		24.02	0.955	0.880	0.868	1.085	1.10
4	0.38	95,200	2,060[c]	61.75	1.04	0.862	0.905	1.195	1.15
5	0.85	91,000		58.90	0.72	0.705	0.868	1.022	0.83
Concrete 4:									
18	0.38	37,600	8,400[c]	5.96	1.167	0.987	0.905	1.183	1.29
19	0.85	37,100		5.94	1.110	0.970	0.868	1.145	1.28
20	1.50	37,100		5.94	1.041	0.948	0.839	1.098	1.24
22	0.38	95,200	8,400[c]	15.10	1.385	0.966	0.905	1.425	1.53
23	0.85	91,000		14.45	1.232	0.928	0.868	1.327	1.42
24	1.50	87,000		13.80	1.007	0.878	0.839	1.146	1.20
(d) Spun Tubular Beams[d] (Král); $j = j_0$									
I_c[e]	0.19	47,025	.....	.....	1.995	.....	0.936	.....	2.13
II_a[e]	1.51	42,902	.....	.....	1.206	.....	0.928	.....	1.30
III_a[e]	3.32	41,992	.....	.....	1.145	.....	0.902	.....	1.27
9_b[f]	0.37	57,307	.....	.....	1.806	.....	0.927	.....	1.95
7_a[f]	1.33	56,311	.....	.....	1.365	.....	0.928	.....	1.47
10_b[f]	2.01	55,316	.....	.....	1.280	.....	0.909	.....	1.41
16_b[g]	0.30	47,025	.....	.....	2.775	.....	0.907	.....	3.07

[a] The corresponding beams with the same steel percentages do not agree regarding loading and arrangement, respectively. [b] T-beams. [c] Cube strength. [d] Beams with the smallest breaking load. [e] Octagonal tube, 11 in. high, mild steel. [f] Octagonal tube, 11 in. high (upper width, b = 4.8 in.), twin twisted steel bars. [g] Spun beam 8.7 in. total height with lower flat base, mild steel.

and 55,300 lb per sq in. (three different percentages of steel, two or three separate results for each percentage, respectively), and the other type between 93,000 and 95,000 lb per sq in. (two different percentages and only one test result each). Table 13(b) contains a selection of the Columbia University tests with two different steel types (yield point stresses 35,600 to 35,800 and 50,470 to 56,330 lb per sq in.), two different percentages in each case, no difference resulting between rectangular beams C_7 and C_{17} and **T**-beams D_3 and D_{13}. In Tables 13(a) and 13(b), also, the concrete cylinder strengths are given. The rectangular beams reported in Table 13(c), with a cross section 8 in. by 9 in., had a span of 8 ft and were loaded with two single loads in a distance of 3.3 ft; four different concrete mixes were tested, each with two different grades of steel (yield point stresses 37,100 to 37,600 and 87,000 to 95,200 lb per sq in.). From Table 13(c) (which contains average values of two specimens each), it can be seen that only concretes 1 and 4 have been taken into account, concrete 1 being of very low strength, much different from the high-grade concrete 4. For comparison the values $\frac{c}{d}$ have been computed on the assumption that $\frac{3}{4}$ of the cube strength given in the Table 13(c) (obtained from 8-in. cubes) is identical with the cylindrical concrete strength. The values j for $n = 15$ and ratios $\frac{c_t}{j}$ have been added in Table 13(c) for purposes of comparison. The tubular spun beams, according to Table 13(d), do not actually agree with this comparison, since they represent a combination of tubes and beams, which works very favorably, the automatic pre-stressing of the transverse reinforcement (owing to the centrifugal force during spinning) being of great advantage in regard to reduction of the principal stresses. Nevertheless, their results have been included to show the steel saving that is possible if consideration is given to the actual behavior of reinforced concrete structures with small reinforcement. In Table 13(d) only the results with the smallest breaking moments have been considered. In this case the percentages have been computed for the upper breadths of the tubular beams, and the ratios $\frac{c_t}{d}$ have been calculated with the assumption of an existing secondary reinforcement of one round bar (area = 0.22 in.) in either lower outside corner of the octagon, the values j_0 representing the ratios of the lever arms of **T**-beams $d - \frac{t}{2}$ and their depths d (t being the thickness of the slab). The tests were conducted on beams of 3.6-ft span and 7.4-ft span, subjected to two single loads in a distance of 1 ft and 1.5 ft, respectively.

From Table 13(b) it is seen that the results of tests made at Columbia University agree very well with those reported in Table 2. The ratios $\frac{c_t}{c}$ vary between 1.06 and 1.185; but the difference is much greater according to tests on slabs (Table 13(a)) and on rectangular beams (Table 13(c)). There is a discrepancy of 50% with a small percentage of medium-grade steel tests (Table 13(a)), whereas with a high-grade steel only a smaller excess of strength of 27.5% occurs. The Viennese tests, conducted by the writer, agree very well

with those of Lehigh University, but in the first case slabs have been tested, and in the second case rectangular beams. Concrete 1, of very low strength, gives only an excess of strength with a small percentage by use of high-grade steel, whereas with a greater percentage no use can be made of the reinforcement, since the concrete strength is too low. In this case mild steel causes better results than high-grade steel similar to the test results (Table 13(*a*)). In neither case was the concrete strength very great. With concrete 4, of great strength, the excess of strength is greater when high-grade steel is used. (Similar results have been obtained with concretes 2 and 3.) The test results of spun tubular beams surpass those of rectangular beams and slabs materially. Attention is drawn to values $\frac{c_t}{j_0} = 2.13$ and 1.95 in the case of octagonal beams and of 3.07 (!) in case of beams with flat lower base, the concrete tensile zone being thus increased. Tests on these beams have proved that the concrete tensile zone, even at breaking stage, still resists to a large extent, in spite of the discontinuity caused by the cracks.

Even if the results of tests on tubular spun beams are eliminated, because of their different behavior, those of the tests in Tables 13(*a*) and 13(*c*) show an excess of strength of up to 50%, which fact is reason enough to study the matter more closely in order to make use of it in design. First, the fact itself may be investigated as to how these results can accord with the assumption that the yield point stress, or one connected with a definite elongation, is the reason for failure, the concrete tensile zone not cooperating due to cracks. The latter will be the case if the bond between concrete and steel is destroyed and the concrete strength is low. Actually, with increased concrete strength, the concrete area in the tensile zone cooperates to some extent, which causes the tensile stress of the reinforcement, between the cracks (especially in cases of small percentages), to be much lower than in the cracks. In the latter it ought to increase to a maximum stress calculated from the breaking moment, the tension in the concrete being neglected. Such a straining would cause the steel stresses between the cracks to increase gradually on both sides of the cracks, owing to the decreased bond, and thus obtain in the points of the cracks their greatest values. Such an irregular straining with maximum points at intervals seems to be unlikely to agree with the reality. It may be expected that a redistribution of stresses takes place and the greatest steel stresses are smaller than those computed for the crack points. This hypothesis was offered by the writer in 1936.[69] Besides, as long as the cracks are not too wide, it can be assumed that the actual steel stresses are materially greater than the yield point stress or its equivalent. Even if elongations of 5% and more are taken into account (the equivalent for yield point straining is only 0.2% and 0.4%, respectively, for permanent and total elongations), no great harm can be caused if the elongation extends only to a small length. The latter depends on concrete tensile and compressive strength, on the distribution of the reinforcement, its diameter and behavior (strain-stress diagram), etc., to avoid wide cracks and to insure a good bonding between the cracks. From this point of view, at failure, even stresses as high as the full strength may act

[69] *Monatsnachrichten Oester. Beton Ver.*, April/Mai 1936.

in the steel at the cracks, as long as the latter have not surpassed a limit width. Besides, there is another possible explanation of the excess. The strain-stress diagrams of steel have been obtained on specimens of a certain length. It may be expected that the behavior of the material is quite different for a gage corresponding with the width of a crack, say, 0.2 in. From all these considerations it will be realized that further investigations would be of the greatest importance in finding the bases necessary for design. These particulars must be dependent on test results. K. Hajnal-Kónyi[70] has examined such a proposition which agreed very well with the tests reported in Table 13(*a*); but in this method only the compressive strength of the concrete is taken into account, and the results can therefore only correspond with the reality as long as a definite ratio exists between compressive and tensile strength, the respective bonding also being of importance. That method, for example, disagrees with the tests in Table 13(*c*). Concretes 2, 3, and 4, of different tensile and compressive strengths, differ little in their carrying capacities.

From this point of view it seems of greatest importance to make further investigations on beams and slabs reinforced with small percentages of steel to find the proper basis for an improvement in design, since small percentages become more and more common, owing to the use of high-grade material and of steel pre-stressing. For a definite concrete grade the basis can be found by a number of tests and can be applied to the design if definite equality of the concrete is guaranteed.

[70] Discussion by K. Hajnal-Kónyi of "A Study of Reinforcement in Concrete Slabs," by Inge Lyse and George R. Wernisch, *Journal*, Am. Concrete Inst., Vol. 33, 1937, p. 16–1.

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DISCUSSIONS

EXPERIENCES IN OPERATING A CHEMICAL-MECHANICAL SEWAGE TREATMENT PLANT

Discussion

By Messrs. Darwin W. Townsend, Lloyd H. Flickinger, Isador W. Mendelsohn, and Ralph E. Fuhrman

Darwin W. Townsend,[8] M. Am. Soc. C. E. (by letter).[8a]—The practice of pollution control by cities on the upper Mississippi River has been marked during the past decade (1930–1940). Modern sewage treatment works are now (1941) in operation in the cities of Minneapolis-St. Paul, and South St. Paul; Rock Island, Ill.; and Davenport, Iowa, as well as in other populated centers of lesser magnitude. The Minneapolis-St. Paul plant, which Mr. Schroepfer's excellent paper treats, and the South St. Paul plant, designed by the writer's firm, are the two most northerly plants of magnitude on the Mississippi River in the foregoing group. Upon the completion of the Newport, Minn., treatment works, the pollution control in the populous and highly industrial Minneapolis-St. Paul metropolitan area will have been effectively accomplished. Of unusual note, perhaps, is the fact that three separate treatment plants will serve this area, these plants being within sight of each other.

It may be of interest, by way of contrast, to comment briefly upon sewage B.O.D. concentrations obtaining in each of the aforementioned instances. Sewage delivered into the Minneapolis-St. Paul treatment plant has a 5-day B.O.D. concentration—approximating 200 ppm, as stated by Mr. Schroepfer. At this concentration, a sewage of average domestic character is indicated, in which case little, if any, difference might be anticipated between human and equivalent populations.

Available dilutions and dissolved oxygen (D.O.) supply afforded by the waters of the Mississippi River for the assimilation of organic pollution indicated that B.O.D. removals from Minneapolis-St. Paul sewage, approximating 70% for ten months and 33% for two months, would prove adequate to main-

Note.—This paper by George J. Schroepfer, Assoc. M. Am. Soc. C. E., was published in January, 1941, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: March, 1941, by Messrs. F. C. Roberts, Jr., and Rolf Eliassen; and April, 1941, by Frederic Bass, M. Am. Soc. C. E.

[8] Cons. Engr. (Consoer, Townsend & Quinlan), Milwaukee, Wis.

[8a] Received by the Secretary March 31, 1941.

tain in the receiving waters a 2.0 ppm D.O. residual at all except infrequent times of unusually low river stage. It is evident from the data presented by Mr. Schroepfer that, when contrasted with the volume of river flows shown, plant removals thus far obtained have been quite adequate to accommodate the D.O. standard established.

The requirements as to degree of treatment for the South St. Paul-Newport area differ vastly from those of the Minneapolis-St. Paul area and call for a much higher degree of treatment. In South St. Paul, as well as in Newport, the meat-packing industry predominates, and as a result the sewage treatment problem is essentially that of treating packing-house and stockyard wastes containing an average B.O.D. concentration approximating 1,000 ppm, the peak concentrations reaching as high as 3,000 ppm. The human population of South St. Paul approximates 15,000, whereas, at times, the equivalent population may exceed 300,000.

To comply with the requirement of the Minnesota Board of Health (namely, that the effluent from the South St. Paul treatment plant contain a B.O.D. concentration that would not be greater per unit of volume than the Minneapolis-St. Paul effluent), it was necessary to design treatment works of biologic character to obtain 90% to 95% B.O.D. removal, these works to include three stages of sedimentation, flocculation, grease removal, and trickling filters. Effluent chlorination is to be practiced during the summer months.

The lead taken by the Minneapolis-St. Paul Sanitary District in pollution control of Mississippi River waters provided the necessary stimulus to downstream cities for subsequent participation in work of similar character, some of which have been referred to herein.

Although Mr. Schroepfer has performed a yeoman's task in preparing and presenting a well-conceived and well-written paper, supplemented with interesting data, the writer takes the liberty of suggesting that, in the closing discussion of his paper, something additional be written regarding the extensive river-water studies made by himself and the late J. A. Childs, M. Am. Soc. C. E., former sanitary engineer for the District. The writer had occasion to study this work while serving the District as consultant, and it is most certain that the results of these water reoxygenation studies would appeal to many sanitary engineers.

Of particular interest to the writer is the subject of sludge disposal as practiced at the Minneapolis-St. Paul plant. This installation is believed to be the most extensive of its particular kind in existence, and may be considered unusual, perhaps, in that raw, rather than digested or otherwise treated, sewage solids are filtered and incinerated.

The partial dehydration of sewage solids by means of vacuum filters has become standard practice during comparatively recent years, the first sizable installation having been placed in operation at Milwaukee, Wis., about 1925. This installation dehydrates activated sludge, which subsequently is heat-dried for use as a fertilizer. The solids filtered at this plant may be classified as raw activated sewage solids, as differentiated from digested solids, and raw untreated solids. They possess radically different filtering characteristics

from those of the two last-named materials. Sludge filtering at Milwaukee, therefore, is not comparable with generally similar work done at the Minneapolis-St. Paul plant.

Many treatment plants throughout the United States now use vacuum filters and appropriately designed incinerators as a means of sludge disposal, and this method appears to be becoming increasingly popular. However, general practice recognizes certain economies indicated and attainable through the use of an intermediate step in sludge-disposal procedure—namely, that of reducing sludge volume through the process of digestion.

Instances may be cited wherein the available power contained in the gas resulting from controlled sludge digestion equals or exceeds the entire power and heat requirements of a treatment plant. Since power is a substantial item of expense in any sewage treatment plant (more so in some than in others, depending upon plant type and prevailing rates), savings that may accrue to a city through the use of gas-engine equipment, motivated by gas produced from the sewage within the plant, have in recent years met with almost universal favor.

Perhaps the author may make some reference in his closing discussion to vacuum filtration of raw solids, and incineration versus digestion and gas utilization as studied in connection with the Minneapolis-St. Paul project. Such references would be most interesting.

LLOYD H. FLICKINGER,[9] ASSOC. M. AM. SOC. C. E. (by letter).[9a]—The subject of sewage treatment has received a valuable contribution by Mr. Schroepfer. Designers and operators of many treatment plants, whether of the chemical-mechanical type or other types, can learn valuable lessons from this paper. When this plant was designed, it was more than twenty times as large as any other one of its type then in operation. Many new ideas were incorporated in the plant and it is gratifying to find that the results have been very satisfactory, with a few minor exceptions.

The writer was employed on the Minneapolis-St. Paul project throughout the design and construction period of the treatment plant, and for a short time after the plant was placed in operation. The purpose of this discussion is to call attention to certain features of the design and operation that merit close attention by future designers of similar plants.

The kind of treatment selected for this project was of a type having great flexibility to provide a degree of treatment in direct proportion to the demands of the Mississippi River. This procedure is not subscribed to by many state boards of health. Too often they require "complete treatment" with little regard for the ability of the stream to absorb a pollution load. It is economically unsound to provide a continued high-grade treatment where a large part of the pollution load can be absorbed by the stream without nuisance or danger to the community. This procedure would be comparable to operating a furnace in a home at full capacity regardless of the temperature outdoors, merely because of the available capacity of the heating plant.

[9] Asst. Civ. Engr., The Panama Canal, Special Eng. Div., The Third Locks Project, Balboa Heights, Canal Zone.

[9a] Received by the Secretary April 16, 1941.

The type of plant selected for this project is economically sound and provides a very high degree of treatment only when it is required. This is sound economics and should be observed.

The plant was designed on a basis of 185 ppm suspended solids in the raw sewage. The actual amount found by operation was about 275 ppm. Contrasted to this the estimated B.O.D. was 200 ppm and the actual was 210 ppm. The unusual amount of suspended solids has never been explained fully. The quantity of sewage in gallons was about 20% less than estimated for the years 1938–1940. This accounts only partially for the high suspended solids content.

In 1933 principal sewers were weired and samples collected for about two weeks. The principal outlets were attended by a sample collector 24 hr per day throughout the test period, and the quality of the analytical work has never been questioned. The only deduction is that something caused a great change in the sewage in the 5-yr interval from 1933 to 1938. However, no substantial industrial change took place except in the brewing and malting plants.

The extra-long settling tanks, almost 300 ft, were selected because they fit the site very well and were lower in first cost than the shorter ones. The estimated removal of suspended solids was 56% and the actual was about 70%. Designers should note that these tanks have a very high weir ratio. The amount of weir length per million gallons of sewage was about halfway between that used for circular tanks and that usually used for rectangular tanks. These tanks are 56 ft by 290 ft and have 480 lin ft of weir per tank. The unusual results from the extra long tanks with high weir ratios should be carefully noted by all designers of settling tanks.

The scum handling methods originally installed proved unsatisfactory in operation. Pneumatic ejectors definitely proved their worth. Line stoppages can be eliminated with this method, if the scum influent lines to the ejector are short and have an appreciable slope.

The plant is arranged so that several kinds of chemical can be used. Engineers have long suspected that the price of many chemicals does not depend on the cost of production or the supply. It is suspected that the price is controlled, perhaps on the basis of all the traffic will bear. Engineers can meet this practice by designing for several chemicals. All the added cost can be saved easily in purchases of the most economical chemical. In other words, while the chemical producers are using chemical prices as so many poker chips in an industrial game of chance, the sanitary engineer can buy the chemical that is in the "bear" market.

The sludge in the concentration tanks actually became too concentrated to be handled at times. The need for concentration can be questioned, but the storage facilities providing a reservoir on the line proved the worth of the tanks. Future concentration tanks can properly include facilities for diluting the sludge when necessary.

Sludge containing 10% to 12% solids needs thinning for proper handling through a piping system of any length. Thick sludge frequently disobeys some of the fundamental laws of hydraulics. Sludge pipe design demands that a little "salt" be taken with theoretical results assumed using hydraulic computations for water.

It is gratifying to note the low chemical dosage required to condition the sludge. The writer recommends no change from the original dosage of 3% ferric chloride and 10% lime, as a basis of design. The chemical demand for the conditioning of sludge usually cannot be determined before the plant is put in operation, with the exception of additions to existing plants and plants other than the first in a city.

The favorable results of vacuum filtration of raw sewage solids at this plant definitely put an end to the argument that digestion is necessary to satisfactory and economical vacuum filtration. Mr. Schroepfer's experiments with different kinds of filter cloth are very enlightening, and similar tests should be made at other plants to confirm the results.

Incineration of sewage solids was in its infancy when the Minneapolis-St. Paul plant designers decided to use this method of sludge disposal in 1936. The use of extensive and delicate heat-recovery units does not appear economically sound in the light of present knowledge. They are very costly and expert operation is necessary to prevent serious damage to the units. The problem at the Minneaplis-St. Paul plant seemed to be one of disposing of excess heat instead of recovering it.

The writer recommends that screenings and grit be disposed of by incineration, omitting the extensive grit washing operation and screening grinders. This waste can be carried direct to the incinerators by one belt conveyer. The belt can be operated manually as required, dumping the unwashed grit and unground screenings on a horizontal belt conveying the sludge cake to the incinerators. Close control of the grit chamber velocities becomes unnecessary because high putrescible contents of grit to be incinerated are not objectionable. Due to existing arrangements at the Minneapolis-St. Paul plant, it is probably not practical to do this, but designers of new plants can well consider this possibility.

The profession is indebted to Mr. Schroepfer and the operating staff of the Minneapolis-St. Paul Sanitary District for the extensive and valuable data obtained from the first two years of operation. The plant is so designed that one half can be used as a control and the other half for experimental work. At the present rate more valuable contributions to the science of sewage treatment can be expected from this plant.

The writer is very pleased to learn of the vast improvement in the condition of the Mississippi River and is particularly pleased to have been a part of the design staff that made this improvement possible.

ISADOR W. MENDELSOHN,[10] M. AM. SOC. C. E. (by letter).[10a]—As expected, Mr. Schroepfer has presented a fitting complement to his notable contribution on "Economics of Sewage Treatment" published in 1938.[11] This paper is more than a repository of interesting experiences for the first two years of operation of the sewage treatment plant serving the Minneapolis-St. Paul

[10] Civ. Engr. in Chg. Sub-Section, Const. Div., Quartermaster Corps, War Dept., Washington, D. C.

[10a] Received by the Secretary April 30, 1941.

[11] *Transactions*, Am. Soc. C. E., Vol. 104 (1939), p. 1210.

Sanitary District. It should prove a source of unending stimulation to operators and designers of sewage treatment plants, large and small, for it is replete with ever-promising, economy-producing experiments and studies in mechanical equipment and plant operation. Throughout all of these studies, the engineers never lost sight of the purpose of producing as economically as possible an effluent that would merge with the varying receiving body of water in a manner not prejudicial to public health and comfort.

The lengthy and remarkable list of tests on plant equipment and operation performed by Mr. Schroepfer and his organization in this period includes:

For screen and grit removal—

(1) Increasing the velocities of flow from 0.75 to 1.20 ft per sec to secure a greater percentage of inert material in the sludge, thereby increasing the solids content of the sludge and reducing the quantity of conditioning chemicals required;

(2) Burial of the proportionately small quantity of screenings with an appropriate depth of relatively clean grit;

(3) Reducing sand wear on the lower and intermediate bearings on the screw conveyer grit washers by installing sealed water-washed and lubricated bearings;

(4) Overcoming difficulty with the grit bucket elevator by installing an air jet to sweep high moisture content, large "fines" percentage grit out of the buckets, and installing sealed bearings of a special type.

For sedimentation—

(5) Careful pumping control of raw sludge from the settling tanks once during each of the three shifts so as to average 7.7% solids (92.3% moisture);

(6) Providing an ejection method of handling scum after its removal from the settling tanks, thereby reducing labor required;

(7) Providing for blowing back sludge suction lines as long as 300 ft with plant effluent or compressed air to handle heavy sludge containing as much as 20% solids;

(8) Carrying a heavy blanket of normal sewage sludge in the tanks previous to storm flows to remove the inert material more effectively because of evident loosening of the mud and silt which ordinarily packs and produces difficult removal problems;

(9) Replacing pistons and collars on sludge pumps with heavier parts made in the Sanitary District shop at a fraction of the cost of such parts from manufacturers to permit resurfacing from time to time, thereby prolonging their life;

(10) Installing chain tighteners on the sludge collection equipment to reduce breakage of links of drive chains.

For magnetite effluent filters—

(11) Overcoming sand erosion and displacement difficulties at greater than average flows by enlarging influent ports and other improvements.

For sludge conditioning—

(12) Shortening the period of sludge conditioning to an average of 6 min with continuous control of filtration by means of funnel tests;

(13) Improving sludge for filtration and at the same time reducing the quantity of chemicals used by agitation with excess low-pressure air from the blowers, provided for removing cake from the filters;

(14) Providing means for distributing the chemicals more uniformly through the conditioning tanks;

(15) Determining the proper sequence of chemical (lime and ferric chloride) additions.

For vacuum filtration—

(16) Recirculation of a percentage of incinerator ash to the raw sewage ahead of the screen and grit chambers to improve vacuum filtration, reduce the quantity of conditioning chemicals, and increase sedimentation efficiency;

(17) Reducing the quantities of chemicals (3% ferric chloride and 10% lime) to less than 2% and 5%, respectively, amounting to a saving of approximately $100 a day in chemical costs alone;

(18) Increasing filter cloth life from about 150 hr of use to more than 300 hr;

(19) Purchasing identical filter cloths locally at a fraction of their original cost;

(20) Sewing filter cloths with a different type of thread and with a lock stitch to eliminate seam difficulties;

(21) Improving the washing of filters by impinging the water at right angles to the cloth on the drum, thereby reducing the washing time to one third, using less water, and eliminating brushing of the cloth;

(22) Installing sloping $\frac{1}{4}$-in. rods on 12-in. centers in the filter take-off plates to break up the cake and prevent stoppages.

For incinerators—

(23) By by-passing preheated air direct to the stack to save rabble arms and other incinerator parts.

For sludge disposal—

(24) Exploring the fertilizer or soil conditioning possibilities of sludge cake as produced by the filters and its use after storing for periods of time;

(25) Experimenting with the production of partly dried material by removing portions of the incinerator top-hearth sludge, using heat from the remainder of the sludge incinerated for drying purposes;

(26) Investigating the use of ash for binder or filler for various products, such as a soil conditioner or an ingredient for certain cements;

(27) Testing the utilization of heat from incineration in the generation of steam for all heating purposes and electric-power requirements.

For flocculation without chemicals—

(28) Testing for a brief period to determine the degree of improvement that may be obtained in sedimentation.

For chemical treatment—

(29) Testing for four days to determine possible sedimentation-removal improvements.

For chlorination—

(30) Experimenting on bacterial removal with chlorine dosages of 6 to 8 ppm with favorable results. A large part of the great reduction (approximately \$330,000) in operation costs of the first 2 yr is attributed to the operation economies effected by the tests mentioned.

The paper is notable also for the detailed operation and maintenance cost data presented. For comparative purposes with other sewage treatment plants, it would be desirable, when more complete operation data are available, to show for each plant unit the construction cost, the operation and maintenance costs, the specific parts-per-million removals in suspended solids and B.O.D., and all three costs per 1% reduction in suspended solids and B.O.D.

The fact that vacuum filtration and incineration of sludge cost \$3.70 per million gal, or approximately 45% of the entire plant operation and maintenance costs, is noteworthy. The relative cost seems high for sludge disposal alone. It presents a great opportunity for improvement. Whether the cost of sludge disposal would have been lower with some other process or combination of processes cannot be determined from the data presented. As bearing on this question, it would be of interest to note what studies were made regarding sludge disposal in the design of the plant, what construction, operation, and maintenance costs were estimated for each method considered, and why vacuum filtration and incineration were adopted.

As between vacuum filtration and sludge incineration operation and maintenance costs, it would be desirable, for comparison with similar data in other plants, to learn the basis of allocating labor and supervision expenditures to each.

With reference to the incinerator, it is surprising to learn that "It was the manufacturers' expectation that the location of burning, and the temperatures of combustion, would be controlled by the use of varying quantities of oil introduced at different points" (see heading, "Sludge Disposal: Incinerators"). Acceptance tests on multiple-hearth sludge incinerators at Auburn and Elmira, N.Y., sewage treatment plants in May, 1937, disclosed excess heat, requiring by-passing of preheated air direct to the stack to prevent damage to incinerator parts, and use of oil only for warming up and maintenance of temperature in off-burning periods.

Mr. Schroepfer refers to extensive guarantee and maintenance bonds of various types included in mechanical equipment contracts for the plant. In view of his experience with such provisions, it would be of value to learn his views as to what kind of guarantees and maintenance bonds should be included in specifications and for what kind of mechanical equipment, what tests should be made to determine whether or not the guarantees are complied with, how the tests should be conducted, and for what time periods.

Before concluding these remarks, it is most important to note that the worthy results obtained in the operation of this sewage treatment plant are

the outgrowth of a capable and energetic operator with an experienced organization, adequate laboratory and plant recording equipment, and above all a body of intelligent, sewage-minded Sanitary District officials, representing a community that desires, and is willing to back, disposal of its sanitary sewage in a manner which will not prove harmful. Similar fully trained, equipped, and cooperating groups are a vital necessity in all communities, however large or small, for sanitary disposal of their sewage.

RALPH E. FUHRMAN,[12] ASSOC. M. AM. SOC. C. E. (by letter) [12a]—In a comprehensive manner, Mr. Schroepfer has described the development of the sewage treatment scheme for the Twin Cities. From the inception of the project, the primary purpose of the plant (that of eliminating pollution from the Mississippi River) has received first consideration.

The design of the plant has, as its fundamental basis, the varying requirements of the receiving stream. This rational basis has provided, through the author's analysis, the degree, and probable duration and frequency of varying degrees of treatment imposed by river conditions. The result, a chemical-mechanical sewage treatment plant, provides the range of treatment and flexibility to relieve the river of pollutants economically.

The District of Columbia sewage treatment plant and receiving stream present some interesting parallels with the Twin City plant and its receiving stream, namely:

Description	Twin City	District of Columbia
Design flow (mgd)	134	130
Full-time operation begun	July 20, 1938	August 1, 1938
Original construction cost	\$3,750,000	\$3,400,000
Construction cost per design (mgd)	\$28,000	\$26,100
Minimum flow of receiving stream (cu ft per sec)	864	700
Maximum estimated flow (cu ft per sec)	107,000	480,000
Annual average flow (cu ft per sec)	8,800	9,480

Similarity ends, however, when the plant processes themselves are considered. At the Twin City plant, sewage is screened, grit is removed, coagulating chemicals may be added prior to metering, flocculation, and sedimentation. When desired, more solids may be removed by use of the effluent filters. At the District of Columbia plant, sewage is pumped, grit is removed, the sewage is metered and aerated for grease separation before sedimentation, the last step in the sewage treatment process. At the Minneapolis-St. Paul plant, raw solids are dewatered on vacuum filters and incinerated. At Washington, D. C., raw solids are digested, digested solids are elutriated and dewatered by vacuum filters. Most of the sludge cake is shipped to the District of Columbia penal institutions for agricultural uses. Between the two plants, practically all current sewage and sludge treatment methods except activated sludge are used.

[12] Asst. Supt., Sewage Treatment Plant, District of Columbia, Washington, D. C.

[12a] Received by the Secretary May 20, 1941.

Since the dates of design and initial operation of the two plants are very close, some operating experiences have been common to both plants. For instance, the failure of the underwater and grit thrust bearings of the grit conveyers described by Mr. Schroepfer was also experienced at Washington, D. C. At the District of Columbia plant this failure occurred at an early date and the difficulty was corrected by installing standard ball thrust bearings at the shaft coupling above the sewage level. Completely satisfactory operation of the conveyers has been gained by this alteration.

Although the sedimentation tanks of the District of Columbia plant are circular, experiences in tank skimming have been similar to those at the Minneapolis-St. Paul plant, except for difficulties caused by climatic conditions. At the Washington, D. C., plant, completely automatic skimming was not provided originally. Collection of the scum from the surface of the sedimentation tank was mechanical and continuous, but removal of the scum was manual and intermittent. New skimmers have been installed which give continuous skimming and removal of scum. Minor stoppages have occurred in the lines conveying scum to the hoppers from which the sludge pumps take suction to pump the scum to the sludge digestion tanks; but as these lines are short, clogging is easily relieved mechanically.

Because of the completely different nature of sludge treatment by the two plants, an operating cost comparison of sludge treatment is interesting. From Mr. Schroepfer's paper, Twin City costs for the 1939 calendar year for vacuum filtration and incineration were \$136,815.43, giving unit costs of \$3.70 per million gallons or \$3.63 per ton of dry solids treated. At Washington, D. C., for the 1939 fiscal year (July 1, 1938, to June 30, 1939), the operating costs for sludge digestion, sludge elutriation, sludge dewatering (vacuum filtration) and sludge cake disposal were \$54,486, giving unit costs of \$1.59 per million gallons or \$4.41 per ton of dry solids removed. These data should not be taken in direct comparison, as they are based on early operating periods of both plants. They do indicate, however, that ultimate operating costs of both methods will be reasonable and of a comparable nature. The final cost analysis must include fixed charges, and energy recoveries by both methods. In the Minneapolis-St. Paul layout this will mean heat from the incinerators and ash value, whereas at Washington, D. C., it will mean sludge gas, both as heat and power and the fertilizer value of the sludge produced.

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DISCUSSIONS

LABORATORY INVESTIGATIONS OF SOILS AT FLUSHING MEADOW PARK

Discussion

BY E. J. KILCAWLEY, ESQ.

E. J. KILCAWLEY,[17] ESQ. (by letter).[17a]—An unusual opportunity is afforded, by this paper, for a comparison of results and conclusions reached under rush conditions and those determined by retesting and study in a more leisurely manner. It represents a comparison and suggested correlations of physical and mechanical test results.

In this later detailed analysis, a method of presenting results of routine tests has been developed and applied. These methods seem to aid materially in visualizing actual field conditions which, in turn, make quick and reasonable conclusions possible. The necessary extent and method of sampling, and further detailed testing to determine and evaluate mechanical characteristics of the deposits, can likewise be done on the basis of the fundamental facts established by simple and quickly performed soil tests.

An excellent example is the work reported in the use of the squeeze test. The modification of the three-dimensional test proposed by Mr. Jürgenson[18] in 1934 offers an efficient and handy tool for selecting samples, and for showing changes in the structure of the deposit. The method of plotting the final average diameter of the specimen measured in centimeters as abscissa for both undisturbed and remolded samples against the depth at which the sample was taken as an ordinate offers a quick and fairly reliable method of evaluating structure. Furthermore, it makes possible a means of estimating the dangers involved in the disturbance. These results, as concluded from the tests, are shown in the graphs for borings Nos. 2 and 17. The plot of final diameter in the squeeze test at the plastic and shrinkage limits, furthermore, shows graphically the range of moisture content involved in the stabilization process.

The records of the driving resistance of the casing shown also reflected

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[17] Prof. of Soil Mechanics and San. Eng., Rensselaer Polytechnic Inst., Troy, N. Y.

[17a] Received by the Secretary May 8, 1941.

[18] "The Shearing Resistance of Soils," by Leo Jürgenson, *Journal*, Boston Soc. of Civ. Engrs., July, 1934, p. 242.

the soft nature of the deposits, as pointed out by the author. A marked increase in the driving resistance is shown as the casings approach the rather highly compacted sand and gravel layers underlying the soft, silty clay. However, in the case of boring No. 2, this resistance increased from about eight blows at a depth of approximately 20 ft to about fifty blows at a depth of approximately 53 ft. Since the peat layer is approximately 10 ft below this point, it would seem that this reflects the fact that quick compression has an appreciable effect even in soils of low consistency.

In the further consideration of consolidation, the author has again emphasized the importance of structure as it affects the consolidation process. The effect of remolding is shown in Fig. 2 by the amount of consolidation which takes place under a loading equivalent to the pre-consolidation load, and by the spread between the consolidation curves for the undisturbed and remolded samples. This spread again offers a measure of the natural structure of the material. The results of repeated compression tests (also plotted in Fig. 2) show graphically the resulting change in structure and internal stress conditions. This part of the paper might well be expanded. Such a method of analysis might be applied to a study of stress history of deposits and of effective stress conditions.

The comparison of pressure-voids ratio curves in Fig. 3 shows both the range and the characteristics of the materials, and confirms the prediction of consolidation characteristics as indicated by the simple soil tests. The data included from the thesis of Mr. Horne[10] show the relation between consolidation characteristics and the liquid limit determination. They further point out the possibility of establishing a reliable guide in making good estimates of mechanical behavior and a means of establishing a maximum range of variation in these values of soil type characteristics. More extended study on this phase of behavior would be valuable.

In the analysis of quick, slow and delayed shear tests, the author has definitely shown the importance of hydrostatic equilibrium and effective stresses in the determination of shearing resistance. The significance of hydrostatic equilibrium as first pointed out by Professor Terzaghi is clearly shown. Volume change and structural adjustment are likewise emphasized and related to the important time factor in shear testing. An important consideration of an evaluation of stress conditions in undisturbed deposits, and in the test before any attempt is made to analyze results, is appropriately pointed out by the author.

In presenting the results of the tests and the correlations with the fundamental theories of Messrs. Terzaghi,[12] Hvorslev,[14] Jürgenson[11] and others,

[10] "Correlation of Compressibility and Atterberg Limits," by C. R. Horne, Jr., *Thesis No. 494*, Dept. of Civ. Eng., Columbia Univ., New York, N. Y., June, 1938.

[12] "The Shearing Resistance of Saturated Soils and the Angle Between the Planes of Shear," by Karl Terzaghi, *Paper No. D-7*, *Proceedings*, International Conference on Soil Mechanics and Foundation Eng., Harvard Univ., Cambridge, Mass., 1936, Vol. I, p. 54.

[14] "The Shearing Resistance of Remolded Cohesive Soils," by M. J. Hvorslev, Assoc. M. Am. Soc. C. E., *Paper No. E-1*, *Proceedings*, Soils and Foundation Conference, U. S. Eng. Dept., Boston, Mass., June 17–21, 1938.

[11] "The Application of Theories of Elasticity and Plasticity to Foundation Problems," by Leo Jürgenson, *Journal*, Boston Soc. of Civ. Engrs., July, 1934, p. 206.

Professor Burmister has made a definite contribution. Certain parts of the paper could have been expanded. More information on the effect of recompression might do much toward explaining varying results obtained on soils which are presumably the same. The underlying lesson taught in this work is the possibility of establishing a maximum range of variation for soil type groups and for the evaluation of physical and mechanical characteristics. Such a possibility deserves the best consideration of all soil physicists.

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DISCUSSIONS

CONCRETE IN SEA WATER: A REVISED VIEWPOINT NEEDED

Discussion

By Messrs. Harry E. Squire, and J. W. B. Blackman

Harry E. Squire,[25] Assoc. M. Am. Soc. C. E. (by letter).[25a]—In purporting to advocate a revised viewpoint in regard to concrete in sea water, Mr. Hadley has actually presented a carefully prepared and plausible argument in favor of one of the oldest traditional viewpoints of this controversial subject—namely, that sea water does not attack "sound" concrete, or embedded steel that is adequately covered with "sound" concrete. The contrasting viewpoint is that sea water does attack portland cement concrete and embedded steel whenever it comes in intimate contact with the cement paste or embedded steel, and that the only difference in durability between "sound" and "unsound" concrete is the time factor and intensity of exposure.

Before little if any concrete had been subjected to the sea water of the Pacific Coast, European engineers with a background of more than fifty years experience in marine concrete construction had likewise divided along almost identical lines of opinion.[26] On the Pacific Coast, the forms had barely been stripped from the early concrete before the same cleavage developed among those charged with responsibility for waterfront structures. The extravagant claims of advocates of the newly introduced "permanent" construction faded under the searching forces of sea-water erosion which laid bare all manner of defects ignored under ordinary atmospheric exposure. Nevertheless, enough resistant concrete remained to encourage the proponents of "sound" concrete in their claim that all deterioration could be explained by the term "defective concrete." Improved understanding of concrete technology combined with improved methods of manipulation and manufacture have eliminated almost entirely the defects of the earlier structures; and these methods insure a service life of present-day concrete equivalent to the commercial life of most marine

Note.—This paper by Homer M. Hadley, Assoc. M. Am. Soc. C. E., was published in January, 1941, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: March, 1941, by Thomas E. Stanton, M. Am. Soc. C. E.; April, 1941, by Messrs. W. F. Way, and Glenn S. Paxson; and May, 1941, by Messrs. Lester C. Hammond, Ladis H. Csanyi, and G. M. Williams.

[25] Asst. Chf. Engr., Board of State Harbor Commrs., San Francisco, Calif.

[25a] Received by the Secretary May 3, 1941.

[26] "Treatise on Dock Engineering," by Brysson Cunningham, 1904, Chapter 4, pp. 123–130.

structures built. The controversy becomes more and more a question of academic rather than practical interest and, in the writer's opinion, might be automatically ended by setting up a definition of "sound" concrete in terms of sea-water exposure—material which would withstand for a stated time (say forty years) a certain type of exposure (say tidal or complete immersion) with a stated loss of calcium (say 5%) or, if preferred, an equivalent gain in magnesium. Every one acknowledges that such concrete has been, and is being, manufactured, and most authorities will agree that such concrete is satisfactory for 90% of the structures built. The energy and enthusiasm of those proponents who refuse to acknowledge the mortality of concrete in sea water could then be concentrated on extending the frontiers of durability to really debatable periods beyond fifty years.

In studying the behavior of "sound" concrete in sea water, the writer agrees that there is no better place to go than to the structures which have stood successfully for periods of from thirty to fifty years. However, the writer suggests that the next time the author emulates the prophet and "goes to the mountain," he take a core boring machine and a laboratory assistant with him. There have already been too many surface inspections of both sound and disintegrated structures and too much speculation as to what is going on under the outer skin of the concrete. This is well illustrated by one of the instances of "sound" concrete cited in the paper—namely, the breakwater test specimens exposed at San Pedro.[10] These experimental blocks are almost an exact replica of similar test specimens exposed by Russian engineers in the breakwater at Libau a decade earlier.[27] The Libau blocks were found to be sound under surface inspection but they were, nevertheless, blasted open and samples of concrete from the outside and interior were analyzed and tabulated to show variation from the original cement. Quoting the concluding sentence of the report:

> "One fact, nevertheless, was certain—that the process of deterioration had already commenced and that it progressed, slowly perhaps, but none the less surely in the case of the mortars considered."

The writer had the good fortune to make a visual inspection of the San Pedro specimens on the contractor's barge during coring operations. The concrete was sound but saturated even in the interior of the blocks. The cores were to be forwarded to the National Bureau of Standards for testing and analysis, but as yet (1941) no record of these tests comparable to the Russian tabulation has come to the writer's attention. The article referred to by the author[10] is merely a report of the surface inspection together with some photographs of the core specimens nicely crated for shipment—hardly sufficient information to warrant the author's rather positive statement that there was "no evidence of sulfate of magnesium attack." Physical and chemical tests of the core samples would give what the lawyers call "the best evidence" as to what attack, if any, has taken place and, if repeated on future cores, will give valuable data on the rate of attack.

[10] *Western Construction News*, June 25, 1932, p. 367.

[27] *Proceedings*, International Assoc. for Testing Materials, 6th Cong., New York, N. Y., 1912, Second Section, XVII.

The writer hopes that one of the organizations having funds available for research will find it advisable to make a systematic investigation of structures, successfully exposed to sea water, by means of core borings and laboratory analysis. The list of structures selected by Mr. Hadley would be satisfactory to begin with. No doubt, most waterfront engineers would like to add some pet structure to the list, in which case the writer would select the Ferry Building Foundation Wharf at San Francisco, Calif. This well-preserved structure, approaching its 50th anniversary and outliving its utility as a passenger terminal, is remarkable in that it violates some of the choicest taboos of sea-water concrete practice. For one thing, the immersed concrete was mixed approximately one to eight and can hardly be considered impermeable; for another, the Dykerhoff cement used was of a reasonably high alumina content (approximately 11%) in contrast to the special cements of low alumina content advocated for sulfate resistance.

Most of the analyses available have been made of concrete that was badly disintegrated. Investigators seem to take a ghoulish interest in these misfortunes but have little time for successful structures. Invariably, these analyses show a partial replacement of calcium by magnesium. This has lead to the hypothesis, questioned by the author, that the disintegration is the result of sulfate of magnesia reaction with the calcium hydrate of the set cement.

Analyses of disintegrated concrete taken from a recent and somewhat disconcerting failure of good concrete in sea water (that of the concrete piles of the Ford Motor Company Plant at Long Beach, Calif.) indicated not only the usual increase in magnesia content at the expense of the calcium, but in addition a marked increase in the alumina content. Search for the source of the alumina led to the discovery of variable percentages of soluble alumina and active silica in samples of fine aggregate taken directly from the gravel pit. Submitted to the state mineralogist, the samples were identified as mixtures of quartz particles with varying percentages of feldspar particles, the feldspar being partly kaolinized.

The disintegration was confined to that part of the pile below the line of immersion, and was spotted in a most irregular manner over the entire wharf structure. The conclusion was inescapable that it was due to chemical reactions caused by the sea water; the variation in the behavior of the piles could readily be attributed to variations in the percentage of deleterious feldspar in the aggregate. A marked characteristic of this disintegration was swelling in volume of the concrete affected. The formation of sulfo-aluminates (alums) with their great capacity for increase in volume by taking up water of crystallization has been suggested as the primary cause of this type of breakdown. No matter what the theory, whether due to replacement of calcium by magnesia, to the formation of sulfo-aluminates, or of complex silicates from the active silica of the sand, the fact remains that exactly the same phenomenon of disintegration was simulated in the laboratory by A. A. M. Russell, Assoc. M. Am. Soc. C. E., using the suspected sand mixed with local cement and

immersion in San Francisco Bay water. The breakdown of the mortar was similar to what occurred in the Long Beach piles exposed to sea water.

The writer questions the author's opinion that disintegration is primarily the result of physical and mechanical forces rather than chemical, but he agrees that most breakdowns occurring in less than 25-yr exposure may be attributed to two kinds of defective concrete:

(*a*) The use of mixes having a deficiency of cement; and
(*b*) The placing of mixes in excessive water (not merely wet mixes but concrete drowned by leaky forms or by an accumulation of water in tight forms).

Both are probably phases of the same fundamental defect—concrete in which the water-cement ratio is so high that water voids or loose crystallization admits the sea water into ready contact with the cement paste.

After studying the breakdown occurring in the piles at Long Beach and also experiments made by Mr. Russell on a large number of puzzolan admixtures which showed that many substances containing active silica or alumina produce unsound briquets, the writer is convinced that a number of failures previously attributed to improperly proportioned or placed concrete have been accelerated, if not primarily caused, by aggregates which yield alumina and silica capable of reacting with the substances in the set cement paste and with the sea water; and that the superior behavior of some successful structures is due in large measure to the inert nature of the aggregate. In this type of failure, the expansion resulting from internal chemical activity seems, progressively, to open up the structure of even superior concrete to the sea-water salts.

In regard to the other ailment which afflicts concrete in sea water—the corrosion of embedded steel—experience has revealed one optimistic fact. Below the level of immersion, corrosion appears to be completely inhibited and the embedded steel appears to be quite as permanent as wood below the ground-water level. In tidal waters this immunity extends to about the mean high-tide line.

Above the high-tide line a very different condition prevails, and when the surface is subjected to intermittent splashing and drying out, superior concrete in thicknesses ordinarily specified in reinforced concrete practice will not protect against cracking and spalling from expansion of the corroded steel. Greater depths of cover may prevent this type of damage because there is a depth at which the shearing resistance of the concrete cover exceeds the pressure of the corroding materials; but the absence of cracking or spalling is no assurance that corrosion is not in process. Where small bars are used, they will frequently rust away entirely without showing surface indication. Furthermore, corrosion is sometimes localized, the same rod rusting entirely away at one point and having full section not more than a few feet away. The writer never has been able to satisfy himself that if the corrosion of a 1-in. rod cracks 1 in. of cover in from ten to fifteen years, it may not crack 2 in. to 3 in. in thirty to forty years or it may even rust off entirely. Here again is a place for field investigation to determine the relation between depth of cover, impermeability, and rate of corrosion. Perhaps the investigators will conclude, as the writer has, that it

is easier to seal the concrete off from sea-water moisture than to obtain a convincing answer.

In the "Summary" Mr. Hadley makes a significant statement which rings responsively in the writer's mind:

> "The harm resulting from misunderstanding a natural phenomenon arises from the likelihood that measures adopted to effect changes and alterations in it will be misdirected."

There are few branches of engineering construction in which methods are more divergent and in which those engaged in the work more individually opinionated than in marine concrete work. Under the guise of producing resistant concrete, engineers resort frequently to the most extreme measures in proportioning, in the use of water, in surface treatments, in manipulation, and in the use of admixtures—measures which are frequently so onerous that they defeat their purpose by diverting attention from simple but necessary requirements. Much of this effort must be misdirected because the writer has rarely seen more than one or two of these extreme requirements applied simultaneously by any one engineering organization.

In addition, many practices are contradictory. Thus, in one case there is the exhibition of two groups of eminent engineers charged with the same problem of constructing bridge piers in harbor waters—the one group, adopting a special cement with a specially low alumina content, and the other prescribing a special puzzolan cement in which an activated alumina silicate is added to the normal alumina content; in another instance, the paradoxical practice of two large harbor organizations in which one protects the concrete under water to the extent of impregnating it under pressure with asphalt but ignores the surfaces above water, whereas the other carefully seals the above-water surfaces but exposes the under-water surfaces without protection to the ocean sea waters.

The best proof of the ability of ordinary, good concrete and standard portland cement to withstand the usual sea-water exposures is the fact that so many structures have given a good account of themselves in spite of extraordinary procedures resorted to in the name of producing resistant concrete. The writer agrees that it is highly desirable to clarify the concrete technologist's viewpoint and attitude when he comes down to the sea; but he believes that the way to do it is to probe into existing structures and discover what changes in structure and chemical content have occurred. Only by knowing what has happened in the first fifty years can one judge what may be impending in the next fifty or hundred.

J. W. B. Blackman,[28] M. Am. Soc. C. E. (by letter).[28a]—Referring to Mr. Hadley's paper, it is noted that observations on marine structures are confined to the Pacific Coast. It is further noted that the paper deals only with disintegration of portland cement structures from the action of sulfate of magnesium. This very much limits the discussion.

[28] Cons. Engr., Long Beach, Calif.

[28a] Received by the Secretary May 13, 1941.

It must be stated that there are very few portland cement concrete structures north of San Francisco. Since the seaports of the Pacific Northwest—in Washington, Oregon, British Columbia, etc.—are in a timber country, creosoted piling is used extensively for construction of their marine works. The San Francisco Bay area and the Los Angeles and San Diego harbors are where the greatest number of concrete structures exist.

Portland cement concrete, when exposed to sea water and sea air, is subjected to many causes of deterioration and disintegration other than the action of sulfate of magnesium (which is generally considered as secondary by engineers engaged on the construction of marine work). Mr. Hadley has mentioned the many other attacks of various kinds.

It may be well to summarize briefly some of the various causes of action that the writer has encountered. In driving pre-cast concrete piles, many hazards are met before the pile reaches its final position. Assume that the concrete mixture has been carefully designed, the mineral aggregates tested, the concrete compacted by careful vibration, a minimum quantity of water used, constant inspection by experienced men, and not less than 3 in. of cover for the steel. Then, with a perfectly impervious and well-protected pile cast ready for transportation, there are yet many hazards to guard against. It must be lifted and carefully handled at the point of driving. It must be placed in the pile-driver leads and driven with a hammer whose weight is proportioned to the size and weight of the pile. The amount of driving that the pile is to be subjected to is to be governed generally by the experience of the engineer and the type of soil it penetrates. Where piles are held entirely by friction, some engineers prefer to jet most of the way down, only driving the last few feet, whereas others prefer to drive the entire distance, if possible.

There is a difference of opinion between some engineers as to the amount of driving that a pile should receive. It is not necessary to elaborate in detail on Mr. Hadley's paper with regard to the many types of failure to which concrete is subjected, but briefly they appear to be as follows: Poor workmanship, faulty mineral aggregates, poorly designed mixtures, too much water, insufficient cover of reinforcement, cracks due to shrinkage, handling, and driving. Porosity is one of the greatest causes of failure. It allows the moisture to gain access to the steel reinforcement, causing ready oxidation and steel expansion which, in turn, spalls the concrete. Many writers have demonstrated that there are two distinct causes of failure—one due to mechanical action and the other due to chemical decomposition.

Porous concrete can be disintegrated by forces exerted by the crystallization of salts in its pores. This condition is accelerated by the constant wetting and drying of concrete exposed to the wave action of sea water. The continuation of this process creates surface cracks, which are caused by this building up of crystals and concentration of the salts, and will in time cause deterioration of the concrete both by mechanical and chemical action. It is apparent, therefore, that all concrete exposed to sea water and sea air must be as impermeable to water as it is possible to make it.

The writer was asked to inspect a wharf built some few years ago in Long Beach (Calif.) Harbor. A large number of the concrete piles were in a serious

stage of disintegration. The concrete in which disintegration showed was swollen and disrupted, and this condition appeared to be more acute on the upper side of the batter piles. Repairs to the structure cost about $250,000. In the opinion of some engineers the disintegration was due to feldspar in the small aggregates. In the opinion of others the damage was due to other causes. It is interesting to note that a large number of the piles adjacent to those disintegrated showed no apparent decomposition. Is it to be assumed that the cause of the disintegration was due to faulty workmanship? For instance, some of the piles may have been compacted carelessly, there may have been too much water, and the porosity may have been much greater than that of the adjacent piles that were not injured. This raises a question that is difficult to answer.

The Portland Cement Association has, with wisdom, recommended[29] as follows: "For unusually severe exposures, as in sea water, etc., a clear cover of 3″ is recommended at least for the portion of the pile where such exposure is encountered." Some cement companies have placed on the market brands of cement such as "sulfate resisting cement," "high-silicate cement," etc. The chemists of cement companies have been endeavoring to find a remedy for attacks on cement concrete exposed to sea water and sea air.

Concrete in the soffits of slabs and beams is just as vulnerable as other parts of concrete structures to disintegration or deterioration from the various causes mentioned. Hence, various methods of protection have been tried, some of which have proved successful.

Reference is made by Mr. Hadley to the small ferry wharf at North Vancouver, B. C., Canada. The writer knows this structure well and has inspected it. He feels that where such examples are cited, for comparison, the following additional information should be supplied:

(1) The structure is built on columns (not piles). Had piles been used instead of columns, they would have been subjected to the stresses of handling and driving, which in many cases creates cracks. This in time readily admits sea air and salt water.

(2) The salinity of the Burrard Inlet (North Vancouver) is much less than that of the open sea because large bodies of fresh water are discharging all the year round from the creeks upstream and downstream from the ferry wharf.

Mr. Hadley's statement that the protective measures of asphalt impregnation of concrete piles, as used in Los Angeles Harbor, are thoroughly effective as to impermeability and density is interesting. A recent inspection of the same type of piles in Richmond Harbor, Calif., and San Diego Bay, shows that these piles are in excellent condition. Some of the piles in Los Angeles Harbor have been in use for twenty years.

Mr. Hadley should be thanked for bringing this interesting matter to the attention of engineers.

[29] "Concrete Piles," Portland Cement Assn., November, 1939, p. 28.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

RIGID FRAMES WITHOUT DIAGONALS (THE VIERENDEEL TRUSS)

Discussion

BY MESSRS. JAROSLAV J. POLIVKA, AND W. A. MILLER

JAROSLAV J. POLIVKA,[18] M. AM. SOC. C. E. (by letter).[18a]—In the "Synopsis" of his paper Professor Baes states that the points of contraflexure in each vertical of a Vierendeel truss, with chords of equal moments of inertia, are at the midheights of the verticals. This is true only for open-web trusses with inclined chords when the "reduced" moments of inertia of the chords in each panel are equal, as mentioned later in the paper. In other words, when the elastic weights of the chords in each panel are equal:

$$G_u = \frac{l_u}{E\,I_u} = G_l = \frac{l_l}{E\,I} \qquad (7)$$

In such a case the reactions due to a vertical displacement (Δ, Fig. 11) of the panel intersect at the midheight of the vertical member. (In Fig. 11, C_v, C_u, and C_l are the points of contraflexure in vertical, upper chord, and lower chord members, respectively.)

The occurrence of equal "reduced" moments of inertia in the chords and verticals is quite exceptional and will not be true in most cases encountered in practice. However, except for this restriction, the assumption of fixed points of contraflexure in the verticals will shorten the computations considerably in many cases.

Eq. 5*a* does not involve the influence of shear and direct stress on deformation. This equation can be derived in a general form by introducing the elastic weights of the structural members, and their relationships with the deformations.[19] This further step enables the designer to analyze open-web trusses, composed of members of any shape, and to consider the effect of direct stress and shear.

NOTE.—This paper by Louis Baes, Esq., was published in January, 1941, *Proceedings*.

[18] Research Associate, Univ. of California, Berkeley, Calif.

[18a] Received by the Secretary May 16, 1941.

[19] "Graphical Analysis of Framed Structures on the Basis of Ellipse of Elasticity," by Jaroslav Polivka, Soc. of Eng. Students, Inst. of Technology, Prague, 1919; also, "Graphical Methods of Analyzing Statically Indeterminate Structures," mimeographed lectures by Jaroslav Polivka, Berkeley, Calif., 1940 and 1941.

The general equation is

$$- U'_n G_{v,n} r^2_{v,n} + U'_n G_n r^2_n - U'_{n+1} G_{v,n+1} r^2_{v,n+1} = \Delta_n \ldots\ldots (8)$$

in which $G_{v,n}$ and $G_{v,n+1}$ are the elastic weights of the vertical members at the joints n and $n + 1$ (Fig. 10) and $r_{v,n}$ and $r_{v,n+1}$ are the respective semi-axes of ellipse of elasticity of these verticals. Furthermore, G_n is the elastic weight of the panel n consisting of two verticals and two chords, considered as being isolated from the remainder of the truss; and Δ_n is the displacement due to the external bending moment of the panel n. Eq. 8 expresses the equality of displacements due to internal and external forces. For constant moments of inertia, and for rectangular panels (and neglecting the influence of direct stress and shear on deformation) the values of Eq. 8 become:

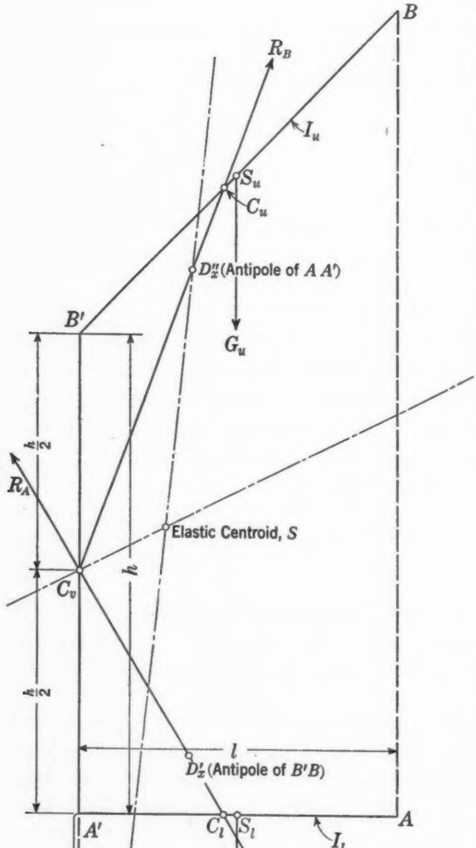

FIG. 11

$$G_{v,n} = G_{v,n+1} = \frac{h}{E\,I_v} \ldots (9a)$$

$$r^2_{v,n} = r^2_{v,n+1} = \frac{h^2}{12} \ldots (9b)$$

$$G_n = \frac{2}{E}\left(\frac{h}{I_v} + \frac{l}{I}\right) \ldots (9c)$$

and (since $I_{v,n} = I_{l,n} = I$):

$$G_n\, r^2_n = 2\left[\frac{h^3}{12\,E\,I_v} + \frac{l}{E\,I}\left(\frac{h}{2}\right)^2\right]. \quad (9d)$$

Eq. 5a is obtained by substituting Eqs. 9 in the general formula, Eq. 8. This general formula can be used for trusses composed of members with variable moments of inertia such as those that result, for example, when the members are connected by brackets or fillets. This is the most frequent case in practice. Elastic constants for several regular shapes of haunched members are compiled in standard tables. From the values u, v, p, and q tabulated by Walter Ruppel,[20] Assoc. M. Am. Soc. C. E., the constants of the

[20] Discussion by Walter Ruppel of "Moments in Restrained and Continuous Beams by the Method of Conjugate Points," by L. H. Nishkian and D. B. Steinman, Members, Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 90 (1927), pp. 167–187.

ellipse of elasticity can be computed by the following relationships:[21]

$$G = (p + q)\, G_B \qquad (10a)$$

$$r^2 = s\,(d - s) \qquad (10b)$$

$$s = \frac{p}{p + q}\, l \qquad (10c)$$

and

$$d = (1 - u)\, l \qquad (10d)$$

The relative displacement (Δ) of the end faces of a structural member, perpendicular to its axis, is caused by a force R passing through the elastic centroid S and having the magnitude

$$R = \frac{\Delta}{G\, r^2} \qquad (11)$$

A general equation similar to Eq. 8 can be derived for panels having members with different moments of inertia.

When the members are relatively slender and the effect of bending is predominant, the points of contraflexure in all members may be obtained by photoelastic investigation and the truss may be considered as statically determinate, having imaginary hinges in all members.[22] The check of the static solution consists in determining the pattern (and even the magnitude, if necessary) of the internal forces by photoelastic investigation.[23] The photoelastic method, however, becomes more complicated whenever the truss consists of deep members. In this case the effect of shear and of stress concentrations, which the fillets cause near the joints, must be considered. The points of contraflexure in the chords approach the joints and in some cases disappear (Fig. 12).

In 1933, and again in 1937 and 1938, the writer applied the foregoing principles in the design of a bridge over the Nusle Valley in Prague, Czechoslovakia,[24] composed of continuous spans of open-web trusses. The results agreed closely with those obtained by means of photoelastic tests. This structure had seven spans, five of which were 246 ft long. The typical 246-ft span was divided into twelve panels of 20.5 ft each, the typical panel opening being 12.31 ft by 7.37 ft. A lower deck was provided for the subway. It was first studied by means of a series of photoelastic tests to determine the most favorable shape for the fillets. Curved fillets gave the best results.

For demonstration purposes, consider the open-web, Vierendeel truss, with six panels, shown in Fig. 13. First determine the characteristic points of each member and the forces R_u and R_l relative to the assumed vertical displacement of the chords (see Fig. 14).[19] Then the shear T in the panel must be distributed

[21] *Proceedings*, Am. Soc. C. E., Vol. 67, April, 1941, p. 710.

[22] "Analysis of Statically Indeterminate Structures on the Basis of Photoelastic Data," by J. J. Polivka and H. D. Eberhart, Assoc. M. Am. Soc. C. E., paper presented before the Structural Engineers Assoc. of Northern California, San Francisco, August 6, 1940 (mimeographed).

[23] "Some Stress Relationships in Photoelasticity," by J. J. Polivka and H. D. Eberhart, *Proceedings*, Tenth Semi-Annual Eastern Photoelasticity Conference, Cambridge, 1939.

[24] "The Vierendeel Truss," by J. J. Polivka, *Technical Encyclopædia*, Vol. XIV, Prague, 1937, p. 730.

between the upper and lower chord according to the ratio

$$\frac{R_u}{R_l} = \frac{T_u}{T_l} = \frac{G_l \, r^2{}_l}{G_u \, r^2{}_u} \quad \text{.....................(12)}$$

in which the values G and r, of the elastically restrained members, are determined from the characteristic points as shown in Fig. 14. The elastic centroid

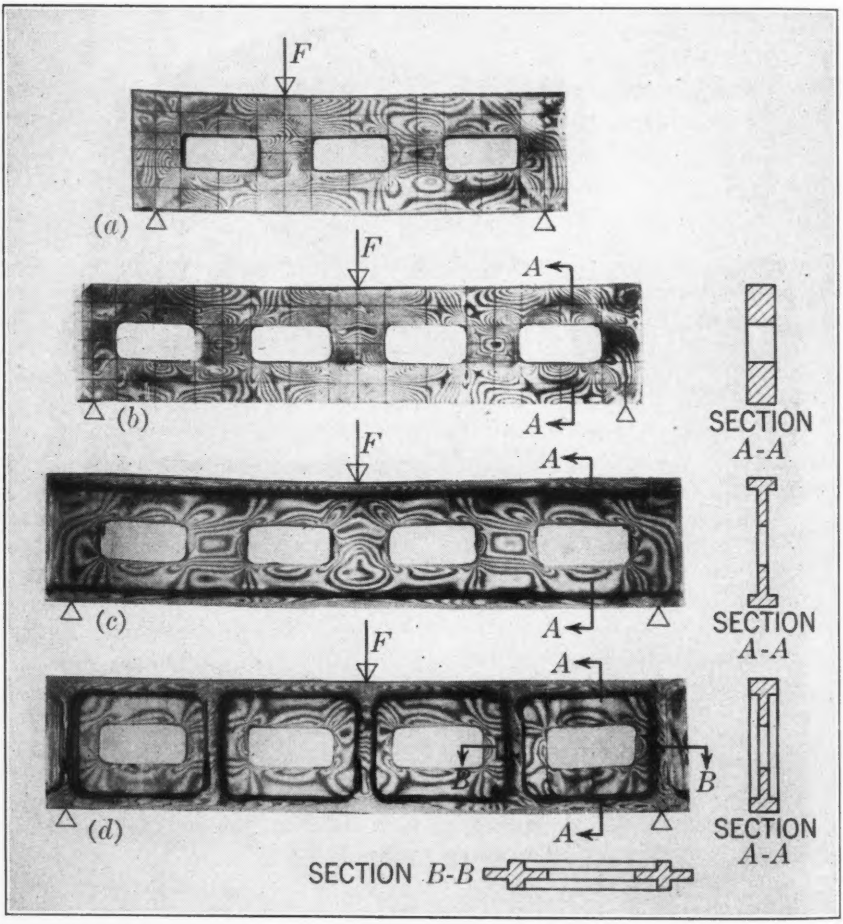

FIG. 12

S and the magnitude of the semi-axis r are found by the intersection of two semi-circles having diameters equal to $l\,(1 - v)$ and $l\,(1 - u)$. The total elastic weight of the member is

$$G = G_0 + G' + G'' \quad \text{.....................(13)}$$

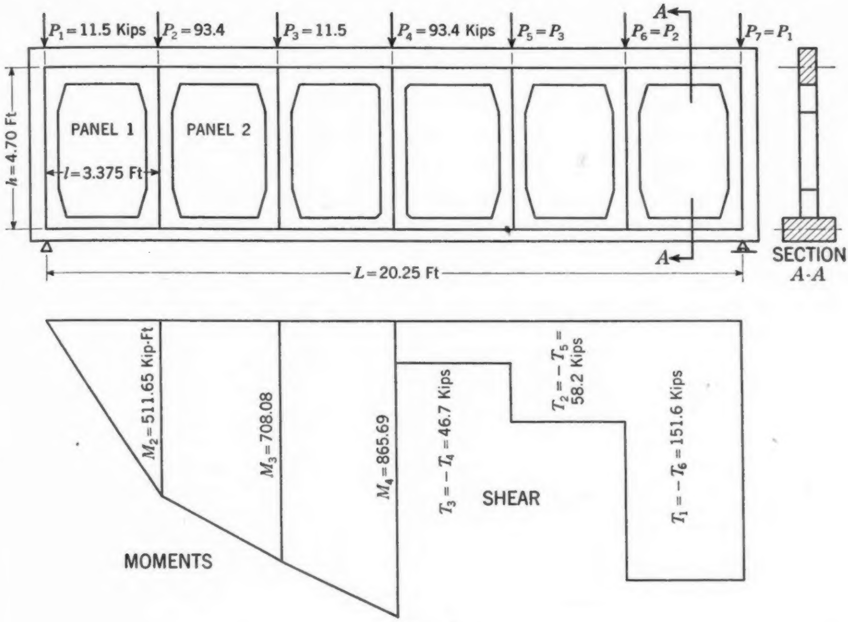
$P_1 = 11.5$ Kips
$P_2 = 93.4$
$P_3 = 11.5$
$P_4 = 93.4$ Kips
$P_5 = P_3$
$P_6 = P_2$
$P_7 = P_1$
A
PANEL 1
PANEL 2
$h = 4.70$ Ft
$l = 3.375$ Ft
$L = 20.25$ Ft
SECTION
A-A
$M_2 = 511.65$ Kip-Ft
$M_3 = 708.08$
$M_4 = 865.69$
MOMENTS
$T_3 = -T_4 = 46.7$ Kips
$T_2 = -T_5 = 58.2$ Kips
$T_1 = -T_6 = 151.6$ Kips
SHEAR

FIG. 13

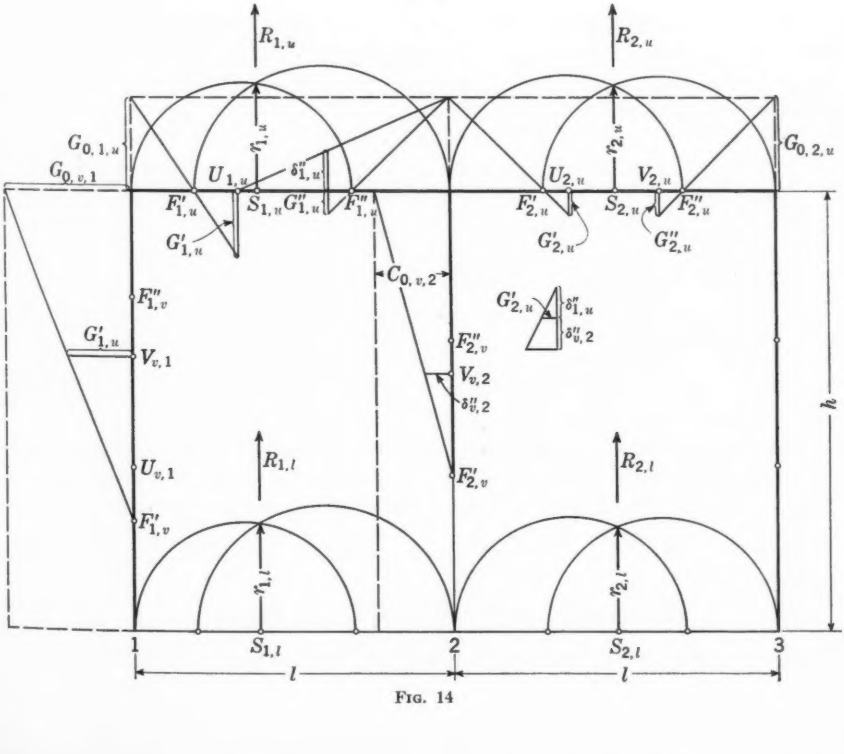
$R_{1,u}$
$R_{2,u}$
$r_{1,u}$
$r_{2,u}$
$G_{0,1,u}$
$G_{0,2,u}$
$G_{0,v,1}$
$U_{1,u}$
$\delta''_{1,u}$
$F'_{1,u}$
$S_{1,u}$
$G''_{1,u}$
$F''_{1,u}$
$G'_{1,u}$
$U_{2,u}$
$V_{2,u}$
$F'_{2,u}$
$S_{2,u}$
$F''_{2,u}$
$G'_{2,u}$
$G''_{2,u}$
$C_{0,v,2}$
$F''_{1,v}$
$G'_{1,u}$
$V_{v,1}$
$U_{v,1}$
$F'_{1,v}$
$G'_{2,u}$
$\delta''_{1,u}$
$\delta''_{v,2}$
$F''_{2,v}$
$V_{v,2}$
$\delta''_{v,2}$
$F'_{2,v}$
$R_{1,l}$
$R_{2,l}$
$r_{1,l}$
$r_{2,l}$
h
1
$S_{1,l}$
2
$S_{2,l}$
3
l
l

FIG. 14

in which G_0 is the elastic weight of the isolated structural member (for constant moment of inertia $G_0 = \frac{l}{E\,I}$), and G' and G'' are the elastic weights of the supports of the elastically restrained structural member.

The shears $R_{1,u}$ and $R_{1,l}$ of the first panel (Fig. 15(*a*)) are carried over to the second panel as the values $R_{1,2,u}$ and $R_{1,2,l}$ (see Fig. 15(*b*)), to the third panel as $R_{1,3,u}$ and $R_{l,1,3}$ (see Fig. 15(*c*)), and so on, these shears being computed from the moments carried over from the first panel to the second panel, then to the third, and so on. Similarly, the values: $R_{2,1,u}$, $R_{2,3,u}$, $R_{2,4,u}$, $\cdots$ etc., and $R_{2,1,l}$, $R_{2,3,l}$, $R_{2,4,l}$, $\cdots$ etc., are obtained by distributing the shears $R_{2,u}$ and $R_{2,l}$ to the other panels (see Table 1).

TABLE 1.—Shear Distribution

Panel	Shear Distributed from the Panel in Col. 1 to:					
	Panel 1	Panel 2	Panel 3	Panel 4	Panel 5	Panel 6
(1)	(2)	(3)	(4)	(5)	(6)	(7)
1	R_1	$R_{1,2}$	$R_{1,3}$	$R_{1,4}$	(vanishing)	(vanishing)
2	$R_{2,1}$	R_2	$R_{2,3}$	$R_{2,4}$	$R_{2,5}$	(vanishing)
3	$R_{3,1}$	$R_{3,2}$	R_3	$R_{3,4}$	$R_{3,5}$	$R_{3,6}$
4	$R_{4,1}$	$R_{4,2}$	$R_{4,3}$	R_4	$R_{4,5}$	$R_{4,6}$
5	(vanishing)	$R_{5,2}$	$R_{5,3}$	$R_{5,4}$	R_5	$R_{5,6}$
6	(vanishing)	(vanishing)	$R_{6,3}$	$R_{6,4}$	$R_{6,5}$	R_6

It can be seen that the values of the shear distributed to the fourth and following panels are negligible. The total shear in each panel is obtained by adding all values of the shear occurring at that panel, and those brought over from other panels, with due regard to sign.

A table similar to Table 1 must be compiled for both upper and lower chords. For symmetrical trusses (geometrically and elastically), only one half of these values are computed. The resulting bending moments in the upper chords due to the actual loading producing shears T_1, T_2, T_3, $\cdots$ in the panel 1, 2, 3, $\cdots$ are obtained by adding the expressions

$$\frac{T_1}{R_{1,u} + R_{1,l}} \left(M_{1,u} + \frac{R_{1,2,u}}{R_{2,u}} M_{2,u} + \frac{R_{1,3,u}}{R_{3,u}} M_{3,4} + \cdots \right) \ldots\ldots (14a)$$

$$\frac{T_2}{R_{2,u} + R_{2,l}} \left(\frac{R_{2,1,u}}{R_{1,u}} M_{1,u} + M_{2,u} + \frac{R_{2,3,u}}{R_{3,u}} M_{3,4} + \cdots \right) \ldots\ldots (14b)$$

etc.

Similar expressions are obtained for the lower chords. The addition of the six expressions produces two final expressions (for the upper and lower chords) that are simultaneously valid for all panels. Their general forms are:

$$M_{n,u\,(\text{total})} = a_u\, M_{1,u} + b_u\, M_{2,u} + c_u\, M_{3,u} + \cdots \ldots\ldots (15a)$$

and

$$M_{n,l\,(\text{total})} = a_l\, M_{1,l} + b_l\, M_{2,l} + c_l\, M_{3,l} + \cdots \ldots\ldots (15b)$$

The values $M_{n,u}$ and $M_{n,l}$ are moments at the joints due to the imaginary

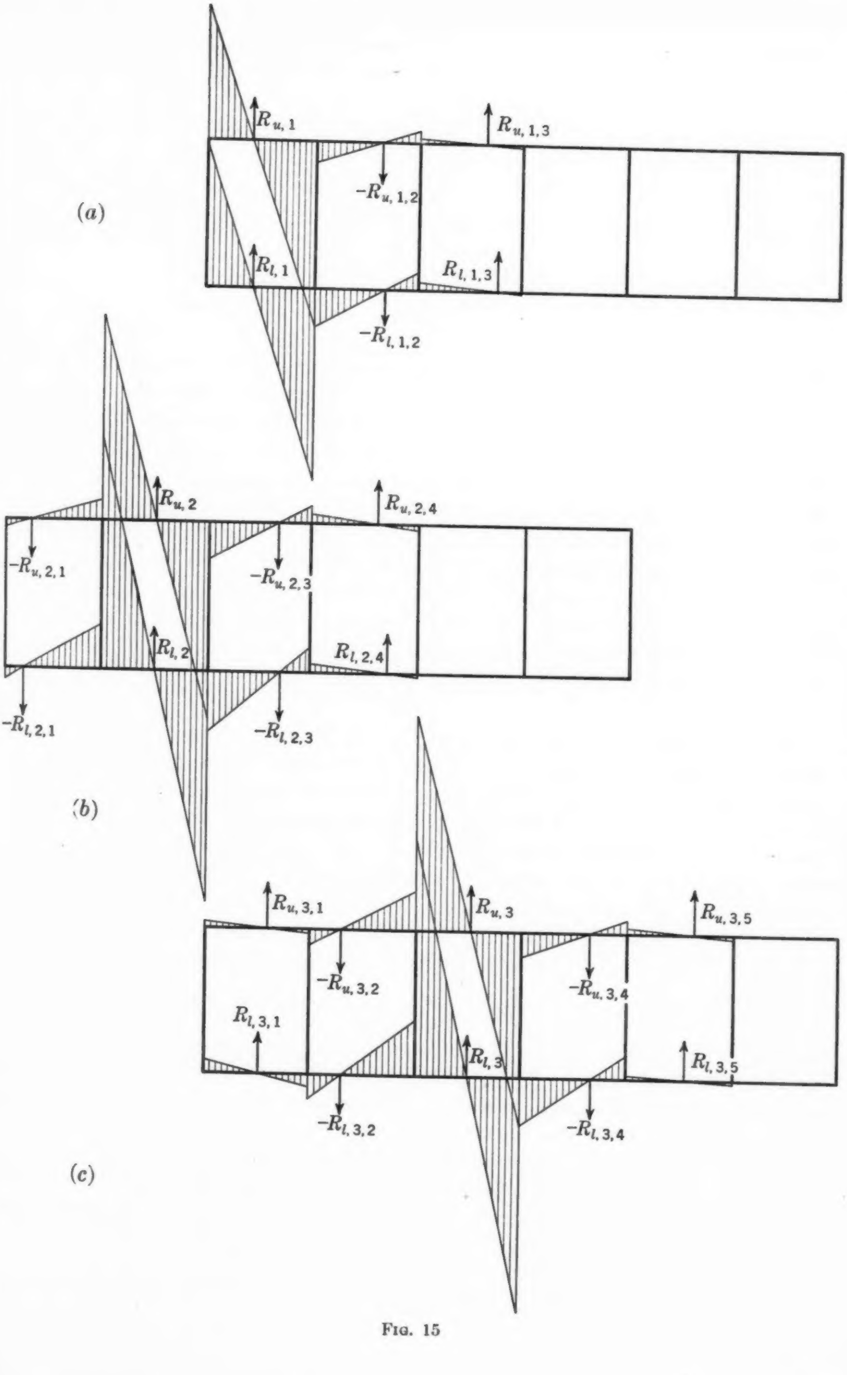
(a)
$R_{u,1}$
$R_{u,1,3}$
$-R_{u,1,2}$
$R_{l,1}$
$R_{l,1,3}$
$-R_{l,1,2}$
$R_{u,2}$
$R_{u,2,4}$
$-R_{u,2,1}$
$-R_{u,2,3}$
$R_{l,2}$
$R_{l,2,4}$
$-R_{l,2,1}$
$-R_{l,2,3}$
(b)
$R_{u,3,1}$
$R_{u,3}$
$R_{u,3,5}$
$-R_{u,3,2}$
$-R_{u,3,4}$
$R_{l,3,1}$
$R_{l,3}$
$R_{l,3,5}$
$-R_{l,3,2}$
$-R_{l,3,4}$
(c)

FIG. 15

forces R_n (Eq. 11), assuming a constant displacement Δ for all chords (for example, $\Delta = 1$).

The moments in the verticals at the joints are then

$$M_{v,n,u} = -(M''_{n-1,u} + M'_{n,u}) \quad \text{(16a)}$$

and

$$M_{v,n} = -(M''_{n-1,l} + M'_{n,l}) \quad \text{(16b)}$$

The horizontal component of the thrust in the vertical is

$$U_n = \frac{M_{v,n,u} + M_{v,n,l}}{h_n} \quad \text{(17)}$$

Usually the loading is nearly symmetrical and the components U_n must satisfy the condition $\Sigma U_n = 0$. For unsymmetrical truss and loading it will be found that $U_n = \Delta U$. In this case $-\Delta U$ must be applied as an external force and its effect combined with the moments due directly to the loading.

If a slight error is permissible, it is not necessary to distribute the shear to the neighboring panels, and the analysis becomes relatively simple. The error decreases rapidly with the number of panels, being less than 5% for the usual type of structure. The adaptation of the method as shown herein makes it possible to analyze highly complicated grid framings of the Vierendeel type.

Conclusion.—The suggestions offered herein extend Professor Baes' analysis to a general form that can be applied to any size, shape, and inclination of truss members and to multiple tiers or stories of panels in the truss, such as may be used for the special design of building frames.

W. A. Miller,[25] Assoc. M. Am. Soc. C. E. (by letter).[25a]—Experiments along lines somewhat similar to those of Professor Baes, conducted in the Department of Civil Engineering, University of Sydney, in Sydney, New South Wales, Australia, by J. I. Miller,[26] verify the position of the point of contraflexure in the verticals of the Vierendeel truss and substantiate some of the other conclusions of the paper. Using steel spline models and macroscopic deformations, Mr. Miller considered cases with parallel as well as non-parallel chords. The moment of inertia of the verticals was taken about one third that of the upper chord, and that of the lower chord was taken to vary from "equal to" to eight times that of the upper chord. For the range of $\frac{I_u}{I_l}$ from 1 to 4, it was found that the departure of the point of contraflexure in the verticals from midpoint did not exceed 1% of their height. Further examination of frames in sheet celluloid, using microscopic deformations with the aid of a standard deformeter gage, supported the findings using the spline models.

As the main object of the investigation undertaken by Mr. Miller was to develop an approximate and dependable algebraic analysis, applicable to the Vierendeel type of truss likely to be encountered in tower and bridge frames,

[25] Prof., Civ. Engr., Univ. of Sydney, Sydney, New South Wales, Australia.

[25a] Received by the Secretary May 21, 1941.

[26] From a thesis presented to the University of Sydney, in 1939, in partial fulfilment of the requirements for the degree of Bachelor of Engineering in Civil Engineering.

that analysis was continued, based on the assumptions that the chords were equal (or nearly so) and that the points of contraflexure of the connecting web members occurred at their midpoints.

Using the U-axis (as did the author) and a Z-axis on any vertical in the panel, the hyperstatic unknowns were applied at the intersection and equations in U, W, and Z derived. It was found finally that the selection of the Z-axis to pass through the elastic center of the panel considerably simplified the form of the equations. For the case in which the author's "modified moments of inertia" are equal, these equations reduce to

$$W_n = \frac{1}{2} M^0{}_n \qquad (18)$$

$$Z_n = \frac{1}{2} V^0{}_n \qquad (19)$$

and

$$- h^3{}_{n-1} U'{}_{n-1} + \left[h^3{}_{n-1} + 2 \frac{I_v}{I_n} (h^2{}_{n-1} + h_{n-1} h_n + h^2{}_n) \right] U'{}_n - h^3{}_{n+1} U'{}_{n+1}$$
$$= 3 l (h_{n-1} + h_n) M^0{}_n \qquad (20)$$

The symbols are those used by the author, except $V^0{}_n$ and $M^0{}_n$, which are the simple beam shear and moment, respectively, on the vertical through the elastic center. Eq. 20 reduces to Eq. 5*a* for a truss with parallel chords.

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DISCUSSIONS

VALUE OF PUBLIC WORKS

Discussion

By Baxter L. Brown, M. Am. Soc. C. E.

Baxter L. Brown,[14] M. Am. Soc. C. E. (by letter).[14a]—Major Hallihan's paper may be termed prophetic for the reason that at the time he prepared it (early in 1940) there did not exist the movement for the tremendous expenditures for national defense that now (June, 1941) is in progress.

The demand for public work will be enormous on account of the large number of men who will be out of employment after the defense program is concluded. Therefore, it is incumbent not only for the government to prepare for federal work, but for the states, counties, and municipalities to do likewise in preparing for non-federal work. There will be a still greater demand for public work when the army is demobilized. Looking to the future, under these conditions, the writer's judgment is that plans should be prepared for work of a non-defense character, which can be postponed until the necessity arises. Naturally, there will be some work that will have to be undertaken as emergency work.

An interesting item that can be used to illustrate this proposition is the project to beautify Memorial Plaza in St. Louis, Mo. The money for this work is available, but it would be of no value in the defense program and, therefore, should be deferred until a later date. The City of St. Louis also has some betterments and enlargements of city-owned hospitals and institutions that should be undertaken with a view to the possible necessity of taking care of sick or injured should the United States enter the war.

On account of prosperity brought about by expenditures for national defense, it probably will be much easier to vote bonds for public works than it has been in the past few years. For instance, in St. Louis, bonds were voted in 1920 and 1923 by much more than the required two-thirds vote.

Note.—This paper by J. P. Hallihan, M. Am. Soc. C. E., was published in February, 1941, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: April, 1941, by Clarence W. Post, M. Am. Soc. C. E.; and May, 1941, by Messrs. Uel Stephens, William J. Wilgus, Bernard L. Weiner, Albert Ed. Scheible, H. B. Cooley, and Philip W. Henry.

[14] St. Louis, Mo.

[14a] Received by the Secretary May 19, 1941.

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DISCUSSIONS

HYDRAULICS OF SPRINKLING SYSTEMS FOR IRRIGATION

Discussion

BY MESSRS. TOM A. BITHER, RALPH W. POWELL, AND HARRY F. BLANEY

TOM A. BITHER,[13] ASSOC. M. AM. SOC. C. E. (by letter).[13a]—This excellent paper deals with a type of irrigation that is becoming more and more extensively adopted for the irrigation of row crops, pastures, and orchards, resulting in a saving in water and more uniform distribution, as well as increased yields. It has been the good fortune of the writer to be located near Davis and the Sacramento Valley where most of Mr. Christiansen's experimental work and field tests have been conducted. The paper does not adequately indicate to the reader the vast amount of experimental work that Mr. Christiansen has done during the eight years since 1933; nor is the author able, in the space available, to cover in full the many tests on sprinklers and sprinkler lines that he has conducted during that period.

Some sprinkler lines have been sold without any consideration of the fundamental principles of design. In some cases, for example, claims have been made that a higher pressure would be obtained at the outer (distal) end of the line than at the inlet end, even though the line was to operate on level ground. It is to be hoped that with the publication of Mr. Christiansen's data and design formulas this practice can be stopped.

In regions where natural rainfall alone is relied upon for crop production, droughts frequently occur. Supplementary irrigation during dry periods will greatly increase crop yields and often will save valuable crops and prevent serious injury to orchards. The average seasonal rainfall is usually more than enough to grow good crops, but the rains do not always come at the right times, with the result that crops suffer if not protected by some type of supplemental irrigation. Any farmer should be able to increase production with the aid of irrigation since he can then assure himself of always having sufficient moisture

NOTE.—This paper by J. E. Christiansen, Assoc. M. Am. Soc. C. E., was published in January, 1941, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: April, 1941, by Arthur F. Pillsbury, Assoc. M. Am. Soc. C. E.

[13] With California Corrugated Culvert Co., Berkeley, Calif.

[13a] Received by the Secretary April 29, 1941.

available when needed. The overhead or sprinkler system of supplementary irrigation has been found to be the most suitable method for many crops and conditions. By this method, water may be applied efficiently and economically since it enables the irrigator to apply a uniform depth of water over his land regardless of topography, and without waste. This method requires less water than surface irrigation methods. Canals and ditches are eliminated, thereby doing away with seepage losses.

On uneven or rolling ground, surface methods are unsatisfactory since they result in heavy applications and water logging in the low spots, and light applications or no water at all on the high spots. A considerable quantity of water is also lost by deep percolation at the upper ends of checks and furrows where surface methods are used. The cost of preparing land for satisfactory surface irrigation is an important consideration. Extensive leveling and grading not only cost money but also have the serious objection of removing the surface topsoil from the high spots and leaving a less fertile subsoil. Where the soil is shallow, the high spots may be completely ruined by deep grading.

Expensive leveling of land is entirely eliminated where sprinkler irrigation is used to apply water. This method applies water in the form of rain and is applicable to all types of soil and to any condition of topography. It permits the utilization of a small head of water that would be entirely inadequate for surface irrigation methods. It insures a uniform distribution of water; permits keeping the soil in the best condition at all times; and, by applying water at the proper rate, prevents serious soil erosion.

Mr. Christiansen should have explained that the sprinkler distribution tests (Figs. 3 to 7) were made using sprinklers that are now considered obsolete and inadequate for agricultural work. These sprinklers were of the reaction-drive type and have been almost entirely supplanted by the impulse-drive sprinkler, a type which rotates at a uniform speed when properly adjusted. These sprinklers do not produce the unfavorable patterns shown in Figs. 3 to 7. Also, as a result of Mr. Christiansen's investigations, these later types of sprinklers produce a cross-sectional pattern approximating that shown in Fig. 9, pattern *b*. Sprinkler manufacturers have also developed agricultural sprinklers operating at lower pressures than those he describes, some of them, for under-tree work, operating at pressures as low as 2 lb per sq in. with satisfactory patterns for this work.

As a result of personal observations, the writer agrees with Mr. Christiansen that the loss due to evaporation in the spray between the nozzle and the ground is negligible. Also, the total losses incurred from the sprinkler method of irrigation are less than that from any surface method of application since water is lost neither by deep percolation at the upper ends of furrows and checks, nor from seepage from ditches.

The writer is of the opinion, as the result of field tests on operating lines, that the pressure losses determined from the logarithmic chart (Fig. 11) and from Eq. 11 are too high. This is undoubtedly due to the assumption that all sprinklers are discharging the same quantity of water, which is not the case, as indicated by Fig. 12. Although a logarithmic chart of the form of Fig. 11 is undoubtedly convenient and useful for determining pressure losses, it has been

found, in the preparation of handbooks and manuals for field men, that tables are more convenient than charts. As a consequence, the writer has computed and compiled tables for various sizes of pipe and of sprinklers placed at several spacings on the line. These tables which give pressures, total line discharge, and equivalent rainfall enable the field man to determine quickly the size of pipe as well as the size and spacing of sprinklers for any particular installation, depending upon the size of the field to be irrigated, the size of the power unit or pump, the amount of water available, the type of soil, or other governing factor.

RALPH W. POWELL,[14] M. AM. SOC. C. E. (by letter).[14a]—The writer wishes to comment on only one section of this interesting paper—namely, that on the hydraulics of sprinkler lines, and principally on only one phase of it, which is the recovery of pressure head in the main line of pipe when some of the flow is drawn off through a side outlet. This has been discussed in the technical press, but as Wallace M. Lansford,[15] Assoc. M. Am. Soc. C. E., has pointed out, no "complete experimental and analytical solution * * * has yet been published." Probably the most thorough experimental study of the question was made by the Bureau of Reclamation at the Fort Collins (Colo.) laboratory in 1932; but their report[16] deals primarily with the energy loss, and the only data explicitly given regarding pressure recovery seem to be those applying to points opposite the exact center of the junctions, where the recovery might be expected to be only half completed, or perhaps less than that.

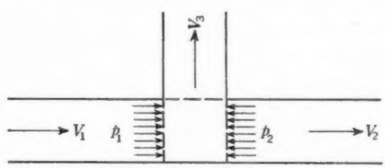

FIG. 13

For many years the writer has been puzzled by the result arising from a momentum treatment of this problem. In Fig. 13, if the unbalanced force acting in the liquid within the dashed lines is made equal to the change in momentum per second in the usual manner,

$$p_2 A - p_1 A = \frac{w\, Q_1\, v_1}{g} - \frac{w\, Q_2\, v_2}{g} \dots\dots\dots\dots\dots (18)$$

which reduces to

$$p_2 - p_1 = \frac{w\, v_1^2}{g}\, (2\, r_q - r_q^2) \dots\dots\dots\dots\dots (19)$$

In Eq. 19, $p_2 - p_1$ is just twice the value given by Eq. 5. The frictional drag around the periphery of the free body has been neglected, but an approximate estimate shows that it would be negligible, and at any rate it is in the wrong direction to explain the discrepancy. One may first be tempted to state that there is a question as to whether Bernoulli's theorem applies between two points at which the quantity of flow is different, and that, since momentum

[14] Associate Prof. of Mechanics, Ohio State Univ., Columbus, Ohio.

[14a] Received by the Secretary May 14, 1941.

[15] *Engineering News-Record,* April 10, 1941, p. 522. For previous discussions of the same subject, see *Engineering News-Record,* November 21, 1940, p. 697, and January 16, 1941, p. 78.

[16] "Model Studies of Penstocks and Outlet Works," *Bulletin No. 2,* Boulder Project Final Repts., U. S. Dept. of the Interior, 1938, especially pp. 49–64. (The reference, on p. 62, to a paper by the writer is unfortunate, as the paper was never published.)

considerations give an exact solution without the loss term which is required in the energy equation, Eq. 19 is more likely to be right than Eq. 5. However, when he investigates the reported experiments, he finds that the actual recovery is less than that given by Eq. 5, rather than more.

The explanation seems to lie in the unequal velocity distribution in the main pipe just downstream from the junction. Following the usual notation,[17] the momentum equation should have been written

$$p_2 A - p_1 A = \frac{w\, Q_1\, \beta_1\, v_1}{g} - \frac{w\, Q_2\, \beta_2\, v_2}{g} \quad \text{(20)}$$

which leads to

$$p_2 - p_1 = \frac{w\, v_1^2}{g}\left[\beta_1 - \beta_2\,(1 - r_q)^2\right] \quad \text{(21)}$$

Then, calling the ratio of recovered pressure head to original velocity head K_2:

$$K_2 = \frac{(p_2 - p_1)\, 2\, g}{w\, v_1^2} = 2\, \beta_1 - 2\, \beta_2\,(1 - r_q)^2 \quad \text{(22)}$$

Table 2 gives the average values of K_2 as reported by Professor Oakey.[5] As the author states, these agree with Eq. 5 better than with Eq. 6. Table 2

TABLE 2.—VALUES OF K_2 AND β_2

Description	PROPORTION OF WATER LEAVING SIDE OUTLET $r_q = Q_3/Q_1$										
	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
Empirical Coefficient K_2:											
John A. Oakey	0.00	0.21	0.38	0.53	0.66	0.74	0.80	0.83	0.82	0.78	0.69
Reclamation Service	0.02	0.21	0.38	0.53	0.66	0.76	0.84	0.85	0.80	0.70	0.56
Coefficient β_2	1.01	1.13	1.30	1.54	1.92	2.56	3.75	6.61	15.5	67.0	∞

also gives the values of K_2 deduced from material published by the Reclamation Service.[18] The agreement between the two sources is very satisfactory. The measurements of the Reclamation Bureau were on a much larger scale and are probably more accurate. The last line gives the values of β_2 necessary to make the Reclamation Bureau values of K_2 satisfy Eq. 22, β_1 being taken as 1.02.

The values of β_2 in Table 2 may seem impossibly high, but it must be realized that, when a large part of the flow is diverted to a side outlet, the remaining flow will be distributed very unequally across the pipe, possibly even to the extent of a reversal of flow on the side toward the outlet. If the velocity averaged zero over half the cross section and $2\,v$ over the other half, β would exceed 2.

It would seem to the writer that in this problem there is no hope of deducing how much head will be recovered and how much lost, but that one must depend on experiments.

[17] "Mechanics of Liquids," by Ralph W. Powell, The Macmillan Co., New York, N. Y., 1940, p. 125. (One w is printed erroneously there as W.)

[5] "Hydraulic Losses in Short Tubes Determined by Experiments," by John A. Oakey, *Engineering News-Record*, June 1, 1933, pp. 717–718.

[18] "Model Studies of Penstocks and Outlet Works," *Bulletin No. 2*, Boulder Project Final Repts., U. S. Dept. of the Interior, 1938, Fig. 30 and the first equation on p. 39 (after supplying a missing parenthesis around the last three terms).

HARRY F. BLANEY,[19] M. AM. SOC. C. E. (by letter).[19a]—Sprinkling irrigation has been used extensively in the United States for many years. The paper on this subject by Mr. Christiansen presents some excellent data but may give some readers the impression that sprinkling systems for irrigation are comparatively new. Information obtained from the 1920 Census of Agriculture, and from other sources, indicates that more than 12,000 acres were thus irrigated in 1919. As early as 1916 the U. S. Department of Agriculture made some general studies on spray irrigation and the results were published.[20] In 1924 the University of California made a study of irrigation by overhead sprinkling [21] in Southern California. However a comparison of these reports with the author's paper will indicate the change that sprinkler equipment has undergone. Tests of typical German and American sprinkler equipment in 1928 by the U. S. Department of Agriculture at the Arlington Experiment Farm under eastern conditions are also interesting.[22]

Many agricultural crops are now being successfully irrigated by portable sprinkling systems at reasonable cost, which was not deemed feasible years ago, and Mr. Christiansen's paper is a valuable contribution to engineering literature on this subject.

The writer agrees with the author's conclusions regarding evaporation losses. In the case of citrus fruits, losses by both surface evaporation and deep penetration below the root zone are unavoidable when water is applied so that the maximum tree growth is obtained. Tests made under the direction of the writer indicate that the efficiency of irrigation under the sprinkler method may not be greater than under surface irrigation. Conditions under which irrigation by sprinkling has an economic advantage over surface irrigation are those of steep or uneven topography. A higher efficiency of irrigation can be maintained under such conditions than is possible with surface irrigation. Under sprinkler irrigation, surface runoff usually is a negligible factor if the water is properly applied.

TABLE 3.—IRRIGATION EFFICIENCIES

Grove	Water applied, acre-in. per acre	Water placed in root zone, acre-in. per acre	Percentage efficiency
A	0.62	0.18	29
B	1.48	0.62	42
B	2.36	1.47	62
C	7.50	3.50	47

The variations of efficiencies of irrigation under overhead sprinkling with light, medium, and heavy applications of water as determined from intensive soil moisture studies,[23] San Diego County, California, 1926, are shown in Table 3.

During the season of 1939 the Division of Irrigation, in cooperation with the Los Angeles Bureau of Water Works and Supply, conducted experiments

[19] Irrig. Engr., Div. of Irrig., SCS, U. S. Dept. of Agriculture, Los Angeles, Calif.

[19a] Received by the Secretary May 23, 1941.

[20] "Spray Irrigation," by Milo B. Williams, *Bulletin No. 495*, U. S. Dept. of Agriculture, 1917.

[21] "Irrigation by Overhead Sprinkling," by H. A. Wadsworth, *Circular No. 4*, California Agri. Extension Service, 1926.

[22] "Tests of Spray Irrigation Equipment," by F. E. Staebner, *Circular No. 195*, U. S. Dept. of Agriculture, 1931.

[23] "Irrigation Water Requirement Studies of Citrus and Avocado Trees in San Diego County, California," by S. H. Beckett, Harry F. Blaney, and Colin A. Taylor, Assoc. M. Am. Soc. C. E., *Bulletin No. 489*, Univ. of California, Coll. of Agriculture, 1930.

on applying irrigation water with low-head sprinklers and by furrows. The average efficiency of irrigation under the sprinkler system was 49% and with the furrow method 47%. Two types of sprinklers were used with pressures varying from 8 to 18 lb per sq in.

Corrections for *Transactions:* In January, 1941, *Proceedings*, page 114, line following Eq. 1, change "d" to "x"; in Eq. 13 change "Q^2" to "$Q_g{}^2$"; in Eq. 16*b* change "Q" to "Q_g"; and on page 125, definition for q should read "= rate of flow, in gallons per minute, through a single sprinkler."

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DISCUSSIONS

FORMULAS FOR THE TRANSPORTATION OF BED LOAD

Discussion

By Joe W. Johnson, Assoc. M. Am. Soc. C. E.

Joe W. Johnson,[14] Assoc. Am. Soc. C. E. (by letter).[14a]—Compliments are due the author on his approach to the problem of bed-load transportation, especially in his dimensionless expressions and the absence of any "critical" term whose value, in general, is difficult to determine accurately.

In connection with investigations[15,16] by the Soil Conservation Service on the transportation of sediment in natural streams, most of the published and unpublished data from the United States and Europe on the transportation of bed load in laboratory flumes have been assembled. For these various data, computations have been made and the results tabulated in a consistent system of units as follows:

(1) Discharge (cu cm per sec)
(2) Flume width (cm)
(3) Mean water depth (cm)
(4) Water-surface slope
(5) Water temperature (°C)
(6) Mean velocity (cm per sec)
(7) Bottom velocity (cm per sec)
(8) Hydraulic radius of bed (cm)
(9) Friction velocity (cm per sec)
(10) Reynolds' number
(11) Manning's n for the bed ($ft^{1/6}$)
(12) Unit discharge (cu cm per sec per cm)
(13) Rate of bed movement (g per sec per cm)
(14) Duration of run (minutes), and
(15) Bed condition.

Items 1 to 7, inclusive, and 13 to 15, inclusive, are observed factors and have merely been converted from the various units of observation to the foregoing units. The hydraulic radius of the bed (item 8) has been calculated by the author's method of eliminating the effects of the flume side-walls. The

Note.—This paper by H. A. Einstein, Assoc. M. Am. Soc. C. E., was published in March, 1941, *Proceedings*.

[14] Hydr. Engr., Sedimentation Div., SCS, U. S. Dept. of Agriculture, Washington, D. C.

[14a] Received by the Secretary May 28, 1941.

[15] "Studying Sediment Loads in Natural Streams," by Gilbert C. Dobson and Joe W. Johnson, *Civil Engineering*, February, 1940, pp. 93–96.

[16] "A Distinction Between Bed Load and Suspended Load in Natural Streams," by H. A. Einstein, A. G. Anderson, and J. W. Johnson, *Transactions*, Am. Geophysical Union, 1940, Pt. 2, pp. 628–633.

other derived terms—that is, friction velocity, Reynolds' number, Manning's n, and unit discharge—are calculated by using the hydraulic radius of the bed.

An off-hand assumption might be that there is available a set of data reduced to a consistent system of units and applying to channels of infinite width. With the effect of flume width eliminated it would appear possible to obtain a relation between the rate of transportation or bed roughness and certain physical and hydraulic factors. However, it is scarcely possible to obtain such a relationship because of certain important factors inherent in the apparatus used in each separate set of experiments. These factors vary from experiment to experiment, and unfortunately cannot be reduced to a definite value as in the case of side-wall effect.

In studying the summary of the various bed-load data, several objections to the experimental apparatus and procedure are apparent and appear to account for the limiting value of the data. The most important objections are: (*a*) Variations in the type and manner of sand feed; (*b*) short flumes; and (*c*) short periods of sand collection. The author has already referred briefly to the problem of sand feed and has stressed its importance. The use of short flumes has the twofold disadvantage that: (1) It is extremely difficult to obtain an accurate measurement of the slope; and (2) the distribution of velocity and flow conditions do not become stabilized in such short reaches. Of interest in respect to this latter factor are certain experiments conducted in the United States, as well as in Europe, with a bed of movable material placed in a recess only a few feet in length located near the center of a flume in which the remainder of the bed is fixed at a different roughness than the sand bed. In such cases the vertical velocity distribution measured above the movable bed is not the distribution resulting from flow over the movable bed but more closely approximates the distribution for the relatively smooth bed upstream. No relation between velocity distribution and bed movement or roughness could be expected in such cases.

The duration of run is of importance in giving information on the relative value of various experiments. In most of the past experiments where short collection periods were used, a considerable scattering of data is noticeable. The author previously[17] has stressed the importance of using relatively long collection periods in studying the rates of bed-load transportation in order to eliminate the short-period fluctuations that are characteristic of most bed-load movement. Also, with short runs the bed condition may not have time to become so adjusted that a calculated roughness factor is indicative of conditions that would prevail after a longer period of operation.

Data from laboratories in the United States as well as in Europe all appear to be limited in value by one or more of the aforementioned factors. The distinct limitations of the various data became evident to the writer when an attempt was made to use certain data to obtain information on the correct value of the grain diameter to use in the author's ψ-functions and ϕ-functions when mixtures are considered. (The author has used the grain diameter at the 40% value in discussing the movement of United States Waterways Experi-

[17] "Calibrating the Bed-Load Trap as Used in the Rhine," by H. A. Einstein, *Schweizerische Bauzeitung*, Vol. 110, No. 14; also, see *Transactions*, Am. Soc. C. E., Vol. 104 (1939), p. 1293.

ment Station mixtures, but admits that the 35% value would have tended to cause better agreement.) Mr. Gilbert's data[8] on the transportation of synthetic mixtures, for instance, should be of value in providing such information, but unfortunately he neglected to observe the depth of flow in these particular experiments. In this case, as well as in many others, extremely important observations have not been recorded because the experimenter apparently failed to realize their importance, or was confining his attention to some other phase of the problem. All pertinent data should be recorded whether or not the observer can see any immediate use of certain observations.

It is strongly urged, therefore, that, before future extensive flume experiments on bed-load transportation are undertaken (with the possible exception of experiments intended for demonstration or qualitative information), the experimenter "take stock" of the limitations of past studies and design his experiments to eliminate most of these objections. Little progress in the technique and range of conditions of laboratory bed-load investigations appears, in general, to have been made since the classic experiments of Mr. Gilbert.[8] Bed-load experiments are both expensive and time consuming and future experiments to improve on those made in the past should be directed to give more accurate data throughout a wider range of conditions, especially in the range $\phi > 10$ where transportation in large streams is perhaps most likely to occur.

[8] "The Transportation of Debris by Running Water," by Grove Karl Gilbert, *Professional Paper No. 86*, U. S. Geological Survey, Washington, D. C., 1914.

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DISCUSSIONS

DESIGN OF ACCELERATION AND DECELERATION LANES

Discussion

BY T. F. HICKERSON, M. AM. SOC. C. E.

T. F. HICKERSON,[13] M. AM. SOC. C. E. (by letter).[13a]—One of the hazardous and annoying features of highway transportation to which even the most careful drivers are subjected is that incident to traffic unexpectedly entering or leaving the highway at speeds different from that of the traffic stream. For the freeway, express highway, parkway, or super-highway of tomorrow, where abutting traffic has no right of access except at widely separated intervals, the foregoing hazards may be greatly minimized if entrance and exit lanes are built in accordance with the general considerations brought out so thoroughly by Mr. Mitchell.

The deceleration equation, Fig. 12, may be simplified perhaps if written as follows:

$$a \text{ (or } -a) = 1.35 + 0.03\, v \qquad (14)$$

Likewise, the acceleration equation, Fig. 13, might be written

$$a = 5.76 - 0.044\, v \qquad (15)$$

In changing from v (feet per second) to V (miles per hour) it is a little confusing to the reader to note the ratio 1.47 (the strictly correct value) in Eq. 9, whereas the approximate value of 1.50 is used in Eqs. 7, 8, 12, and 13. Obviously the difference is of no practical significance.

Since $v = \frac{ds}{dt}$ and $a = \frac{dv}{dt}$; by division, $\frac{v}{a} = \frac{ds}{dv}$; or, $ds = v\,\frac{dv}{a}$. Hence, Eq. 8 may be obtained directly from Eq. 6, by only one integration, instead of from Eq. 7. In a similar manner, Eq. 13 may be derived from Eq. 12 by one integration instead of two, as suggested by the author. These latter comments are trivial and are merely offered for what they may be worth as constructive criticism of a timely and valuable paper.

NOTE.—This paper by Adolphus Mitchell, Assoc. M. Am. Soc. C. E., was published in March, 1941, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: May, 1941, by Messrs. H. F. Holley, D. W. Loutzenheiser, Hawley S. Simpson, and Milton Harris.

[13] Prof. of Applied Math., Univ. of North Carolina, Chapel Hill, N. C.

[13a] Received by the Secretary April 22, 1941.

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DISCUSSIONS

ANALYSIS OF BUILDING FRAMES WITH SEMI-RIGID CONNECTIONS

Discussion

BY WAYNE W. SMITH, LEONARD P. ZICK, JR., JUNIORS, AM. SOC. C. E., AND CONRAD C. WAN, ESQ.

WAYNE W. SMITH,[5] LEONARD P. ZICK, JR.,[6] JUNIORS, AM. SOC. C. E., AND CONRAD C. WAN,[7] ESQ. (by letter).[7a]—As the authors have shown, there are reasonable possibilities for a more economical design of frame construction by considering the semi-rigidity of connections. As they have stated, their method, in practice, is too cumbersome to be used directly. Their suggested procedure using simple beam moments and reduction constants thus appears to be the most practical method of design. However, it should be emphasized that every continuous structure or frame is an individual problem and should be dealt with as such. Graphs, charts, and simple formulas may be used to simplify certain designs, but a thorough knowledge of their limitations must guide their use.

According to the authors' test results, the maximum moment developed was about 274 kip-in. (Fig. 14(*d*)). For the 10-in., 25.4-lb, I-beam used, this meant that a stress of 11 kips per sq in. was developed, a stress considerably below the elastic limit of steel. The use of the connection constant γ seems quite proper within the "design range," but as seen in Fig. 15 the constant changes rapidly beyond this range. This design range is not clearly specified in the paper. Making the logical assumption that the design range is one half the elastic limit, there is an "overload range" for which γ is no longer constant. Failure should not occur until the elastic limit is reached, which limit is the farther boundary of this overload range. Realizing that the maximum moment does not necessarily occur at the joint, the question arises as to how important is the discrepancy due to assuming γ constant when the member is stressed

NOTE.—This paper by Bruce Johnston, Assoc. M. Am. Soc. C. E., and Edward H. Mount, Esq., was published in March, 1941, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: May, 1941, by Maurice P. van Buren, Assoc. M. Am. Soc. C. E.

[5] Asst., Illinois Inst. of Technology, Chicago, Ill.

[6] Asst., Illinois Inst. of Technology, Chicago, Ill.

[7] Graduate Student, Illinois Inst. of Technology, Chicago, Ill.

[7a] Received by the Secretary May 14, 1941.

nearly to the elastic limit. To ignore this point may reduce the factor of safety for the member.

It seems that the authors' suggestion of neglecting the sidesway moments due to vertical unsymmetrical loadings is contradictory to the degree of refinement in their analysis. As shown in Table 5, the sidesway moments range from 6.74 to −12.25 kip-in., which, in the latter case, amounts to a decrease of about 29%.

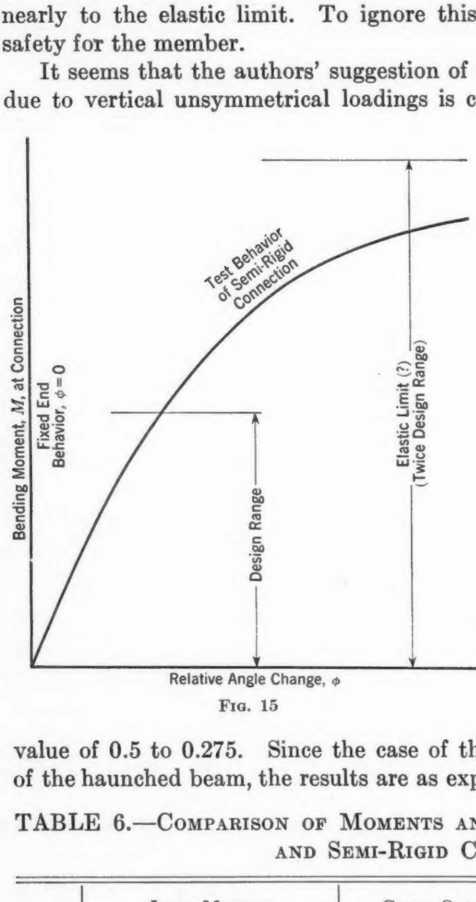

FIG. 15

In the comparison by the method of moment distribution of a rigid structure and a semi-rigid structure, it is very interesting to notice the changes in the carry-over factors and the end rotation stiffness (distribution factor).

The solutions in Table 6 are for the test frame shown in Fig. 8 assuming rigid connections and neglecting joint width. The authors' results are taken from Fig. 10. As shown, the carry-over factor for the beams was decreased from the usual value of 0.5 to 0.275. Since the case of the semi-rigid beam is the opposite of the haunched beam, the results are as expected. The changes in the carry-over factors for the columns are due to the consideration of the joint width only. The results given in Table 6 for the analysis with joints 100% rigid is merely additional information. The authors have already shown the large

TABLE 6.—COMPARISON OF MOMENTS AND STIFFNESS FACTORS FOR RIGID AND SEMI-RIGID CONDITIONS

Member	Joint Moment		Carry-Over Factor		Distribution Factor	
	Authors	100% fixed	Authors	100% fixed	Authors	100% fixed
1–2	−1.09	7	0.275	0.500	0.211	0.443
1–3	1.09	−7	0.542	0.500	0.789	0.557
2–1	34.06	93	0.275	0.500	0.182	0.362
2–4	88.43	91	0.542	0.500	0.685	0.456
2–7	−122.49	−184	.....	.....	0.133	0.182
3–1	57.51	83	0.542	0.500	0.447	0.358
3–4	−135.11	−187	0.275	0.500	0.120	0.284
3–5	77.60	104	0.449	0.500	0.433	0.358
4–2	−19.38	−40	0.542	0.500	0.411	0.313
4–3	130.10	198	0.275	0.500	0.110	0.249
4–6	−92.23	−113	0.449	0.500	0.399	0.313
4–8	−18.49	−45	.....	.....	0.080	0.125
5–3	38.72	52	0.449	0.500	.....	.....
6–4	−46.02	−56	0.449	0.500	.....	.....

disagreement considering 100% rigid connections (see heading "Comparison Between Theoretical Analysis and Test Results").

The authors are to be commended for the close agreement between the theoretical and experimental results that they obtained. Also, in their consideration of the joint width, their results showed that for large ratios of the joint width to the member length some adjustment should be made.

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DISCUSSIONS

SURFACE RUNOFF DETERMINATION FROM RAINFALL WITHOUT USING COEFFICIENTS

Discussion

By L. L. Harrold, Assoc. M. Am. Soc. C. E.

L. L. Harrold,[16] Assoc. M. Am. Soc. C. E.[16a]—To obtain data for the design of urban storm sewers, the authors have undertaken to utilize both hydrologic and physical data to develop "in place" surface runoff, and then have shown how the "in place" surface runoff can be routed through gutter systems by established hydraulic principles.

The paper infers that these hypotheses can be adapted to natural watersheds. It appears to the writer that there is no reason why they are not applicable as long as the natural watersheds produce only surface runoff. Many drainage areas are of this class, whereas the hydrology of other areas is complicated by the presence of ground-water flow and subsurface storm flow. By "subsurface storm flow" is meant that water which filters through the soil surface, forming temporary perched water, and flows laterally and appears at the ground surface lower down the slope. This subsurface storm flow, which joins the stream while surface flow is still in the drainage system, has not the characteristic of true ground-water flow because of its quick return to the surface and its short duration.

Even though the application of these hypotheses may be limited to those drainages where surface runoff constitutes the entire storm flow, there is still a wide field of possible use.

For example, if the hydraulic engineer in charge of designing the size of highway bridge openings knows the hydrologic and physical data of the natural drainage above the site (as did the authors for the urban block), the corresponding hydraulic design data can be derived. To the extent that certain watershed changes, such as the introduction of different land use practices or the construction of conservation structures, increase or decrease the surface retention and infiltration capacity of the basin, they will affect the rates and

Note.—This paper by W. W. Horner, M. Am. Soc. C. E., and S. W. Jens, Assoc. M. Am. Soc. C. E., was published in April, 1941, *Proceedings*.

[16] Hydr. Engr., SCS, U. S. Dept. of Agriculture, Washington, D. C.

[16a] Received by the Secretary June 2, 1941.

amounts of runoff from the area. The hypotheses presented furnish a vehicle for computing this effect at various points in the drainage system.

The problem can be related more closely to the example of the urban block described by the authors. If the 46.4% area of impervious surface were increased to 70%, could the sewer system handle the runoff? By applying the method to obtain the revised hydraulic design data, the engineer could tell the zoning commission that, if the district is to become "commercial," additional impervious surfaces are possible, and a certain sum of money will be necessary to enlarge the storm sewer system.

The writer has not overlooked the fact that these hypotheses must be checked in the field. It soon will be possible to perform the check over widespread areas. The Geological Survey, U. S. Department of the Interior, is collecting topographic and runoff data. The Weather Bureau, U. S. Department of Agriculture, is collecting rainfall data. The Department of Agriculture also is engaged in obtaining data on soils, land use, and infiltration. Several federal agencies are engaged in computing infiltration-capacity values for large areas by correcting the runoff record for channel storage and by relating the resulting direct runoff value to the precipitation data. At this point the writer would like to emphasize the importance of using the precipitation data separately for each gage, rather than combining the records for several gages. This is especially important when some of the rain falls at rates less than the infiltration capacity. When the rainfall rate exceeds the infiltration rate at all points, average basin rainfall rates can be used. The following example explains the reason for this caution.

Assume that in a given watershed or drainage area (see Table 6) there are four rain gages serving equal areas of the basin. Each area served by a rain gage has similar physiographic characteristics. The infiltration rates for each area in Col. 3, Table 6, give an average rate on the entire watershed of 1.02 in. per hr. The corresponding rainfall intensity for each area is shown in Col. 2, Table 6, from which the rain excess over infiltration, in a 15-min period, is found to be $(3.11 - 1.02)\,\frac{15}{60} = 0.52$ in., the average for the watershed. On the other hand, the 15-min runoff values in Col. 5, Table 6, indicate a total excess of 0.64 in.

TABLE 6.—RAINFALL INFILTRATION AND RUNOFF—A HYPOTHETICAL EXAMPLE

Area[a]	Inches per Hour			15-min runoff (inches)
	Rainfall intensity	Infiltration	Excess rain	
(1)	(2)	(3)	(4)	(5)
A	8.0	2.00	6.00	1.50
B	4.0	1.00	3.00	0.75
C	0.40	0.80	0.00	0.00
D	0.04	0.30	0.00	0.00
Aver.[b]	3.11	1.02	2.25	....
15 min[c]	0.78	0.26	0.56	0.56

Average rain intensity on watershed, in in. per hr	3.11
Average infiltration on watershed, in in. per hr	1.02
Average rain excess on watershed, in in. per hr	2.09
Total runoff for 15 min, in in.	0.52

[a] Equal size. [b] Weighted average. [c] Amounts for 15 min.

Assuming that the true runoff was 0.52 in., the computed infiltration is

(0.78 − 0.56) = 0.22 in., or 0.88 in. per hr instead of the weighted average of 1.02 in. per hr. For storms having rainfall rates over parts of the area less than the corresponding infiltration capacity, the computed value of infiltration capacity for the drainage basin will always be less than the actual.

From plats and small watersheds the infiltration capacities for various surface conditions are obtained. If engineers are to use the methods set forth in the paper for applying these infiltration-capacity values to larger drainage systems, it seems imperative that the rainfall records for each gage be applied, through the use of isohyetals for short-time intervals, to the separate drainage units having uniform infiltration capacities.

Corrections for *Transactions:* April, 1941, *Proceedings*, in Fig. 4, change the caption to read "Storms of March 30 and 31, 1938, at Edwardsville, Ill.; and change the denominator of Eqs. 3 to read "$60\, l^{0.5}$."

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DISCUSSIONS

EXTENSOMETER STRESS MEASUREMENTS, NORTH AVENUE BRIDGE, CHICAGO, ILL.

Discussion

By Messrs. Arthur R. Lord, and J. Charles Rathbun

Arthur R. Lord,[4] M. Am. Soc. C. E.[4a]—There is a strange lack of information, in this paper, as to the early state of the art of stress measurement. The statement is made under the heading "Field Measurement of Stress" that:

> "In all of these investigations, which constitute probably the entire scope of the work done in this field until about 1917, no stress measurements were made to compare computed and measured stress, or to determine dead-load stress; and no measurements were made on statically indeterminate structures. * * * The use of the extensometer in the systematic investigation of true field stress and the verification of structural design theory applied to an existing structure were of no consequence until 1917 when D. B. Steinman, M. Am. Soc. C. E., conducted his extensive systematic stress measurements on the Hell Gate Bridge."

The Bibliography given in the Appendix shows a serious lack of important references. To correct this inaccurate record it should be stated that:

(1) The first test for actual stresses in a structure was made on the Deere-Webber Building in Minneapolis, Minn., in the fall of 1910 and described by the writer in a paper published that year.[5] This paper involved extensometer measurements in a statically indeterminate structure and comparison of actual with computed stresses, and it laid the foundation for the design of the reinforced concrete flat-slab construction in the United States and other countries.

(2) Many other tests, each involving thousands of extensometer readings under heavy loads in actual service structures, were made in the years between 1910 and 1917, by the writer[6] and by many others, and were published and

Note.—This paper by Lawrence T. Smith, M. Am. Soc. C. E., and Paul Lillard, Esq., was published in May, 1941, *Proceedings*.

[4] Mgr., Reports Section, Progress Division, Bureau of Yards and Docks, U. S. Navy, Washington, D. C.

[4a] Received by the Secretary May 20, 1941.

[5] "Measurement of Stresses in Flat Slab Building Under Actual Load," by Arthur R. Lord, *Proceedings*, National Assn. of Cement Users (later Am. Concrete Inst.), 1911.

[6] "Measurement of Actual Stresses in a Cantilever Flat Slab Reinforced Concrete Floor Having Rectangular Panels," by Arthur R. Lord, *Proceedings*, National Assn. of Cement Users, Vol. IX, 1913, p. 127.

discussed throughout the United States. The bulletins of the Engineering Experiment Station at the University of Illinois, Urbana, Ill., give the full record of some of these tests. The *Proceedings* of the American Concrete Institute give others. The *Engineering News-Record* contained abstracts of several. In other words, there is a mass of scientific testing, of precisely the type reported by the authors, that preceded the test in 1917. Dead-load stresses were measured, as well as live-load stresses. Stresses (deformations) developing over periods as long as a year, in structures subject to load and temperature effects, were determined. Effects of "plastic flow" of concrete were noted and evaluated. In fact, the entire scientific design base for hundreds of millions of dollars worth of concrete buildings was developed from such tests. The authors may be justly criticized for ignoring this prior record when essaying to write an historical summary such as is presented in the opening paragraphs and in the bibliography of this paper.

It is sincerely hoped that the authors will correct these shortcomings in their closing discussion. For example, the works[7] of Arthur N. Talbot, Past-President and Hon. M. Am. Soc. C. E., and the late Willis A. Slater, M. Am. Soc. C. E., as well as that of the writer and many others, antedate any references that appear in the paper.

J. CHARLES RATHBUN,[8] M. AM. SOC. C. E.[8a]—The title of this paper does not indicate that it is an attempt at verification of one method of analyzing the stresses in a skew arch; nor does the "Synopsis" so indicate. It is only at the bottom of the third page that the most important phase of the paper appears (see heading "Description of Bridge: The North Avenue Rigid-Frame Bridge").

The "Synopsis" of this paper indicates that field tests were made in order to verify certain very critical structural assumptions "which were believed to be quite valid but which lacked definite quantitative substantiating evidence." The writer takes issue with this statement as well as with the second paragraph of the "Synopsis."

The type of design and method used were fully covered from a theoretical standpoint in a paper[9] by the writer published in 1924; and the results of the authors' test could all have been anticipated in terms of that theory. The general case of the skew arch ring was analyzed, as well as the special case considered by the authors. All test loads used by Messrs. Smith and Lillard were included in this special case. Therefore, it seems fair to state that the action of the North Avenue Bridge under a more general loading condition has not been shown experimentally.

Although the authors may have been justified in using it for other reasons, the slab-and-girder type of construction has two objectionable features in the case of this bridge. The deep girders cut down the headroom materially

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[7] "Tests of Reinforced Concrete Buildings Under Load," by Arthur N. Talbot and Willis A. Slater, *Bulletin No. 64*, Eng. Experiment Station, Univ. of Illinois, Urbana, Ill., January, 1913.

[8] Associate Prof., Civ. Eng., College of the City of New York, New York, N. Y.

[8a] Received by the Secretary May 27, 1941.

[9] "Analysis of the Stresses in the Ring of a Concrete Skew Arch," by J. Charles Rathbun, *Transactions*, Am. Soc. C. E., Vol. LXXXVII (1924), p. 611.

compared to that of a simple barrel arch; and this type of construction increases the difficulty of analysis and the uncertainty of stress conditions materially. The writer knows of no formula for torsional rigidity or stress distribution for a shaft of the cross section given in Fig. 9(*d*). There are several for a simple rectangle of all degrees of side ratio, including the extreme case used in the skew barrel arch and the case of the skewed rib used by the authors.

At the time that the skew arch theory[9] was first published (1924), there was some criticism of it because of the length of the equations involved and because the results were not in accord with the then popular idea of the action of a skew arch. Since the publication of this paper, a number of papers have been written on the subject and many models have been tested until now (1941) it has been definitely substantiated that the theory is sound. The writer conducted a series of tests and published the results in 1930.[10] Later he tested[11] a multiple skew arch designed by an extension of this same theory by the method developed by the late George E. Beggs, M. Am. Soc. C. E. These tests proved the validity of the theory and also the extension of the theory to the case in which there is more than one arch ring. In 1926 the Society's Special Committee on Concrete and Reinforced Concrete Arches took issue[12] with the findings of the first paper.[9] Professor Beggs was instructed to conduct a series of experiments subjecting four skew arches of varying widths to all possible conditions of vertical load. After more than a hundred loadings were investigated, the Committee came to the conclusion that the theory as published is correct and leads to correct results.[13]

At present the theory is recognized as correct and is being used in design throughout the United States. The writer has not heard of any engineer, who has designed by it, indicating that it is incorrect or leads to incorrect results; or who feels that, when used understandingly, it is too long and involved. The writer, therefore, takes issue with the second sentence and the second paragraph of the "Synopsis" on the ground that quantitative, computable evidence was available, and that no test loads were applied that fell outside the special case in which the authors' assumptions were valid.

It will probably be of interest to compare the elastic theory of the skew arch with the authors' series of tests and their method of design to encourage a better understanding of the skew arch and its analysis. The problem is essentially one of space analysis. If the reaction at a section across the crown or at an abutment is considered, six components of this reaction must be evaluated—forces in the direction of each of the assumed axes and moments about the line of action of these forces. This makes the arch ring statically indeterminate with six redundants.

Six equations of elastic deformation are required to evaluate these unknown moments and forces. If the deformations due to ring shortening and shear, both horizontal and radial, are considered, these equations become quite

[10] "Crown Stresses in a Skew Arch," by J. Charles Rathbun, *Transactions*, Am. Soc. C. E., Vol. 94 (1930), p. 135.

[11] "An Analysis of Multiple-Skew Arches on Elastic Piers," by J. Charles Rathbun, *loc. cit.*, Vol. 98 (1933), pp. 26–31.

[12] *Proceedings*, Am. Soc. C. E. (Society Affairs), March, 1926, p. 142.

[13] *Transactions*, Am. Soc. C. E., Vol. 100 (1935), p. 1573.

lengthy. Therefore, the work required to make a complete analysis is prohibitive, and the practicing engineer leaves out the terms that have little effect on the final result. In the case of the symmetrical arch, even when unsymmetrically loaded, these six equations split into two groups—one of four, and the other of two, equations. In the case of certain types of symmetrical loading (those used by the authors), the one group of two equations gives zero results—that is, the vertical shear and the moment about the vertical axis at the crown are zero.

Although it has little bearing on the present problem, consider the case in which the skew is zero. These six equations split still further into two equations of one unknown each and two sets of equations of two unknowns each. If the loading is symmetrical about the center line of the upper roadway (the common case in arch analysis), three equations have zero results and are not considered by designers. The other three consist of two of two unknowns and one of one unknown. This is the case considered in textbooks on arches. If, still further, the load is symmetrical about the crown, one equation (for vertical shear) disappears. The two remaining equations can be reduced to one by placing hinges at the abutments. This is the case used by the authors in testing the central rib. The very narrow skewed arch acts the same as a right arch with a span of the skewed length.

Returning to the six equations of the skewed arch, assume a system of coordinates with the origin at the crown. The crown section is taken parallel to the abutment and radial (vertical for symmetrical arches). The x-axis is taken perpendicular to this section, the y-axis is vertical (radial), and the z-axis is perpendicular to these two, and, therefore, is horizontal and in the crown section. The general solution requires the finding of the forces in the direction of these coordinates and the moments about them and requires six equations.

If the arch is symmetrical about the two vertical planes through the y-axis, the one through the z-axis and the other through the center line of the upper roadway (the usual case), and, in addition, if the loads are also symmetrical about these planes, the crown force in the direction of the vertical axis and the moment about this axis both become zero. At the same time two equations disappear, and four equations and four unknowns remain. Next make the rather questionable assumption that the thrust at the crown and the horizontal shear at the crown are in the same ratio to each other as the tangent of the angle of skew—in this case, 35° (the complement of the 55° given by the authors). Also assume that the moments about these axes, the z-axis and the x-axis, are in this same ratio. The problem is now reduced to two unknowns and two equations, and the force line follows the central arch rib. If the abutments are designed as hinges, this reduces the problem to one equation in one unknown (which is the case assumed by the authors), and the formula is as given by Eq. 1.

The two assumptions involving the ratios of the crown forces and moments are fulfilled only under certain conditions, the most important being that the arch and loads must be placed symmetrically about the two vertical planes through the crown indicated previously. The test loads all fulfilled these

conditions. In addition, the shortening of the arch rib, and its deformation due to shear, must be neglected. Another necessary condition is that the abutments do not rotate. Unfortunately, the rotation of these abutments was not measured, but from the results obtained the rotation must have been small and in a direction that tended to correct the other effects. Another assumption involved in the loading is that the earth-fill pressure is parallel to the direction of the upper roadway. This pressure is questionable in its nature. The usual assumption is that the pressure is perpendicular to the abutment face and therefore contributes to skew action.

It is to be noted that the readings given are all on the central rib where any skew effect would be a minimum. The readings on the outside girder are not given in the paper, but the authors indicate that some skew effect exists. The slab joining the several ribs evidently has sufficient rigidity to produce this effect. The torsional stresses in the slabs due to the complicated crown section are difficult to evaluate or observe; those due to horizontal shear are not so difficult. Observations on the slabs might show interesting results. Of course, these stresses are not dangerous because, if they are beyond the allowable limit, adjustment will take place since the slab is not a vital part of the arch system. If they had been made with shear joints or expansion joints (as was done on a skew arch bridge in Tacoma, Wash., with disastrous results),[14] the structure would still have acted as anticipated, but probably with no skew effect.

The writer feels that the objects of making the tests (see "Extensometer Tests: Object") have not all been obtained. For example (Object (1)), the true stress conditions at the questionable places have not been measured. The central arch rib will follow the approximate theory; but will the slab or the outer ribs follow this theory under skew loading? Failure to measure the abutment rotation prevents a check by the theory of the skew arch ring. Unless the authors refer to their "elastic theory," a comparison of true and theoretical stresses has not been made except to a very limited degree. It is to be noted that no effects of skew have been given (Object (2)). This effect is the important and dangerous condition in a skew arch. The elastic action of the system has been studied to a very limited degree only (Object (3)). The data secured (Object (4)) is valuable and adds one more test to the verification of the theory of the skew arch. That no observations were made on the rotation of the hinges detracts very much from its value, however.

The idea of cutting a bar and noting its change of length in the unstrained condition requires considerable caution in its use. It assumes that the bar has been placed in the structure in an unstrained condition, which assumption is far from the truth. For example, assume a 1-in. bar which, due to normal handling, is slightly bent with a radius of curvature of 1,200 ft. When laid in the forms, it is forced into line and wired, but it has a bending stress of 1,000 lb per sq in. If measurements are taken on one side of this bar only, after which the bar is released, there may be an error of 1,000 lb per sq in. in the steel measurement and a corresponding error in the concrete. This error cannot be detected by observing the bar and noting if it straightens up when

[14] *Engineering News-Record*, February 22, 1923.

released because, assuming the foregoing values in a 10-in. gage length, the deflection from a straight line is only $\frac{1}{600}$ in., which is too small for field measurements. If the radius were taken as 120 ft and the deflection as $\frac{1}{60}$ in., the condition would not be unusual and the error could be 10,000 lb per sq in. for the steel. Therefore, it becomes necessary to take measurements on opposite sides of the bar. This involves a great deal of concrete chipping, unless the gage is constructed for this type of observation. The authors should clarify this point. The cutting and resplicing of the stressed bars cause a redistribution of stress among the several bars of the bridge that might be objectionable to the owners but do not affect the validity of the observation. This "cut-bar" method has been suggested several times in the writer's experience but was rejected for the aforementioned reasons.

An alternate method (which has not been used to the writer's knowledge) is to take a small bar (the diameter being governed by its slenderness ratio when acting as a column), suspend it on two supports so adjusted that the effect of bending moment is zero over the gage lengths, and embed this bar in the concrete at the point to be measured. The gage points are placed on the bar before embedment and are well greased. The distance between them is measured before and after embedment. This initially unstressed bar then can be measured for strain at any time, and finally it can be taken out and the gage length checked. The bar must be long enough to insure sufficient embedment on either side of the gage points. Even in this case gage points on opposite sides of the bar are desirable.

The authors are incorrect in their remark that few measurements have been taken on structures in service where the extensometer has been used. The writer can recall several without taking the trouble to make a bibliography. Although the point is of little value, since the question has been raised it might be of interest to see such a bibliography on the subject. It would include the field observations and extensometer measurements by Clyde T. Morris and also by S. C. Hollister, Members, Am. Soc. C. E. The Progress Report of the Committee on Concrete and Reinforced Concrete Arches, dated January 20, 1926, contains more than ten references to field measurement projects.

The bibliography of this paper appears to be one on the use of the strain gage, supplemented by a few standard works on structural analysis and a few references on impact. The important phase of the paper, the stresses in a skew arch, is not touched in this bibliography, although several papers have been written on this subject between the dates given by the author (1917 to 1941), and these papers include strain measurements on existing structures. The title of the list should be less inclusive or a number of papers on the skew arch should be included, preferably the latter because literature on this subject apparently needs to be brought more to the attention of the profession.

The writer wishes to comment on the authors' conclusions as follows:

(4) The low magnitude of temperature stresses in the structure is probably due to the fact that the abutments moved horizontally with the expansion

and contraction of the arch due to temperature changes, thus tending to annul the stresses.

(6) That the arch ring follows the approximate method of skew arch analysis has been proved for the central girder only.

(7) The reasons why the basic design assumptions have not been verified have been explained at length.

(9) That the arch follows the authors' "elastic theory" has not been shown except for the central girder. They neglect the stresses due to skew which have not been touched upon in the paper. This is a very dangerous and incorrect conclusion.

(10) and (11) The conclusion that there is lateral distribution of load is correct. It is this lateral distribution through the slab that makes the skew arch analysis a problem. The complex torsional condition resulting from the skew of the structure produces the dangerous stresses, if any. The remark that "The objectives of this investigation did not embrace a study of this phase in the facia and outer frames" should be emphasized to the extent that, with one reading of the paper in order to secure data on the skew arch analysis, one will then realize that this paper is primarily one of testing and that the authors are not putting forth the idea that the approximate method of skew arch design applied to the two-hinged skew arch has proved satisfactory. It is a dangerous method and could have been unsatisfactory had the hinges acted as designed.

(15) These tests definitely do not warrant the conclusion that the approximate theory (the authors' elastic theory) is correct for hinged skew arches; nor do the tests prove that the theory is incorrect.

(17) The conclusion that the central girder functions as a simple arch is approximately correct, theoretically, if the abutments do not rotate. The other girders will also function in this manner under certain definite conditions of loading.

(18) The data from which this conclusion was drawn are of value, and it is hoped that they will be published in the closure.

(20) In the "cut-bar" technique the authors should show how they avoided stresses due to bending the bar. If they measured both sides of the bar to secure the average strain in the center, this might well be mentioned as a warning to those who use the method in the future.

The writer is very much impressed with this paper from several angles. The method in which the obtuse abutments carried most of the load (which antedates the modern theory) has not been used, and seems at last to have been abandoned in skew arch analysis. The more simple method of using the skew arch and treating it as a right arch is nearer the truth if the hinges do not exist or act.

Although this paper by title and synopsis is one on stress measurement, it actually is one on checking the method of analysis of an arch with a 35° skew. The writer feels strongly that the work performed by the Society's Committee on Concrete and Reinforced Concrete Arches, and the several papers published by the Society, should have been considered more carefully before the

measurements were taken, and that the literature on skew arches should have been given a place in the bibliography. If this had been done, the stresses in the structure as a skew arch might have been given more weight in the paper.

In closing, the writer wishes to urge that designers of the skew concrete arches investigate carefully and read the literature on the stresses due to skew. A statically indeterminate structure with six redundants is comparatively safe; but should not the stresses induced by loads be analyzed carefully so that a lack of understanding of the problem may not outweigh the factor of safety? It is hoped that this paper will revive an interest in the skew arch problem and that this structure will obtain the popularity it deserves.

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DISCUSSIONS

CONSUMPTIVE USE OF WATER FOR AGRICULTURE

Discussion

BY E. B. DEBLER, M. AM. SOC. C. E.

E. B. DEBLER,[43] M. AM. SOC. C. E.[43a]—In regard to the orchard areas discussed by the authors, the writer made some study of the Kings River area in California, which is probably the most typical upon which information is available in that region. There seems to be no material departure from the curve of Fig. 2. In other words, crops are a corollary to the number of thousands of heat units. They go with that factor, and, although the authors state that the consumptive use in orchards is low, the writer believes that, in areas where cover crops are maintained to produce fertility, consumptive use will lie on this general graph (Fig. 2). The areas that are getting along with a lower consumptive use are, as a rule, the so-called clean cultivated orchards, and agriculturists now agree that such a condition can be only more or less temporary, and that with the maintenance of fertility those areas, in time, must go to cover crops.

It may appear rather extraordinary that so few areas were selected in what might be called the more highly developed, long-irrigated regions, such as the South Platte and Arkansas River areas in Colorado, and others. The situation is that, almost without exception, the older irrigated sections are subject to chronic shortages of water. The writer doubts very much that the irrigated area of the South Platte River Basin enjoys a water supply that (on the average) is more than two thirds of a full supply. Two difficulties were encountered in that area:

(1) The situation presents an almost insurmountable condition for the correction of data to represent consumptive use with a full supply, as records on water deliveries are lacking; and

(2) Reliable records on irrigated areas are not available, a situation that could not be overcome without considerable expense.

NOTE.—This paper by Robert L. Lowry, Jr., M. Am. Soc. C. E., and Arthur F. Johnson, Assoc. M. Am. Soc. C. E., was published in April, 1941, *Proceedings*.

[43] Hydr. Engr., U. S. Bureau of Reclamation, Denver, Colo.

[43a] Received by the Secretary May 27, 1941.

The available information was limited to reports of the water commissioners, and there are too many of those reports made with an eye to future water-rights adjudication suits! For these reasons, in areas where water is most valuable and where there is a fight for water from year to year, engineers are handicapped in arriving at consumptive use.

As to the possibility of using annual temperatures: The classic example encountered by the writer is a comparison between the Klamath Project in Oregon and a project in eastern Montana. They have about the same annual temperature. The Klamath Project has an average temperature even higher than the one in eastern Montana; yet it is subject to frost in every month of the year. Naturally, its consumptive use is very much less than that of the eastern Montana Project, where the temperatures are extremely low in the winter, but the summer temperatures are as high as they are in Texas.

In other words, what the engineers were attempting to arrive at was a correlation applicable in every location rather than an easier correlation that might not have general correlation for the United States as a whole. The Klamath Project will show a frost-free period on an average of less than 100 days. The project in Montana will show a frost-free period of 150 days. The results just are not comparable. Notwithstanding this difference in the records, the Klamath area grows plenty of potatoes, which cannot be done in some areas showing a much higher average temperature. The writer is only hopeful that more information can be developed. He found an extreme dearth of records, faulty in many particulars—most commonly with reference to the irrigated areas. In some areas the consumptive use is so small in proportion to the inflowing and outflowing waters that no one can be sure of results, as a very minor error in stream flow may equal the entire consumptive use!

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DISCUSSIONS

MOMENTS IN CONTINUOUS RECTANGULAR SLABS ON RIGID SUPPORTS

Discussion

By Maurice P. van Buren, Assoc. M. Am. Soc. C. E.

Maurice P. van Buren,[9] Assoc. M. Am. Soc. C. E.[9a]—A simplified method of analysis for slabs supported on four sides is contained in this interesting paper. By the assumption that edge slopes and support moments are distributed according to a sine curve, formulas for moments at any point are obtained. Unfortunately, the derivations of Eqs. 2 and 5 are not given, and it is not clear what fundamental relationships connecting slope, load, and moment were used by the authors in their analysis, or whether the effect of torsional rotation has been included. It is to be hoped that this omission will be remedied in the closing discussion.

Values given in the text for support moments in slabs fixed at one edge, and the curves in Fig. 8(*a*) for positive moments in simply supported slabs, agree closely with the theoretical values obtained from elastic analysis using Fourier series by H. M. Westergaard,[10] M. Am. Soc. C. E., in 1921; but it should be pointed out that, in the long direction of oblong panels, the moment M_y (Fig. 8(*a*)) at the center of span may be less than the theoretical maximum. For example, when the ratio of spans is 2, maximum moments occur at about the 0.3 point of the span from each end with a depression of the moment curve at midspan. However, in 1926, Dean Westergaard further modified his formulas[11] to agree with tests by taking into account redistribution of stress. As the load on a panel is gradually increased from zero, bending moments will increase more rapidly in the stiffer parts of the slab than in the more flexible parts. This involves a transfer of bending moment from the center strips to the side strips with a consequent flattening out of the curve of moment distribution across the panel width. During this action there also may be a change in the proportion of load carried by each direction of the slab, and also to some extent a transfer

Note.—This paper by L. C. Maugh, Assoc. M. Am. Soc. C. E., and C. W. Pan, Jun. Am. Soc. C. E. was published in May, 1941, *Proceedings*.

[9] Cons. Engr. (J. Di Stasio & Co.), New York, N. Y.

[9a] Received by the Secretary June 2, 1941.

[10] "Moments and Stresses in Slabs," by H. M. Westergaard and W. A. Slater, *Proceedings*, A. C. I., 1921, p. 415.

[11] "Formulas for the Design of Rectangular Floor Slabs and the Supporting Girders," by H. M. Westergaard, *loc. cit.*, 1926, p. 26.

of moment from the support to the center of span. Dean Westergaard's conclusions indicated a reduction in maximum bending moments due to redistribution of 28%. From these considerations it follows that, although a sine-curve distribution may portray conditions satisfactorily at the beginning of the loading, it does not represent the true conditions in the later stages and yields moments considerably too high for purposes of design.

As one of the authors of the A. C. I. regulations for slabs supported on four sides,[12] the writer wishes to correct the impression that these provisions are merely a beam strip method in which "the only correction for the slab action is made by an empirical distribution of the load between the two hypothetical beams" (see "Introduction"). A brief description of the basis of the A. C. I. formulas will be given. This method of analysis is devised for practical use, and simplifying approximations and empirical constants are introduced within the limits of required accuracy without sacrificing the rational form of the equations. Under the A. C. I. regulations, consideration is given to the action of the entire panel as a unit, and the proportion of the total panel load to be carried in each direction is determined by the relative stiffness of the two spans. The distances between lines of inflection when uniform load is applied separately to each span form convenient measures of the relative stiffness of the two directions. For a panel $A \times B$, this results in a load distribution formula:

$$r_A = \frac{1}{1 + \left(\frac{F_A\,A}{F_B\,B}\right)^3} = 1 - r_B \qquad (15)$$

in which r_A and r_B are the proportions of total panel load carried in each direction, and $F_A\,A$ and $F_B\,B$ are distances between lines of inflection as defined above. The factors $F_A\,A$ and $F_B\,B$ have small variation for usual conditions, and the effect of loads on adjacent panels should be neglected in their determination. It is to be noted that Eq. 15 is not the fourth power distribution, based on the deflections of two intersecting center strips commonly used in the "beam strip" methods, but is a third power distribution that implies the equality of deflections in each span at all corresponding points in the panel.

In addition to distributing the total load between the two directions of the panel, the variation in intensity of loading along each span is also provided for. For instance, at the center point of a square symmetrical panel, one half the unit load is carried in each direction, but, at a point near a support, practically all of the unit load goes directly to the support. Load distribution coefficients are therefore modified by equivalent uniform load factors:

$$e_A = \frac{2}{4 - \frac{F_B\,B}{F_A\,A}} \qquad (16a)$$

and

$$e_B = \frac{2}{4 - \frac{F_A\,A}{F_B\,B}} \qquad (16b)$$

[12] "Slabs Supported on Four Sides," by J. Di Stasio and M. P. van Buren, *Proceedings*, A. C. I., 1936, p. 350.

These factors vary from $\frac{2}{3}$ for a symmetrical square panel to 1 for a panel in which the rectangularity is 2, indicating a uniformly distributed load that approaches the condition of one-way construction.

All required values for F_A, r_A, e_A, etc, are tabulated for ready reference in the A. C. I. Code, from which equivalent uniform loads $e_A\, r_A\, w$ and $e_B\, r_B\, w$, to be carried in each direction, can be obtained directly. Bending moments are then computed exactly as for one-way construction by loading successive spans in each direction with their equivalent uniform loads. This procedure is theoretically feasible because of the phenomenon of redistribution of stress that results in the curve of bending moments across the width of the panel being so flattened out that designs may be based on the average moment that corresponds to about 72% of the maximum theoretical moment without this provision.

The A. C. I. Code also covers the design of the supporting beams on the statical principle that beams and parallel slab together carry the total load in each direction. Thus, corresponding equivalent uniform unit loads for beam design are $(1 - e_A\, r_A)w$ and $(1 - e_B\, r_B)w$.

An interesting comparison results from an extension of Table 2 to include the authors' values reduced for redistribution of stress (see Table 4). It is to be noted that the averag e ordinate of a sine curve is about 64% of the maximum. Therefore, a 36% reduction of moments derived by the sine-curve method may be more nearly correct. Both values are shown. The reasonably close agreement between either of these values and the A. C. I. Code is apparent, and affords definite evidence of its accuracy and reliability in the practical design of unequal panels.

TABLE 4.—EDGE MOMENTS REDISTRIBUTED FOR STRESS

Description	M_{12}	M_{23}	M_{45}	M_{56}	M_{14}	M_{25}	M_{36}
Authors' "exact" formulas	20.56	12.14	3.95	5.40	12.17	10.05	4.26
Authors' "exact" formulas; 28% reduction	14.80	8.74	2.84	3.89	8.76	7.23	3.06
Authors' "exact" formulas; 36% reduction	13.16	7.77	2.53	3.46	7.79	6.43	2.73
A. C. I.	13.18	9.20	3.54	2.94	8.18	5.84	2.80

Sine-curve distribution as developed by the authors is a noteworthy contribution to the analysis of continuous slabs supported on four sides. It provides a method by which the moments in a series of unequal panels of varying degrees of rectangularity may be solved with relative simplicity. The discussion of the effect of Poisson's ratio on positive moments is of technical interest, but has limited scope in reinforced-concrete design where the slabs are subject to tension cracking under working loads. If the formulas are modified to include the effect of redistribution of stress, a useful tool will result for checking practical design constants and methods.